

## Asymptotic analysis of a diffusion flame established in a porous medium

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**Abstract:** *In the present work, the aspects of a diffusion flame established inside a porous medium are analyzed. A stream of hot oxidant is injected into the porous matrix and it impinges on a pool of liquid fuel. The liquid fuel evaporates and in the region in which the mass fluxes are in a stoichiometric proportion the flame is established. A low porosity medium is assumed and the gas is considered incompressible. In this analysis, a low volatile fuel is considered, such that the vaporization rate is small. This work is analyzed through the Schvab-Ze'ldovich formulation, in terms of the mixture fraction and of the excess enthalpy variables in order to eliminate the strong non-linear reaction term from the conservation equations. Own to the nature of the flow, boundary layers are observed in the problem. With the aid of the asymptotic theory these boundary layers are analyzed separately. Hence, the perturbation method is utilized in each boundary layer to obtain the relevant profiles, so as the flame position and its temperature, both eigenvalues of the problem.*

### 1 Introduction

Combustion phenomena have been studied for decades [2, 7, 8]. Such interest may be explained by the large amount of technologic and scientific problems which are still open in this area. Combustion processes confined in porous medium have some different characteristics than those observed in free combustion systems. These new characteristics are consequence of the combination of the thermophysical properties of the solid phase and gas/liquid phase found in porous systems: long range conduction, convection and radiation. The large contact area between solid and gas/liquid and the radiation field, created by the heated solid, contribute to improvements in the interphase heat transfer in these systems. Combustible gases flowing and reacting through the porous medium releases heat. The amount of heat released is intimately coupled not only to the energy and mass transfer in the gas phase but also to that in the solid phase. Under these new conditions, porous medium flames exist far below the flammability limit for free flames (the term “free” is used to denote flames established outside of a porous medium) [1, 10].

Applications ranging from compact combustion chambers, food baking, drying of paper and wood, heavy oil thermal recovery and stability of solid propellants decomposition are of the interest in the area of combustion in porous medium. Fundamental analyses of these relevant problems can be achieved by means of the physical-mathematical study of such systems.

Under such motivation, the present work presents a formulation for the diffusion flame established inside a porous medium, with the fuel vapor being supplied by the evaporation of a low volatile liquid fuel subjected to a stream of hot oxidant. Own to the strong non-linear reaction rate term, the Schvab-Ze'ldovich formulation is proposed [6], and the problem is analyzed through the mixture fraction and excess enthalpy variables. Due to the existence of

physical processes occurring at different length scales [9, 10], the asymptotic theory is employed in order to solve the problem in each length scale. The appearance of a large non-dimensional parameter (the ratio between solid and gas phases thermal conductivities) justifies the use of the perturbation theory in the non-dimensional conservation equations.

## 2 Mathematical formulation

The non-dimensional and normalized formulation is presented following previous works [3, 4]. For the gas-solid region, the conservation equations for mass, momentum, species (fuel and oxidant) and energy (gas and solid) are given respectively by:

$$U = \frac{df}{d\eta}, \quad (1)$$

$$\frac{Pr}{\Gamma} \frac{d^3 f}{d\eta^3} + f \frac{df^2}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 - \Gamma \varepsilon \beta Pr \frac{df}{d\eta} = -\varepsilon Pr (1 + \beta \Gamma), \quad (2)$$

$$\frac{1}{\Gamma} \frac{d^2 y_f}{d\eta^2} + Le_F f \frac{dy_f}{d\eta} = Le_F \frac{w_F}{a}, \quad (3)$$

$$\frac{1}{\Gamma} \frac{d^2 y_O}{d\eta^2} + Le_O f \frac{dy_O}{d\eta} = S Le_F \frac{w_F}{a}, \quad (4)$$

$$0 = \frac{\varepsilon}{\Gamma} \frac{d^2 \theta_g}{d\eta^2} + \varepsilon f \frac{d\theta_g}{d\eta} + \varepsilon q \frac{w_F}{a} + \Gamma n_g (\theta_s - \theta_g), \quad (5)$$

$$0 = (1 - \varepsilon) \frac{d^2 \theta_s}{d\eta^2} - \Gamma n_g (\theta_s - \theta_g), \quad (6)$$

in which  $\varepsilon$  is the medium porosity,  $\Gamma \equiv \bar{\lambda}_s / \bar{\lambda}_g \gg 1$ ,  $Le_i$  is the  $i$ -species Lewis number, defined as  $Le_i \equiv \alpha_i / D_i$ , the ratio between the thermal and mass diffusivities,  $Pr \equiv \nu / \alpha$  is the Prandtl number, a ratio between the viscous diffusion rate and the thermal diffusion rate, and  $\eta$  is the modified vertical coordinate. The chemical reaction is accounted through the reaction rate term,  $w_F$  and the non-dimensional heat released,  $q$ , while  $S$  is the non-dimensional stoichiometric coefficient.

The last term in Eqs. (5) and (6) accounts for the heat exchange between solid and gas phases, and is of the order of  $\Gamma \gg 1$ .

The vaporization rate,  $\dot{m}$ , is related with  $f|_0$  through  $-a^{1/2} f|_0 = \dot{m}$ , in which  $a$  is the non-dimensional strain-rate. Below the gas-liquid interface the following governing equations are valid (mass, energy for liquid and for solid, respectively):

$$\varrho_l v_l = \dot{m}, \quad (7)$$

$$\varepsilon J \frac{d^2 \theta_l}{dz^2} - \varepsilon M \frac{d\theta_l}{dz} = -\Gamma^2 n_l (\theta_s - \theta_l), \quad (8)$$

$$(1 - \varepsilon) \frac{d^2 \theta_s}{dz^2} = \Gamma^2 n_l (\theta_s - \theta_l), \quad (9)$$

in which  $J \equiv \bar{\lambda}_l / \bar{\lambda}_s$  and  $M \equiv \dot{m} (c_l / c_p)$ . The heat exchange between solid and liquid is considered to be of the order of  $\Gamma^2 \gg 1$ .

