11th International Conference on Vibration Problems
Z. Dimitrovová *et.al.*(eds.)
Lisbon, Portugal, 9-12 September 2013

ATTITUDE DETERMINATION AND GYRO CALIBRATION FOR CBERS-2 SATELLITE

William R. Silva*[1], Hélio K. Kuga[1], Roberta V. Garcia[1], Maria C. Zanardi[2]

[1]Instituto Nacional de Pesquisa Espaciais (INPE), Av. dos Astronautas, 1758, São José dos Campos, 12227-010, SP, Brasil.
reis.william@gmail.com
hkk@dem.inpe.br
rvelosogarcia@yahoo.com.br

[2]Universidade Estadual Paulista (UNESP), Av. Ariberto Pereira da Cunha, 333, Guaratinguetá, 12516-410, SP, Brasil.
cecilia@feg.unesp.br

Keywords: Attitude determination, Gyro calibration, Nonlinear least-squares.

Abstract. *This work is applied to the dynamics of rotational motion of artificial satellites, that is, its orientation (attitude) with respect to an inertial reference system. The attitude determination involves approaches of nonlinear estimation techniques, which knowledge is essential to the safety and control of the satellite and payload. Here one focuses on determining the attitude of a real satellite: CBERS-2 (China Brazil Earth Resources Satellite). This satellite was launched in 2003 and were controlled and operated in turns by China (Xi'an Control Center) and Brazil (Satellite Control Center). Its orbit is near polar sun-synchronous with an altitude of 778km, crossing Equator at 10:30am in descending direction, frozen perigee at 90 degrees, and providing global coverage of the world every 26 days. The attitude dynamical model is described by nonlinear equations involving the Euler angles. The attitude sensors available are two DSS (Digital Sun Sensor), two IRES (Infra-Red Earth Sensor), and one triad of mechanical gyros. The two IRES give direct measurements of roll and pitch angles with a certain level of error. The two DSS are nonlinear functions of roll, pitch, and yaw attitude angles. The gyros furnish the angular measurements in the body frame reference system. Gyros are very important sensors, as they provide direct incremental angles or angular velocities. They can sense instantaneous variations of nominal velocities. An important feature is that it allows the replacement of complex models (different torques acting on the space environment) by using their measurements to turn the dynamical equations into simple kinematic equations. However gyros present several sources of error of which the drift is the most troublesome. Such drifts yield along time an accumulation of errors which must be accounted for in the attitude determination process. Herein one proposes to estimate the attitude and the drift of the gyros using the Least Squares Method. Results show that one can reach accuracies in attitude determination within the prescribed requirements, besides providing estimates of the gyro drifts which can be further used to enhance the gyro error model.*

1 INTRODUCTION

Attitude estimation is a process of determining the orientation of a satellite with respect to an inertial reference system from attitude sensor data. After given a reference vector, the attitude sensor measures the orientation of this vector with respect to a fixed reference to the satellite system. It is possible to estimate the orientation of the satellite processing computationally these vectors using attitude estimation methods.

There are a large number of estimation methods, each method being suitable for a particular type of application. Thus it is necessary to evaluate the processing time and precision that you want to achieve. In addition, all estimation methods need observations that are obtained from sensors installed on the satellites.

In this work the attitude is represented by Euler angles, due to its easy geometrical interpretation, with state estimation performed by the least squares method. This method is capable of estimating nonlinear systems states from data obtained from different sensors of attitude. It was considered real data supplied by gyroscopes, infrared Earth sensors and digital sun sensors. These sensors are on board the CBERS-2 satellite (China-Brazil Earth Resources Satellite), and the measurements were recorded by the Satellite Control Center of INPE (Brazilian Institute for Space Research).

2 ATTITUDE REPRESENTATION BY EULER ANGLES

The attitude of a satellite is defined by a set of parameters that allow, uniquely, correlating, in an instant of time, a fixed coordinate system of the satellite (which accompanies his movement of rotation and translation) to another inertial system, which is usually related to the Earth [1]. In general it is considered inertial or near-inertial, which means that its movement in relation to the system truly inertial is despicable, when compared with the movement of the body itself. A way to represent the attitude is by Euler angles, which will express the relationship between two coordinate systems, one of them fixed on satellite and other associated to an inertial system.

In order to preserve the mission success, the satellite is stabilized in a nominal attitude. In the case of CBERS-2, attitude stabilization is done in three axes namely geo-targeted, and can be described in relation to the orbital system. In this frame, the movement around the direction of the orbital speed is named *roll* (φ), the movement around the direction normal to the orbit is called *pitch* (θ), and finally the movement around the Zenith/Nadir direction is called *yaw* (ψ).

