



## STOCHASTIC CELLULAR AUTOMATA MODEL FOR WILDLAND FIRE SPREAD DYNAMICS

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**Abstract:** A stochastic cellular automata model for wildland fire spread under flat terrain and no-wind conditions is proposed and its dynamics is characterized and analyzed. Each cell is characterized by one of the three states that are: *vegetation cell*, *burning cell* and *burnt cell*. The dynamics of fire spread is modeled as a stochastic event with an effective fire spread probability  $S$  which is a function of three probabilities: the proportion of vegetation cells across the lattice, the probability of a burning cell become burnt, and the probability of the fire spread from a burning cell to a neighbor vegetation cell. A set of simulation experiments are performed to analyze the effects of different values of the three probabilities in the fire pattern. The effective fire spread probability is obtained from Monte-Carlo simulations and a critical line that separate the set of parameter or which a fire can propagate from those for which it cannot is obtained. Finally, the relevance of the model is discussed in light of results of simulation experiments carried out that illustrate the capability of the model catches both the dynamical and static qualitative properties of fire propagation.

**Keywords:** wildland fire spread, percolation theory, cellular automata.

### 1. INTRODUCTION

The wildland fire spread is a combustion reaction where the ingredients necessary for its occurrence are: the vegetation, which provides the combustible source for the reaction; the oxygen in the air, which actuate as an oxidizing agent; and a heat source responsible by the initiation and the self-sustainability of the reaction [1]. The fire spreads across the landscape consuming the vegetation and this process can be decomposed into four combustion phases, the so called: pre-heating, ignition, combustion and extinction [1]. The fire front is the region of intense flaming combustion where a large quantity of heat released. Part of this heat released is transmitted to the vegetation that yet is not burning, heating it until reaches the ignition temperature. When the vegetation reaches the ignition temperature, the flames rise and the fire front occupies a new position ahead. The flames remain as the vegetation is burnt out.

In this work is proposed a simple model for wildland fire dynamics under flat terrain and no-wind conditions. The model formulation is based on stochastic cellular automata

and its dynamics is analyzed qualitatively and quantitatively. Cellular automata are models which assume space, state and time discrete [2]. The space is represented by a square lattice and each element that constitutes the lattice is called cell. Each cell has a neighborhood, set of internal states variables, and a set of rules, called state transition functions, that describes the evolution of their states and define the future state as a function of the cell present state and the neighborhood present states. In stochastic cellular automata the state transition function is performed by means of probabilities. The transition functions of the proposed model are defined stochastic with the intention to represent the vegetation heterogeneity and to include random component in the dynamics of the vegetation combustion and ignition process during the fire spread.

The paper is structured as follow. In the Section 2 de modeling approach and the model parameters are described. In the section 3 the model dynamics is characterized and analyzed. Finally, in the last section, the model relevance is discussed.

### 2. MODEL DESCRIPTION

The model is based on the spatially explicit representation and the landscape is depicted as a square and two-dimensional lattice  $\mathcal{L}$  of dimensions  $L_x \times L_y$ . Each cell is defined by:

- (i). its discrete position  $(i, j)$  in the lattice, where  $i=1, \dots, L$  is the column and  $j = 1, \dots, L$  is the row;
- (ii). the finite set of internal states variables that describes the possible behavior of the cells in a given time step  $t$  which are  $S_{(i,j)}^t \in [E, V, F, O]$  where:  $E$  is an *empty cell*, which denotes unburnable cells or without vegetation;  $V$  is a *vegetation cell*, with denote cells with potential to burn;  $F$  is *burning cell*, which denotes a cell whose the vegetation in its inside is burning; and  $O$  is *burnt cell*, which denote vegetation cell that is burned by the fire;
- (iii). the set of finite neighborhood cells  $\mathcal{N}(i, j)$ , where the Moore neighborhood, as illustrated in the Figure 1(a), represents the neighborhood relations in the model and comprises the eight

- cells surrounding  $(i^*, j^*)$  of a central cell  $(i, j)$  according with the definition  $\mathcal{N}(i, j) = \{(i^*, j^*): |i - i^*| \leq 1, |j - j^*| \leq 1\}$ ;
- (iv). the transition function that calculate the future cell state as a function of the present cell state and present neighborhood cell states  $f: S_{(i,j)}^t \times S_{\mathcal{N}(i,j)}^t \rightarrow S_{i,j}^{t+1}$ , where the time  $t$  is also represented by discrete values or time steps. Thus, the time evolution of the model is driven by the interaction between the cell states and the cell neighborhood states. Starting from a given configuration of cells initial states, the cellular automaton self-replicates the sequent cell states. The cellular automata model is stochastic because the state transition function is performed according to probabilities values.