The boundary conditions are given by:

$$\left. \frac{df}{d\eta} \right|_{+\infty} = U_\infty, \quad \theta_g(\eta \rightarrow +\infty) = \theta_s(\eta \rightarrow +\infty) = 1, \quad y_F(\eta \rightarrow +\infty) = y_O(\eta \rightarrow +\infty) - 1 = 0,$$

$$\theta_g(\eta_f) - \theta_f = y_F(\eta_f) = y_O(\eta_f) = 0,$$

$$f(0) - f|_0 = \frac{df}{d\eta}\bigg|_{\eta=0} = 0, \quad \theta_g(0) - \theta_B = \theta_s(0) - \theta_{s0} = 0, \quad y_F(0) - y_{F0} = y_O(0) = 0,$$

$$\theta_l(z \rightarrow -\infty) - \theta_{-\infty} = \theta_s(z \rightarrow -\infty) - \theta_{-\infty} = y_F(z \rightarrow -\infty) - 1 = y_O(z \rightarrow -\infty) = 0, \quad (10)$$

in which  $\eta_f$  is the flame position,  $\theta_f$  is the flame temperature,  $\theta_B$  is the boiling temperature of the liquid fuel,  $\theta_{s0}$  is the solid phase temperature at the interface and  $y_{F0}$  is the fuel mass fraction at the interface.

Mass and energy must be conserved at the interface following:

$$\frac{1}{\Gamma} \frac{1}{Le_F} \frac{dy_F}{d\eta}\bigg|_{\eta=0^+} = (1 - y_{F0}) f|_0, \quad (11)$$

$$\frac{\varepsilon}{\Gamma} \frac{d\theta_g}{d\eta}\bigg|_{\eta=0^+} = -l f|_0 + \varepsilon \frac{J}{a^{1/2}} \frac{d\theta_l}{dz}\bigg|_{z=-z_b} - \Gamma^2 \frac{n_l}{a^{1/2}} \int_{-z_b}^0 (\theta_s - \theta_l) dz. \quad (12)$$

The flame temperature,  $\theta_f$ , is obtained from the matching condition between the zones, and the flame position,  $\eta_f$ , is obtained from the mass conservation at the interface. The solid phase temperature at the interface is obtained by the continuity of the solid phase heat flux and the fuel mass fraction at the interface is obtained through the mass conservation.

### 3 Methodology

The non-dimensional reaction rate term  $w_F$  that appears in Eqs. (3), (4) and (5) is strongly non-linear. This fact imposes difficulties in solving the governing equations.

In order to avoid this strong non-linearity, Eqs. (3), (4) and (5) are combined in such a way that the reaction term is eliminated from the equations. Defining the new variables  $Z \equiv Sy_F - y_O + 1$  and  $H \equiv (S + 1)\theta_g/q + y_F + y_O$  and considering the Lewis number for the oxidant and for the fuel equal of unity, the following equations are found in the gas-solid region:

$$\frac{1}{\Gamma} \frac{d^2 Z}{d\eta^2} + f \frac{dZ}{d\eta} = 0, \quad (13)$$

$$\frac{\varepsilon}{\Gamma} \frac{d^2 H}{d\eta^2} + \varepsilon f \frac{dH}{d\eta} + \Gamma n_g (\theta_s - \theta_g) \frac{(S + 1)}{q} = 0. \quad (14)$$

The new variables are the mixture fraction (Z) and the excess enthalpy (H). This formulation in which the strong non-linear term does not appear is called the Schvab-Ze'ldovich formulation.

In the process of obtaining the governing equations in their non-dimensional form, the large parameter  $\Gamma$  appears. This parameter, the ratio between solid and gas phases thermal conductivities, motivates the use of the perturbation theory in order to seek solutions for the problem.

### 4 Analysis

The proposed problem presents different boundary layers: a macroscopic boundary, own to the viscous flow, a Darcy boundary layer, own to the fluid attachment at the pore walls, and two thermal boundary layers.

The existence of boundary layers in the problem provides the ground for utilizing the method of matched asymptotic expansion [11]. In the gas-solid region three different length scales are observed: outer zone (governed by the Darcy flow and thermal equilibrium between gas and solid), flame zone (where the exothermic reaction consumes oxidant and fuel and releases heat) and inner zone (in which viscous effects arise, thermal non-equilibrium is observed and the diffusion of the vaporized liquid is dominant). The analyzed case is different from "free" impinging flames, as they occur in the inner zone [5], and in the present work the flame is localized in an intermediary zone.

The last term in the left-side of Eq. (14) represents the heat exchange between solid and gas phases, it is possible to observe that due to the high value of this term (of the order of  $\Gamma \gg 1$ ), thermal equilibrium is observed in most part of the gas-solid region. From Equation (14) it is possible to observe the existence of a sink of enthalpy for  $\theta_s > \theta_g$  and of a source of enthalpy for  $\theta_s < \theta_g$ .

From the energy conservation at the interface, given by Equation (12), it is possible to observe that the heat exchange between solid and an liquid phases, expressed by the last term in the right-side of this equation, is the leading order term. Such term controls the vaporization rate of the low volatile liquid fuel. Hence, the addition of the porous medium enhances the vaporization process.

The solid phase temperature at the interface,  $\theta_{s0}$ , that controls this heat exchange between solid and liquid phases, is obtained by imposing the continuity of the heat flux at the interface.

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