The transformation matrix R, which lists the fixed coordinate system in the satellite's body (x, y, z) coordinate system orbital (x_o, y_o, z_o), has its elements described in terms of Euler angles (φ, θ, ψ). The rotation sequence adopted in this work was the 3-2-1, according to [2], and the following sequences of rotations are used:

- 1st rotation of an angle ψ (*yaw* angle) around the axis z_o;
- 2nd rotation of an angle θ (*pitch* angle) around an intermediate axis y';
- 3rd rotation of an angle φ (*roll* angle) around the x axis.

Thus, we have that:

$$R = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi & \sin\phi\sin\theta\sin\psi - \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi - \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} \quad (1)$$

In turn, the kinematic equations of Euler angles are given by [3,2]:

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\cos\theta\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} \tag{2}$$

where, ω_x, ω_y e ω_z are the components of the angular velocity of the satellite in *roll*, *pitch* and *yaw*.

By representing the attitude of a satellite with Euler angles, the set of kinematic equations are given by [2, 4]:

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\cos\theta\end{bmatrix}\left\{\begin{bmatrix}\widehat{\omega}_x\\ \widehat{\omega}_y\\ \widehat{\omega}_z\end{bmatrix} - R\begin{bmatrix}0\\ \omega_o\\ 0\end{bmatrix}\right\} \tag{3}$$

where ω_o is the orbital angular velocity; $\widehat{\omega}_x$, $\widehat{\omega}_y$ and $\widehat{\omega}_z$ are the components of the angular velocity of the satellite on the satellite system.

Defining the state vector composed by: Euler angles (φ, θ, ψ) and the components of the gyros bias $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$; and assuming that ϕ and θ are small angles, soon, the differential equations of State for attitude and bias of the gyros are modeled as follows [4]:

$$\begin{aligned}
\dot{\phi} &= -\omega_o \sin\hat{\psi} + \widehat{\omega}_x + \hat{\theta}\widehat{\omega}_z\\
\dot{\theta} &= \omega_o \cos\hat{\psi} + \widehat{\omega}_y - \hat{\phi}\widehat{\omega}_z\\
\dot{\psi} &= \omega_o\big(\hat{\phi}\cos\hat{\psi} - \hat{\theta}\sin\hat{\psi}\big) + \widehat{\omega}_z + \hat{\phi}\widehat{\omega}_y\\
\dot{\varepsilon_x} &= 0\\
\dot{\varepsilon_y} &= 0\\
\dot{\varepsilon_z} &= 0
\end{aligned} \tag{4}$$

where $\hat{\phi}$, $\hat{\theta}$, $\hat{\psi}$ are the attitude angles obtained by some estimation process.

3 MATHEMATICAL MODELS OF ATTITUDE SENSORS

One way and as certain the attitude of an artificial satellite is necessary to use some attitude sensors. In this section is described the mathematical model of the attitude sensors used in this research for the determination of attitude: gyros, digital sun sensor and infrared Earth sensor.

3.1 Mathematical model of gyroscope

The main advantage of using gyros is that they can provide the angular displacement and/or the angular velocity of the satellite directly. Its mechanism has a wheel that rotates at high speed that responds to changes in the inertial guidance of its axis of rotation which is aligned to the axis of rotation of the satellite.

In this work the gyros (*Integration Rate-Gyros-RIG's*) are used to stop measuring the angular velocity axis of *roll*, *pitch* and *yaw* of the satellite. In addition, the drift errors (bias), due to minor imperfections of its mechanism, are included in the State vector to be estimated. The RIG's model is given by [3]:

$$\Delta\boldsymbol{\Theta}_{\boldsymbol{i}} = \int_0^{\Delta t}(\boldsymbol{\omega}_{\boldsymbol{i}} + \boldsymbol{\varepsilon}_{\boldsymbol{i}})\,dt, \qquad (i = x, y, z) \tag{5}$$

where, $\Delta\boldsymbol{\Theta}$ are the angular displacements measured in the axes of the satellite in a time interval Δt, $\boldsymbol{\omega_i}$ are the components of the angular velocity of the satellite system and ε_i are the components of the gyro bias.