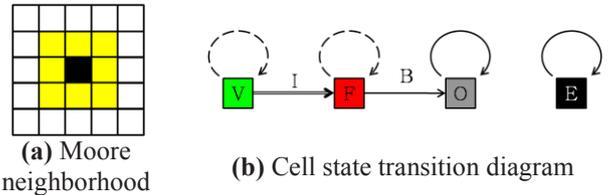
The fire spread is governed by the heat transfer from burning regions to non-burning regions. Thus the fire spread is modeled as a set of ignitions of non-burning regions as the burning regions persist. Stochasticity is used to include the heterogeneity of spatial conditions present in real vegetation patterns and to include random component in the dynamics of combustion and ignition process [4,5,6]. Thus, the dynamics of fire spread is modeled as a stochastic event with an effective fire spread probability  $S$  which is as a function three probabilities, which are:

- (i). the probability  $D$ , that determine the proportion of cells with vegetation across the lattice in the model initialization. Thus, for each cell, there is a probability  $D$  to its state is *vegetation cell* and the probability  $1 - D$  to it is *empty cell*.
- (ii). the probability  $B$ , that models the combustion, where, in each time step, a *burning cell* has a probability  $B$  to change its state to *burnt cell*.
- (iii). the probability  $I$ , that models the ignition, where, there is a probability  $I$  for the fire spreads from a *burning cell* to a neighbor *vegetation cell*.

The transition functions between the states are performed according to these probabilities values. The cell state transition diagram is showed in the Figure 1(b). An *empty cell* is unchangeable and always remains in this state. The fire spread is considered a diffusion contagious process and the fire can spreads only from a *burning cell* to a neighbor *vegetation cell*. Thus, the transition  $V \rightarrow F$  is conditioned for a *vegetation cell* that has at least one *burning cell* neighbor. Given two neighbors cells, one *burning cell* and the other a *vegetation cell*, in each time step, there is a probability  $I$  for the *burning cell* ignites the neighbor *vegetation cell*. Once ignited, in each time step, there is a probability  $B$  for the *burning cell* remain burning, otherwise its state changes to *burnt cell*, which is the transition  $F \rightarrow O$ .

The model input parameters are the probabilities  $D$ ,  $B$  and  $I$ , the lattice size, and the maximum time step  $NSTEPS$ . A complete visit to all cells of the lattice is called a sweep. A simple simulation is performed in the two stages: initialization and fire spreading algorithm. The initialization stage includes: (1) define the model input parameters; (2) execute a sweep and for each cell and sets this state to *vegetation cell* with probability  $D$  or *empty cell* with

probability  $1 - D$ ; and (3) select one or more vegetation cells and change its state to *burning cell*. In the fire spreading algorithm, for each time step  $t = 1, \dots, NSTEPS$  execute a sweep and: (1) for each *burning cell*, evaluate the transition  $B \rightarrow O$ ; (2) for each neighbor of a *burning cell* evaluate the transition  $V \rightarrow B$ . In the end of each time step count the number of burning cells. Time simulation stop if  $t = NSTEPS$  or if there are none burning cell in a given time step.



**Fig. 1. (a) The Moore neighborhood comprises eight cells (yellow cells) which surround the central cell (black cells). (b) In the cell state transition diagram, arrows indicate the state transitions paths. The double arrow indicates that the transition depends on the neighbor cell state. The round dashed arrows indicate that the state transitions are conditioned by the values of other probabilities.**

### 3. SIMULATIONS AND RESULTS

#### 3.1 Qualitative analysis of the fire patterns

The effective fire spread probability  $S$  describes the fire behavior across the lattice as an function of the probabilities  $D$ ,  $B$  and  $I$ . Different fire patterns, with different size and shape, can be obtained varying the values of these probabilities. The Figure 2 characterizes some fire patterns using different values of  $D$ ,  $B$  and  $I$ , for a lattice of size  $201 \times 201$  and the fire starting from a cell at the middle of the lattice  $S_{(100,100)}^0 = F$ . Each cell state along the lattice is represented by colors that which are, empty cell (black), *vegetation cell* (green), *burning cell* (red) and *burnt cell* (gray).

