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EFFECTS OF THE ATMOSPHERIC DRAG IN CLOSE APPROACHES FOR A CLOUD OF PARTICLES

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The goal of this research is to study close approaches between a planet, which is assumed to have an atmosphere, and a cloud of particles. This cloud of particles is formed during the passage of a spacecraft by the atmosphere, due to its explosion. The complete system is formed by two main bodies (the Sun and the planet), that are assumed to stay in circular orbits around their center of mass, and the spacecraft, that is then transformed in a cloud of particles. This spacecraft is moving under the gravitational attractions of the two main bodies when it makes a close approach with the planet in such a position that it passes inside the atmosphere of the planet and then is transformed in a cloud of particles. The motion is assumed to be planar for the spacecraft and all the particles and the dynamics is given by the well-known planar restricted circular three-body problem plus atmospheric drag. For the simulations shown here the planet Jupiter is used as the body for the close approach, but the method works well for any planet. The initial conditions for the spacecraft and the particles of the cloud are specified at the periapsis, because it is assumed that the fragmentation of the spacecraft occurs at this point.

I. INTRODUCTION

The Swing-By maneuver is a very important topic in astrodynamics. A large number of spacecrafts used this technique to save fuel to complete their missions. The Voyager mission, that has just completed 35 years of success, visited the outer Solar System using successive Swing-Bys to gain energy [1]. Several other important applications exist for this maneuver, like: the use of Swing-Bys with the inner planets to travel to the outer Solar System [2][3][4][5]; the use of Venus to perform a swing-by to make a return visit to Mars [6][7][8]; the use of a non-planar close approach with Jupiter to obtain a plane change to send the spacecraft to an orbital plane perpendicular to the ecliptic to collect data from the poles of the Sun [9]; the use of the Moon to help a spacecraft to escape from the Earth [10]; the use of sucessive Swing-Bys to obtain trajectories linking the primaries of the system and the Lagrangian points [11],[12]; the use of the Moon to keep a desired trajectory for the spacecraft in the Earth-Moon system [13]; the use of multiple Swing-Bys to optimize space trajectories [14],[15]; etc. A more detailed description of this maneuver can be found in references [16] and [17] for the basic maneuver, while reference [18] explains the powered version of the maneuver, reference [19] considers a model that takes into account the eccentricity of the primaries, reference [20] makes a comparison of the "patched-conics" approximation with the restricted problem of three bodies, reference [21] shows for applications of Swing-Bys using the planet Jupiter and reference [22] shows similar applications using the Moon. Regarding similar studies considering a cloud of particles, but without the effects of drag, there are also several studies available in the literature, as shown by references [23] to [26].

The Swing-By is known by the astronomers for more than 150 years. Laplace was one of the first to understand its mechanism. After him, some more researchers obtained analytical equations and/or numerical results for this maneuver, in particular considering the problem of capture and escape of comets by Jupiter (see references [16] for more historical information).

In the present paper, the effects of a planetary atmosphere during a close approach between a cloud of particles and a planet is studied. Only planar motion is considered. The mathematical model is based in the restricted planar circular three-body problem, with the extra forces that come from the atmospheric drag. This cloud of particles is assumed to be formed by the explosion of a spacecraft during the passage by the periapsis. This assumption is based in the idea that this is the instant that the spacecraft has the strongest effect of the atmosphere, since the density of the atmosphere has its largest value.

For each single particle generated by the explosion of the spacecraft, we integrate numerically the equations of motion forward in time, until the particle is at a distance that we can consider it far enough from the planet, such that we can neglect the planet's effect and consider the system formed by the Sun and the particle as a two-body system. At this point we can use the two-body celestial mechanics to compute velocity, energy and angular momentum after the close approach for each particle. From those values, the semi-major axis and the eccentricity of each particle is obtained. Then, the orbit of the spacecraft is integrated backwards in time, as a single body, since it is assumed that the explosion occurred at the periapsis of the trajectory. So, this problem is similar to the standard Swing-By maneuver, where a spacecraft comes from a long distance, passes close to a planet and then leaves it again. The planet is supposed to be in a circular orbit around a central body and any type of orbit is assumed for the spacecraft (elliptic, circular or hyperbolic). At the points A and B the spacecraft (or the particles resulted from the explosion) can be assumed to be far enough from the planet and the system can be modeled as a two-body problem (the spacecraft or one of the particle and the central body). The difference from the standard Swing-By maneuver is that the planet is assumed to have an atmosphere and it generates a drag force in the spacecraft that causes the explosion and also changes the trajectories of the particles. The equations of motion for the spacecraft and the particles are the ones valid for the restricted circular planar three-body problem [27] with the inclusion of the atmospheric drag, similar to what was done for a single particle in reference [28]. The drag force is modeled by the standard form proportional to the square of the velocity and the density of the atmosphere is supposed to vary exponentially with the altitude.

The main objective is to understand the change of the orbit of this cloud of particles after the close approach with the planet. It is assumed that all the particles that belong to the cloud come from a single body that existed before the close approach with the planet. It is desired to known the orbital parameters of the particles after the close approach. Then, the effects are compared with a similar maneuver performed assuming that the atmosphere is not present.