The measurement of the components of the angular velocity of the satellite is represented as [3]

$$\widehat{\boldsymbol{\omega}} = \frac{d\boldsymbol{\Theta}_i}{dt} - \hat{\boldsymbol{\varepsilon}}_i - \boldsymbol{\eta} = \boldsymbol{g} - \hat{\boldsymbol{\varepsilon}}_i - \boldsymbol{\eta} \tag{6}$$

where, $\boldsymbol{g}(t)$ is the gyro output vector and $\boldsymbol{\eta}$ represents the white Gaussian noise process, which covers all remaining non-modeled effects.

3.2 Measuring model of Infrared Earth Sensor

The horizon Sensor is an optical instrument used to detect the light emitted by the edge of the Earth's atmosphere. Infrared sensors are used to detect the heat from the Earth's atmosphere, which is very hot compared to the cold of space, in this way are called Infrared Earth sensors (*Infrared Earth Sensors-IRES*). The *IRES* determine the angle between the direction of an axis of symmetry of the satellite and the direction from the center of the Earth.

When using the *IRES*, it may help to estimated drift errors present in gyro [6]. In this work, two sensors are used, where one measures the *roll* angle and the other measures the *pitch* angle.

The equations of measurements for Infrared Earth sensors (Infrared Earth Sensors-IRES) are given by [2].

$$\begin{aligned} \phi_H &= \phi + \nu_{\phi_H} \\ \theta_H &= \theta + \nu_{\theta_H} \end{aligned} \tag{7}$$

where ν_{ϕ_H} and ν_{θ_H} are white noise that represent small remaining effects of misalignment during installation and/or by assembly of sensor. These errors are assumed Gaussian ones.

3.3 Measuring model of Digital Sun Sensor

The Solar Sensor is an optical mechanism that detects the Sun and sets the position of one of the main axes of symmetry of the spacecraft relative to the direction in which the Sun was detected. The Digital Sun Senor (*DSS*) of the CBERS-2 is not able to measure the *yaw* angle, this is, these sensors do not provide direct measures, it measures the coupled *pitch* angle (α_θ) and *yaw* angle (α_ψ). The equations of measurements for the *Digital Sun Sensors (DSS*) are obtained as follows [2,4].

$$\alpha_\psi = \tan^{-1}\left(\frac{-S_y}{S_x \cos 60° + S_z \cos 150°}\right) + \nu_{\alpha_\psi} \tag{8}$$

when $|S_x \cos 60° + S_z \cos 150°| \geq \cos 60°$, and

$$\alpha_\theta = 24° + \tan^{-1}\left(\frac{S_x}{S_z}\right) + \nu_{\alpha_\theta} \tag{9}$$

when $\left|24° + \tan^{-1}\left(\frac{S_x}{S_z}\right)\right| < 60°$, where ν_{α_ψ} and ν_{α_θ} are the white noise and represent a small effects remnants of misalignment during installation and/or by sensor assembly. Just as the Infrared Earth Sensor, these errors are assumed Gaussian ones.

The conditions must be such that the solar vector is in the field of sight, sensor and S_x, S_y, S_z are the components of the unit vector associated with the solar vector satellite system and date by:

$$\begin{aligned} S_x &= S_{0x} + \hat{\psi} S_{0y} - \hat{\theta} S_{0z} \\ S_y &= S_{0y} - \hat{\psi} S_{0x} + \hat{\phi} S_{0z} \\ S_z &= S_{0z} - \hat{\phi} S_{0y} + \hat{\theta} S_{0z} \end{aligned} \tag{10}$$

where S_{0x}, S_{0y}, $S_{0z}$are the components of the solar vector in orbital coordinate system [2] and $\hat{\phi}$, $\hat{\theta}$, $\hat{\psi}$ are the Euler angles, which represent the estimated attitude.

4 LEAST SQUARES METHOD

This method used in this research is an alternative to the criterion of minimum variance estimation. The approach requires an assumption about the sources of uncertainty in the statistics of problem, assuming the State's dynamic model is perfect, that is, without noise.

The estimator of non-linear least squares assumes the following nonlinear dynamic system [6,7]:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}) \\ \boldsymbol{y_k} &= \boldsymbol{h_k}(\boldsymbol{x_k}) + \boldsymbol{\nu_k} \end{aligned} \tag{11}$$

where $\boldsymbol{f}(\boldsymbol{x})$ represents the non-linear vector function of state $\boldsymbol{x}$, $\boldsymbol{y_k}$ represents the vector of observation of the sensors, $\boldsymbol{h}(\boldsymbol{x_k})$ is the function associated with the model of sensor observations, $\boldsymbol{x_k}$ represents the State vector at instant t_k and $\boldsymbol{\nu_k} = N(0, R_k)$ represents the vector associated with the noise of the observations at this point.