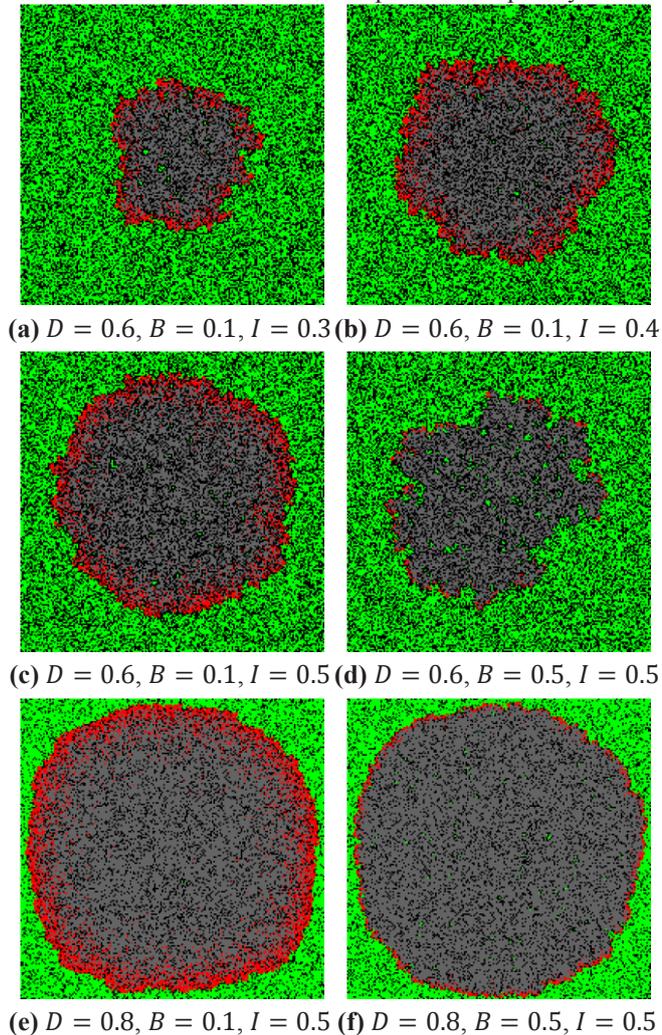
The proportion of cells with vegetation across the lattice determines the spatial distribution of available fuel along the lattice. Higher values of  $D$  implies in more quantity of available fuel along the landscape and therefore the fire propagates with more facility. This effect can be observed comparing the Figures 2(f) and 2(d), when in the Figure 2(f) the burned area is larger than in the Figure 2(d).

The probability  $B$  asserts the combustion latency for a *burning cell* ( $B^{-1}$  is a conception for the mean reaction time). A *burning cell* with a high value of  $B$  burns most quickly (i.e., in less time steps) than those that have a low value. This behavior can be observed comparing the Figures 2(e) and 2(f). Furthermore, how smaller is the values of  $B$ , more high is the probability of the fire spread from the burning cell to neighbors vegetation cells, because the cells remain burning in more time steps. This effect can be observed comparing the Figure 2(c) and 2(d).

The ignition probability  $I$  determines how lightly the fire spreads along the lattice. The effects of different fire rates of spread can be observed by the different burned areas



comparing the Figures 2(a), 2(b) and 2(c). Higher values of  $I$  are related with fire fronts which spread most quickly.