The linearized dynamic equation by the expansion of $\boldsymbol{f}(\boldsymbol{x})$ in Taylor series with truncation on linear term is given by:

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{f}(\overline{\boldsymbol{x}}) + \left\lfloor \frac{df}{dx} \right\rfloor_{x=\bar{x}} (\boldsymbol{x} - \overline{\boldsymbol{x}}) \tag{12}$$

By setting the following deviations

$$\begin{aligned} \delta\boldsymbol{x} &\equiv \boldsymbol{x} - \overline{\boldsymbol{x}} \\ \delta\dot{\boldsymbol{x}} &\equiv \dot{\boldsymbol{x}} - \dot{\overline{\boldsymbol{x}}} = \dot{\boldsymbol{x}} - \boldsymbol{f}(\overline{\boldsymbol{x}}) \end{aligned} \tag{13}$$

the following expression arises

$$\delta\dot{\boldsymbol{x}} = \boldsymbol{F}\delta\boldsymbol{x} \tag{14}$$

where F is the Jacobian matrix given by:

$$\boldsymbol{F} = \left\lfloor \frac{df}{dx} \right\rfloor_{x=\bar{x}} \tag{15}$$

Of course this linearization is valid only while $\delta\boldsymbol{x}$ is small.

The observation equation can also be linearized by the expansion of $\boldsymbol{h_k}$ into Taylor series with truncation on linear term:

$$\boldsymbol{y_k} = \boldsymbol{h_k}(\overline{\boldsymbol{x}}) + \left\lfloor \frac{d\boldsymbol{h_k}}{dx} \right\rfloor_{x=\bar{x}} (\boldsymbol{x} - \overline{\boldsymbol{x}}) + \boldsymbol{\nu_k} \tag{16}$$

The residues are defined by $\delta \boldsymbol{y_k} = \boldsymbol{y_k} - \boldsymbol{h_k}(\overline{\boldsymbol{x}})$, such that the observation equation is given by the linearized equation:

$$\delta \boldsymbol{y_k} = \boldsymbol{H_k} \delta \boldsymbol{x_k} + \boldsymbol{\nu_k} \tag{17}$$

where H_k is given by:

$$\boldsymbol{H_k} = \left\lfloor \frac{d\boldsymbol{h_k}}{dx} \right\rfloor_{x=\bar{x}} \tag{18}$$

The a-priori information to the state and covariance in instant t_0 given by: $\widehat{\boldsymbol{x}}_{\boldsymbol{0}} = \widehat{\boldsymbol{x}}_{\boldsymbol{0}}(t_0)$ and $\widehat{\boldsymbol{P}}_{\boldsymbol{0}} = \widehat{\boldsymbol{P}}_{\boldsymbol{0}}(t_0)$. The method of nonlinear least squares must be implemented in an iterative way, which refines the variations instead of states [7]. By setting the following deviations:

$$\begin{aligned} \delta \overline{\boldsymbol{x}}_{\boldsymbol{k}} &\equiv \widehat{\boldsymbol{x}}_{\boldsymbol{k}} - \widehat{\boldsymbol{x}}_{\boldsymbol{0}} \\ \delta \widehat{\boldsymbol{x}}_{\boldsymbol{k}} &\equiv \widehat{\boldsymbol{x}}_{\boldsymbol{k}} - \widehat{\boldsymbol{x}}_{\boldsymbol{k-1}} \end{aligned} \tag{19}$$

the equations that implement the estimation algorithm for least squares method are [7]:

$$\widehat{\boldsymbol{P}}_{\boldsymbol{k}} = \left(\widehat{\boldsymbol{P}}_{\boldsymbol{0}}^{\,\boldsymbol{-1}} + \boldsymbol{H^T R^{-1} H} \right)^{\boldsymbol{-1}} \tag{20}$$

$$\delta \widehat{\boldsymbol{x}}_{\boldsymbol{k}} = \widehat{\boldsymbol{P}}_{\boldsymbol{k}} \left(\overline{\boldsymbol{P}}_{\boldsymbol{0}}^{\,\boldsymbol{-1}} \delta \overline{\boldsymbol{x}}_{\boldsymbol{k-1}} + \boldsymbol{H^T R^{-1}} \delta \boldsymbol{y} \right) \tag{21}$$

Normally the iterations continue until convergence. Basically, the most widely used criterion to terminate the algorithm is to verify when the deviation $\delta \widehat{\boldsymbol{x}}_{\boldsymbol{k}}$ becomes small enough, in this work we use six iterations of least squares method for State estimation.

In this way the final solution to the States will be

$$\widehat{\boldsymbol{x}}_{\boldsymbol{k}} = \widehat{\boldsymbol{x}}_{\boldsymbol{k-1}} + \delta \widehat{\boldsymbol{x}}_{\boldsymbol{k}} \tag{22}$$

with covariance $\widehat{\boldsymbol{P}}_{\boldsymbol{k}}$ given by Eq. (20).

5 COMPUTER SIMULATION AND RESULTS

Here, the results and analysis of algorithms developed to estimate the attitude are presented. To validate and analyze the performance of estimators, actual data sensors Satellite CBERS-2 were used. The CBERS-2 satellite was launched on October 21, 2003. The measures are for the day April 21, 2006, being collected by the ground system at a sampling rate of about $8.56s$ for about $10\ min$ of observation.

In fact, the ACS (Attitude Control System) on board the satellite has full access to sensor measurements sampled at a rate of 4 Hz for the three gyros, to axes x, y, z of the satellite; $1Hz$ for the two Infrared Earth Sensors to the angle ϕ (*roll)* and θ (*pitch)*; and $0.25\ Hz$ for both Digital Sun Sensors, related to the angles of *pitch* (α_θ) and *yaw* (α_ψ). However, due to the limited the telemetry system can only recover telemetries of sensors at $9s$ sampling only when the satellite passes over the tracking station. This means that the ground system does not have the full set of measurements that are available to the ACS on board [8].

In total, we have a set of 54 measurements from $13h\ 46min\ 25s$ until $13h\ 55min\ 27s$, and measurements are spaced by $10s$ on average. For ease of visualization, we graphically represents measurements of the sensors *DSS* and *IRES* in Figure 1 and measures the gyroscope in Figure 2, both shown below.

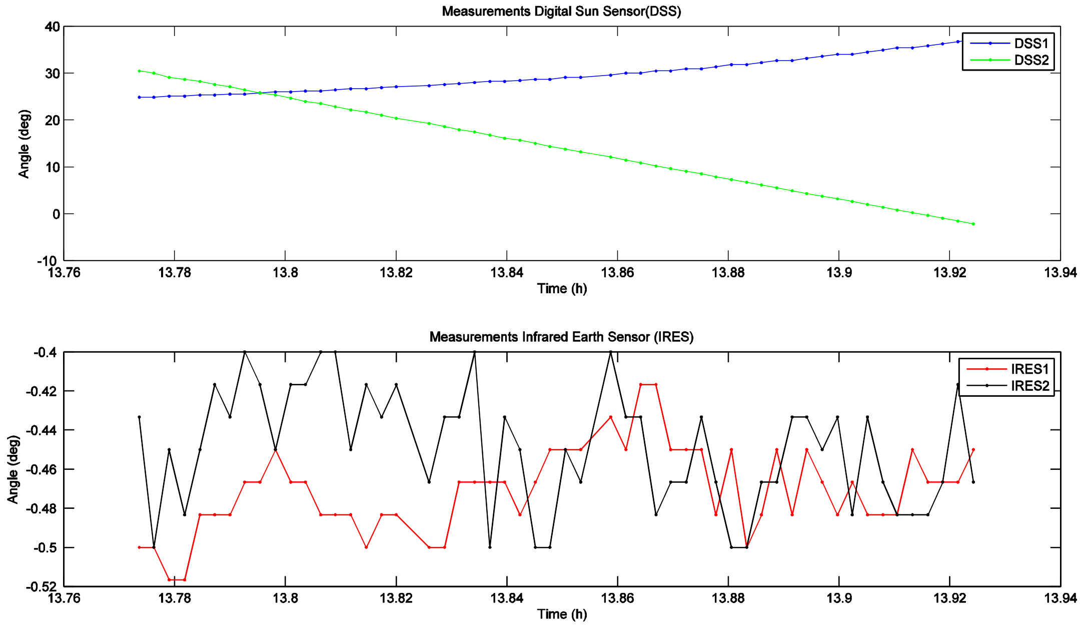

Figure 1.Graphical representation of the real data from the sensors DSS and IRES from CBERS-2