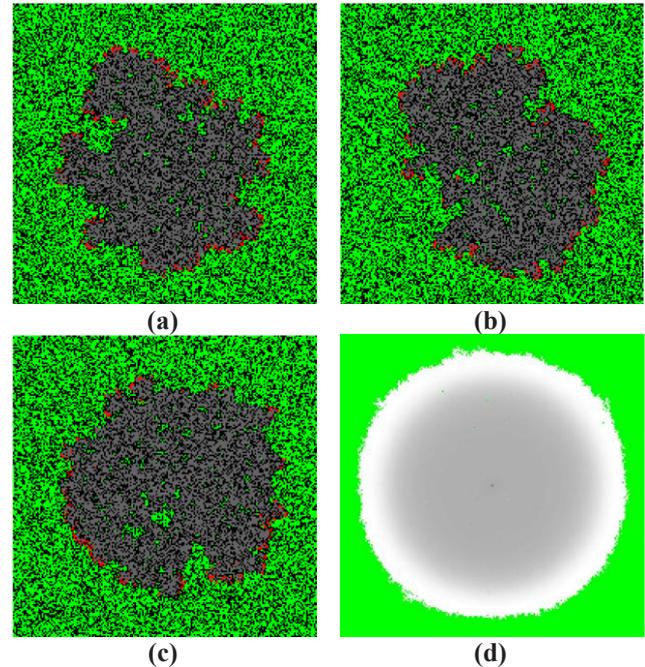


**Fig. 2.** Different fire patterns for  $t = 100$  using a lattice with size  $201 \times 201$  and the fire starting from the middle cell positioned at  $(i, j) = (100, 100)$ . The parameters values are showed immediately bellow the figures.

### 3.2 Monte-Carlo simulations

Because the natural model stochasticity, a same set of parameters values can generate fire patterns slightly different, according showed in the Figure 3(a), 3(b) and 3(c). Thus, is necessary to obtain the mean behavior computed during a large quantity of simulations based on different sequence of generated random number. This is the objective of the Monte-Carlo simulations (MCS). For a given set of model input parameters, a large quantity of simulations are carried out and for each cell is computed the number of times that it burns. The number of time that a cell burn divided by the total number of MCS is the estimative of the cell burning risk. The Figure 3(d) shows the cell

burning risk computed by a MCS for  $D = 0.6, B = 0.5, I = 0.5$  and  $N=100$ .



**Fig. 3.** (a)-(c) Fire patterns slightly different for a lattice with size  $201 \times 201$ ,  $D = 0.6, B = 0.5, I = 0.5, t = 100$  and the fire starting from the middle cell positioned in  $(i, j) = (100, 100)$ . (d) Burning risk for  $N = 10000$  Monte-Carlo simulation. The color map in the figure varies from 0 (black) to 1 (white), and the cells that not burn (green cells).

### 3.3 Calculating the value of $S$ and its critical line

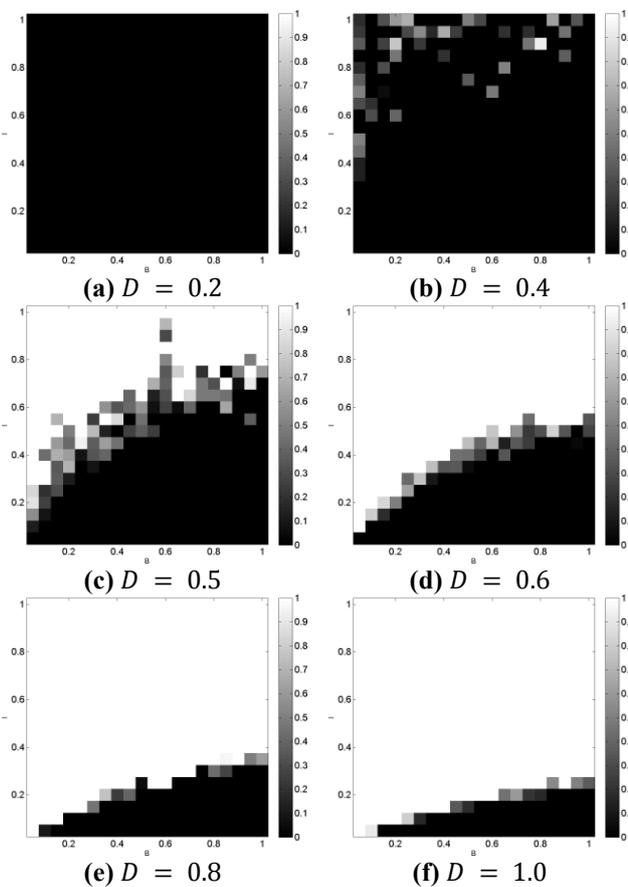
The fire spreads along the lattice following a pathway of interconnected cells which varies as a function of the effective probability  $S$ . Studies in percolation theory corroborate that there is a critical value  $S^*$ , called percolation threshold, so that when  $S > S^*$  always there is this pathway for the fire spreads from a starting cell to some other point inside the lattice [3]. The main question here is how to characterize the probability  $S$  in the model.