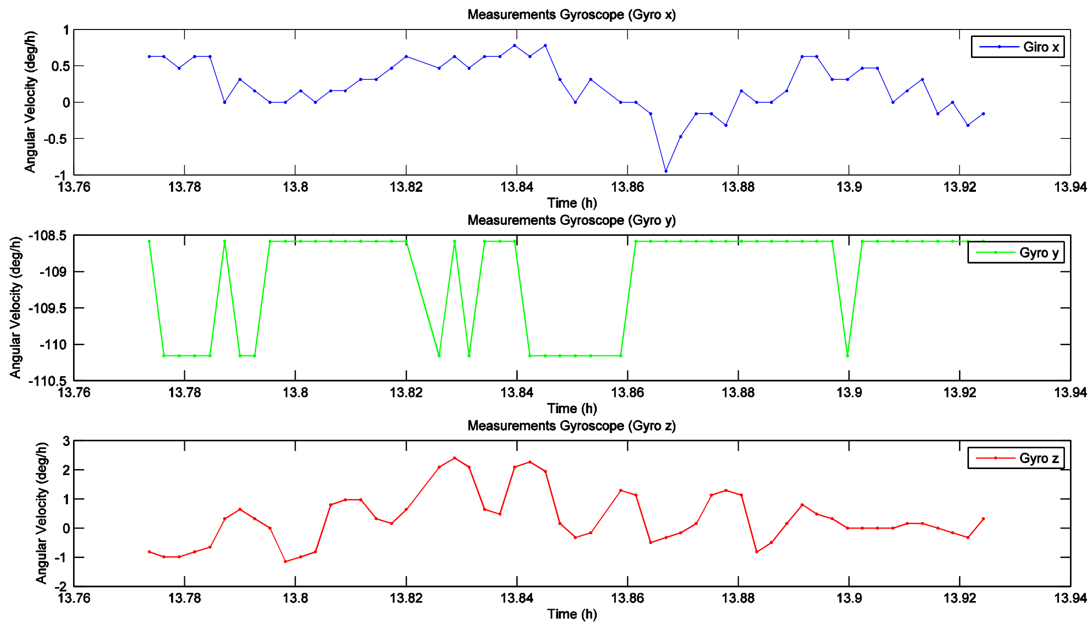

Figure 2.Graphical representation of the real data of the gyroscopes from CBERS-2

The implementation of the algorithm of state estimation by the Least Squares Method was performed using MatLab® software. Using the representation of attitude by Euler angles, the following initial conditions were used:

$\phi\ (deg)$	$\theta\ (deg)$	$\psi\ (deg)$	$\varepsilon_x\ (deg/h)$	$\varepsilon_y\ (deg/h)$	$\varepsilon_z\ (deg/h)$
−0.5	−0.3	−0.5	5.76	220.00	2.68

Table 1. Initial attitude and initial bias of the gyros

${\sigma_\phi}^2\ (deg)$	${\sigma_\theta}^2\ (deg)$	${\sigma_\psi}^2\ (deg)$	${\sigma_{\varepsilon_x}}^2\ (deg/h)$	${\sigma_{\varepsilon_y}}^2\ (deg/h)$	${\sigma_{\varepsilon_z}}^2\ (deg/h)$
0.025	0.025	1.0	2.0	2.0	2.0

Table 2. Values of the main diagonal of the initial covariance matrix $\boldsymbol{P_0}$

${\sigma_{DSS1}}^2\ (deg)$	${\sigma_{DSS2}}^2\ (deg)$	${\sigma_{IRES1}}^2\ (deg)$	${\sigma_{IRES2}}^2\ (deg)$
0,6	0,6	0,06	0,06

Table 3. Values of the main diagonal of the error matrix observation $\boldsymbol{R}$

Below we present in Figure 3 residue for sensors IRES and DSS for the sixth iteration of Least Squares method.

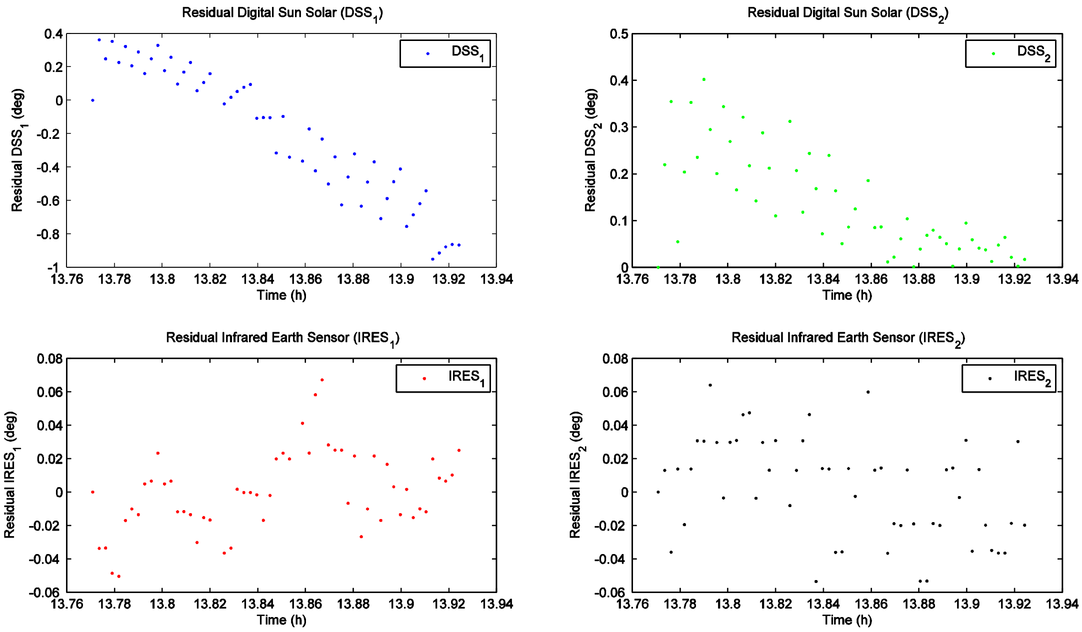

Figure 3.Residues from the sixth iteration of the least squares method

It is observed that when analyzing the residues of the first and sixth iteration of the method of least squares, the residue increasingly approximates the average zero as the number of iterations increases.

In Figures 4 and 5, the following are the states (attitude and bias of the gyro) estimated for six iterations of the method, the graphs also present the estimated states with $\pm\,\sigma$.