The existence of the percolation threshold and the consequent description of the critical line as a function of the probabilities  $D, B$  and  $I$  are investigated using MCS. A set of  $N$  MCS are performed using identical lattices and different values of  $D, B$  and  $I$ . The fire starts at the left border of the lattices and during the  $N$  simulations is computed the number of times that the fire reaches the right border of the lattice. If the fire propagates from one side to the other then the fire percolate the lattice. Thus, the approximation of  $S$ , denoted by  $\langle S \rangle$ , is calculated as:

$$\langle S \rangle = \frac{1}{N} \sum_{j=1}^N C_j, \quad (1)$$

where  $C_j = 1$  if the fire percolate the lattice and  $C_j = 0$  otherwise.

The Figure 3 characterizes the values of  $\langle S \rangle$  for different values of  $D$  varying the values of  $B$  and  $I$ . For each set of parameters values a set of  $N = 1000$  MCS are carried out using one lattice of size  $52 \times 52$ . The color map displays values varying from  $\langle S \rangle = 0$  (black) to  $\langle S \rangle = 1$  (white). If  $\langle S \rangle = 0$  the fire not percolates the lattice, in other words, the fore extinction regime predominates. Otherwise, if  $\langle S \rangle = 1$ , the fire propagation regime predominates and the fire spreads incessantly across the lattice. The Figs. 4(a)-(f) indicate the existence of a critical line that define a partition on the model parameter space and separates the set of parameters for which a fire can propagate from those for which it cannot. The curve position changes as a function of  $D$ . The Figs. 4(a)-4(f) characterizes this critical line in different values of  $D$ .



**Fig. 4.** Values of  $\langle S \rangle$  for different values of  $D$  varying the values of  $B$  and  $I$ . A total of  $N = 1000$  Monte-Carlo simulations are carried out for each set of values  $D$ ,  $B$  and  $I$  using a lattice of size  $52 \times 52$ .

### 3.3 The fire rate of spread

The fire rate of spread  $R$  is a common measure of its behavior. To compute its value and the relation between the rate and the model parameters, a MCS is used to compute the mean fire front edge position, computed in the central cells in the direction of spread, for fires initiated at the left edge of the lattice. The time evolution of the fire front edge position is compatible with a linear fit, and thus a rate of

spread can be defined as the angular coefficient of the linear fit. The rate of spread unit is calculated in cell dimensions per time steps, assuming that the cells have the same dimensions.

The fire pattern generated by the model is resultant of two coupled processes: fire diffusion and fire persistence. The fire diffusion process is the dynamics of fire transmission between the cells, and it is governed by the ignition probability  $I$ . The fire persistence process supports the fire remaining in a *burning cell* and is responsible by the dynamics of fire pattern formation during the time and drives the fire diffusion process.

The combination of these two processes can generate a large quantity of different fire patterns formations and the values of  $I$  and  $B$  are the weights of each one. One higher fire persistence and one lower fire diffusion provides fire fronts that propagate slowly during the time. Keeping constant the persistence and increasing the diffusion, the fire spread increases. This effect can be observed comparing the dependence of rate of spread on  $I$  in the Figs. 5(a), 5(b), 5(c) and 5(d).

Although the probabilities  $B$  and  $I$  are important to the fire pattern formation in the model, the fire pattern existence is conditioned to the value of probability  $D$ . De facto, the diffusive fire spread require a continuous pathway or clusters of *vegetation cells* and the distribution of vegetation cells across the lattice depends on  $D$ . Lower values of  $D$  generate a fuel distribution sparse enough to the fire not propagate across it, and that can be observed in the Fig. 4(a). As  $D$  is increased, the vegetation cells cluster take place and there is fuel enough to occur the fire spread. Thus, there is a critical value  $D^*$ , such that for  $D > D^*$  there is the consequent formation of a percolation cluster and the fire can propagate across it. In the Figs. 4(a)-4(f) can be observed that when  $D$  increases, also increases the proportion of the set of parameters for which the fire can propagate.

## 3. FINAL CONSIDERATIONS

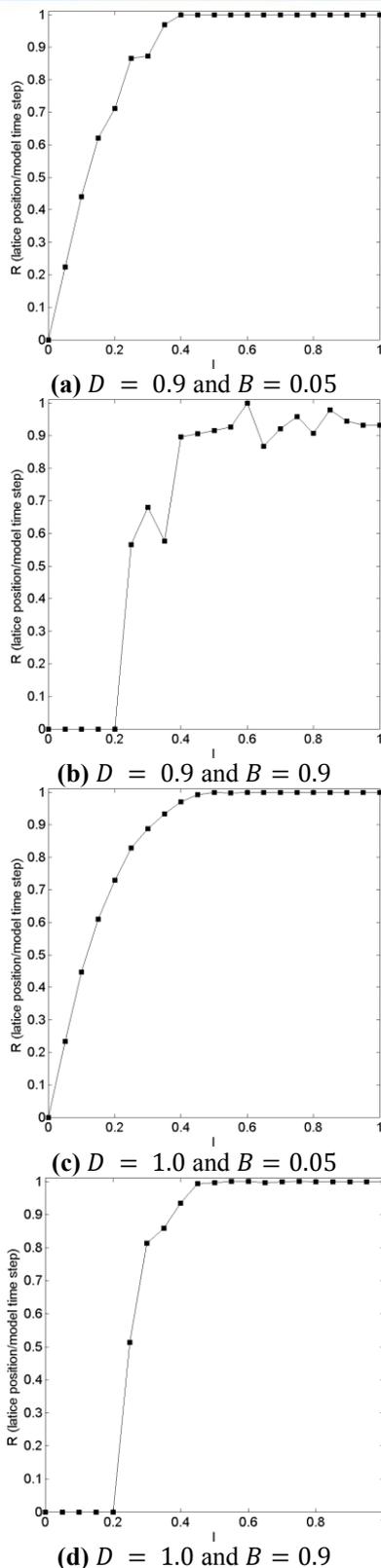
Although the model formulation include only fire spread dynamics under flat terrain and no-wind conditions, the qualitative and quantitative analysis performed in this paper indicate that this model constitutes a qualitative framework for wildland fire spread dynamics simulation. However, for further ecological applications of this model, the relation of the model parameters with meteorological, vegetation and topographical factors remain to be quantitatively established.

The effects of wind and slope on ignition process can be represented according to the incorporation of a directional bias that proportions an anisotropic diffusive process across the lattice [6]. Different values of vegetation density  $D$  can be used to represent several phyto-physiognomy vegetation clusters. The probability  $B$  can be adjusted to include different values of fuel load over the surface and different fuel moisture conditions. This model parameterization consists in finding an explicit expression between the model parameters and the environmental conditions of historical and documented forest fires.



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**Fig. 5.** Dependence of the rate of spread on  $I$  for different values of  $D$  and  $B$ .

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## REFERENCES

- [1] Pyne, S. J., P. L. Andrews, and R. D. Laven. 1996. Introduction to wildland fire. John Wiley and Sons, New York, New York, USA.
- [2] Schiff, J. L., Cellular Automata: A Discrete View of the World (Wiley Series in Discrete Mathematics & Optimization), 2007.
- [3] Stauffer, D. and Aharony, A. Introduction to Percolation Theory, 2nd ed. London: Taylor & Francis, 1992.
- [4] Hargrove, W. W., Gardner, R. H., Turner, M. G., Romme, W. H., Despain, D. G. "Simulating fire patterns in heterogeneous landscapes". Ecological Modelling, 135: 243 -263 (2000).
- [5] Nahmias, J., Téphany, H., Duarte, J., Letacounoux, S. Fire spreading experiments on heterogeneous fuel beds. Applications of percolation theory. Canadian Journal Forest Research, 30: 1318-1328 (2000).
- [6] Favier, C. Percolation model of fire dynamic. Physics Letters A, 330: 396-401 (2004).