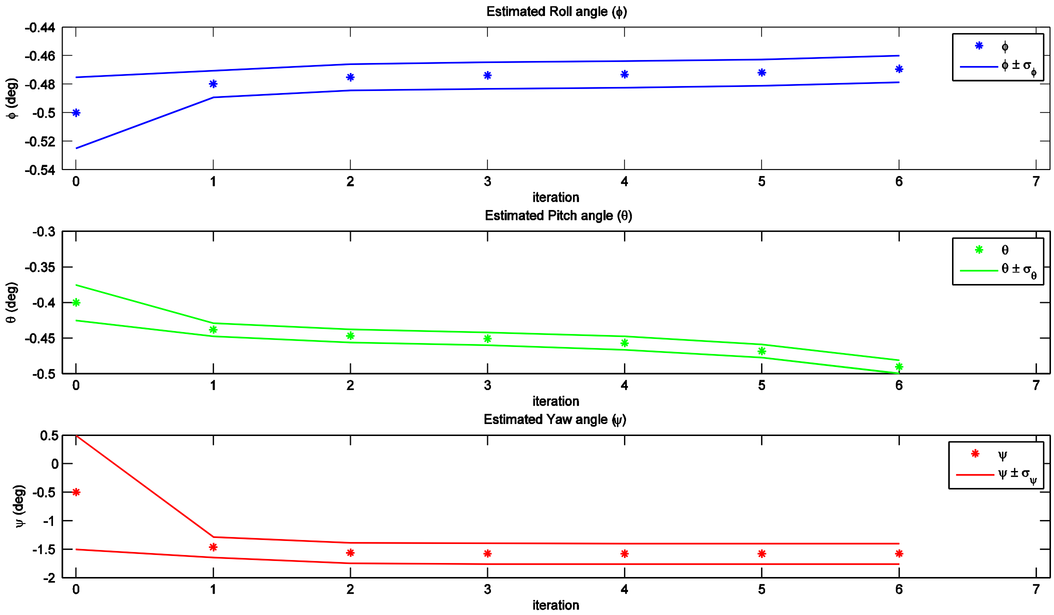

Figure 4. Estimated Attitude

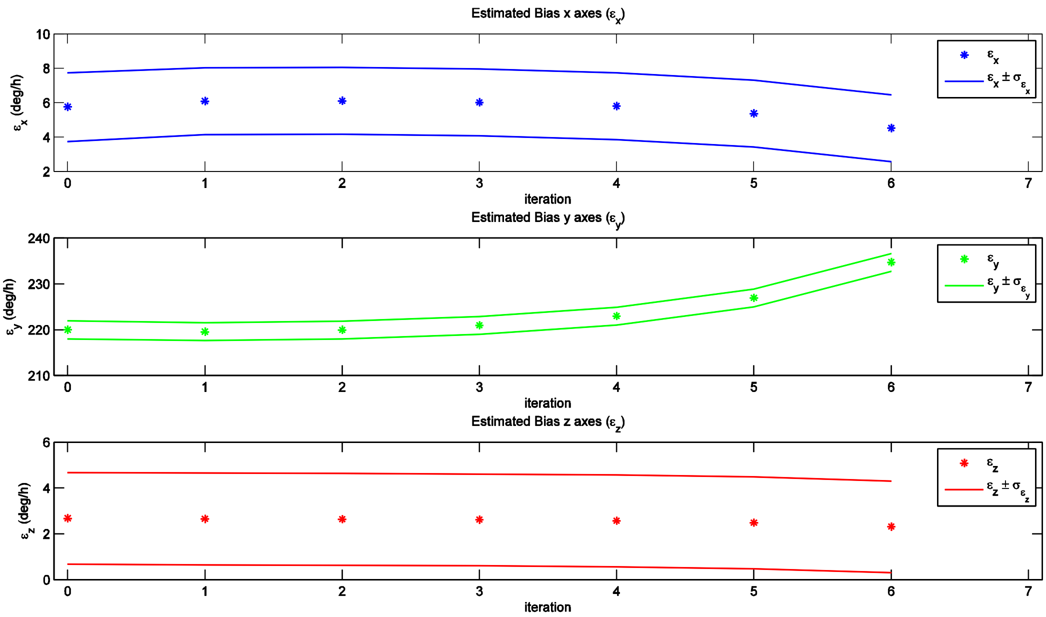

Figure 5. Estimated gyros Bias

6 CONCLUSIONS

The main objective of this study was to estimate the attitude of a CBERS-2 like satellite, using real data provided by sensors that are on board the satellite. To verify the consistency of the estimator, the attitude was estimated by least squares method.

The usage of real data from on-board attitude sensors, poses difficulties like mismodelling, mismatch of sizes, misalignments, unforeseen systematic errors and post-launch calibration errors. However, it is observed that the attitude estimated by the least squares method are in close agreement with the results in previous works [4,5] which used the Extended Kalman Filter (EKF) for estimation of attitude.

However, checking the robustness of the estimation method (LSM), it was noted that the greater the deterioration of the initial conditions, the LSM took longer to achieve convergence compared to [4,5]. In this case the linearization performed by LSM were not making approximations algorithm accurate when the initial conditions are far from the real value, but if these initial conditions are close to the real value, the LSM produces satisfactory results for attitude estimation.

It can be concluded that the algorithm of the LSM converges, for initial conditions close to the actual values, providing a kinematic attitude solution besides estimating biases (gyro drifts).

7 ACKNOWLEDGMENTS

The authors would like to thank the financial support received by CAPES, FAPESP (grant #2012/21023-6), CNPQ (grant #303119/2010-1), and the partial support from project SIA-DCTA-INPE under contract FINEP 0.1.06.1177.03.

8 REFERENCES

[1] W. R. Silva: *Estudo da estabilidade do movimento rotacional de satélites artificiais com variáveis canônicas*. Dissertação de mestrado, UNESP – Universidade Estadual Paulista, Guaratinguetá, S.P. 2012.

[2] H. Fuming, H. K. Kuga: *CBERS Simulator Mathematical Models.* CBTT Project, CBTT /2000 /MM /001.INPE, São José dos Campos. 1999.

[3] Wertz, J. R.: *Spacecraft Attitude Determination and Control*, D. Reidel, Dordrecht, Holanda. 1978.

[4] R. V. Garcia: *Filtro Não-Linear de Kalman Sigma-Ponto com Algoritmo Unscented Aplicado a Estimativa Dinâmica da Atitude de Satélites Artificiais*. Tese de Doutorado, INPE, São José dos Campos, 2011.

[5] R. V. Garcia, H. K. Kuga, M. C. Zanardi: “Unscented Kalman Filter for Spacecraft Attitude Estimation Using Quaternions and Euler Angles”. 22nd International Symposium on Space Flight Dynamics, São José dos Campos, 2011.

[6] A. F. B. A. Prado, H. K. Kuga: *Fundamentos de Tecnologia Espacial*. São José dos Campos, INPE, 2001. 32, 36

[7] H. K. Kuga, *Estimation techniques on practice* (in Portuguese). Class Notes. INPE, São José dos Campos, 2005.

[8] R. V. F. Lopes, H. K. Kuga: *CBERS-2: On Ground Attitude Determination From Telemetry Data.* São José dos Campos, INPE, Internal Report C-ITRP, 2005.