2. DEFINITION OF THE PROBLEM AND MATHEMATICAL MODEL

It is assumed that a spacecraft comes from a long distance, enters the atmosphere of the planet, explodes and the particles that were formed by this explosion leaves the planet. The planet is in a circular orbit around the Sun and the spacecraft can be in an elliptic, circular or hyperbolic orbit. Fig. 1 shows this situation. At the point A the spacecraft is assumed to be far from the planet, such that the system can be considered a two-body problem (spacecraft and the Sun). The planet is assumed to have an atmosphere that generates a drag force. This force changes the orbit of the spacecraft and causes its explosion at the periapsis. From this point, a cloud of particles is formed and the atmosphere changes the form of the cloud. The equations of motion for the spacecraft (and the particles) are the ones used in the restricted planar circular three-body problem plus the atmospheric drag. For the drag force, the standard form that assumes a dependency with the square of the velocity multiplied by the density of the atmosphere is used. The density of the atmosphere is assumed to vary exponentially with the altitude. So, the equations of motion are:

$$\ddot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} + F_x = \frac{\partial \Omega}{\partial x} + F_x \qquad (1)$$

$$\ddot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} + F_y = \frac{\partial \Omega}{\partial y} + F_y \qquad (2)$$

where Ω is given by:

$$\Omega = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \qquad (3)$$

and F_x and F_y are the components of the drag force, given by:

$$\vec{F} = \frac{-C_D A V \rho_0 e^{\frac{-(h-h_0)}{H}}}{2m}\vec{V} \qquad (4)$$

where C_D is the drag coefficient (that depends on the form of the spacecraft or the particle), A is the area of the cross section of the spacecraft, $\vec{V}$ is the velocity of the spacecraft with respect to the atmosphere, m is the mass of the spacecraft, ρ_0 is the density of the atmosphere at an altitude h_0, h is the altitude of the spacecraft, H is a constant that specifies the velocity of the decay of the density with the altitude.

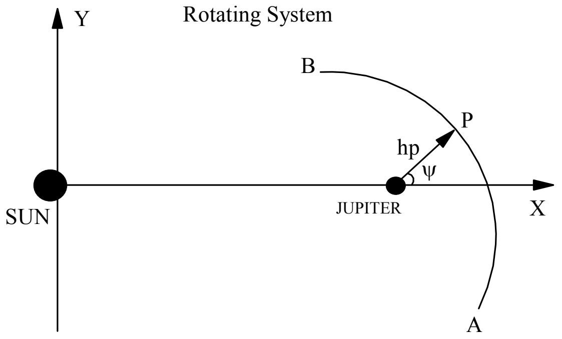

Fig. 1: Geometry of the close encounter.

Another result used in the present paper is the "Jacobian Integral", that is a constant of motion in the restricted three-body problem. It is given by:

$$J = E - \omega C = \frac{\dot{x}^2 + \dot{y}^2}{2} - \frac{x^2 + y^2}{2} - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} \quad (5)$$

where E is energy, C is angular momentum, r_1 is the distance between the Sun and the spacecraft, r_2 is the distance between Jupiter and the spacecraft; ω is the angular velocity of the Sun-Jupiter system. The energy and angular momentum are given by:

$$E = \frac{(x + \dot{y})^2 + (\dot{x} - y)^2}{2} - \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \quad (6)$$

$$C = x^2 + y^2 + x\dot{y} - y\dot{x} \quad (7)$$

where x, y and its time derivatives are the coordinates of the spacecraft measured in the rotating system.

III. PARAMETERS TO SPECIFY ONE TRAJECTORY

Several choices can be made to specify each trajectory. In the present paper the following ones are used [16]:

a) ψ, the angle of approach;

b) J, the Jacobian constant;

c) h_p, the periapsis altitude of the trajectory around Jupiter.

It is necessary to take into account that J is not constant when the drag is included and also that ψ and h_p are difficult to define precisely. So, the method used to compare those trajectories (with and without drag) is:

i) For the trajectory without drag, the numerical integration of the trajectory starts at periapsis and is performed backward in time until the point A (Fig. 1) is reached. Then, the calculations before the close approach in made at this point for the spacecraft. For the part of the trajectory after the passage by the periapsis, a cloud of particles are mounted by applying small changes to the variables ψ and h_p that belongs to the cloud. It means that all the particles will have the values $\psi \pm \delta\psi$ and $h_p \pm \delta h_p$ for those two variables. The Jacobian constant is assumed to be the same for all the particles. Then the same procedure is used and numerical integrations are performed in forward time for each particle;

ii) The maneuver including drag is similar. The numerical integration starts at periapsis, keeping the same three parameters (J, ψ, h_p) of the maneuver without drag. The trajectory is them integrated backward in time for the spacecraft including the drag force in the dynamics, until the point A is reached. For the particles, the numerical integration is performed forward in time for each particle with the drag force active until the point B is reached.

The quantities calculated to measure the influence of the atmosphere are the energy, angular momentum, semi-major axis, eccentricity and Jacobian constant before and after the Swing-By. This is done with and without the drag force, so it is possible to identify its influence. The results are presented in plots showing the semi-major axis and eccentricity of the cloud of particles after the passage by the atmosphere and the same values for the spacecraft before the passage.

IV. RESULTS

The results are presented in plots showing the semi-major axis and eccentricity of the cloud of particles after the passage by the atmosphere and the same values for the spacecraft before the passage. Two different values were used for the Jacobian constant J: 0 and 1. This is equivalent of using two values for the velocity at the periapsis. Eight values were used for the angle of approach ψ: 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°. Figure 2 shows the results. Blue represents that particles for the maneuver where the drag is included and black represents the situation where drag is not included.

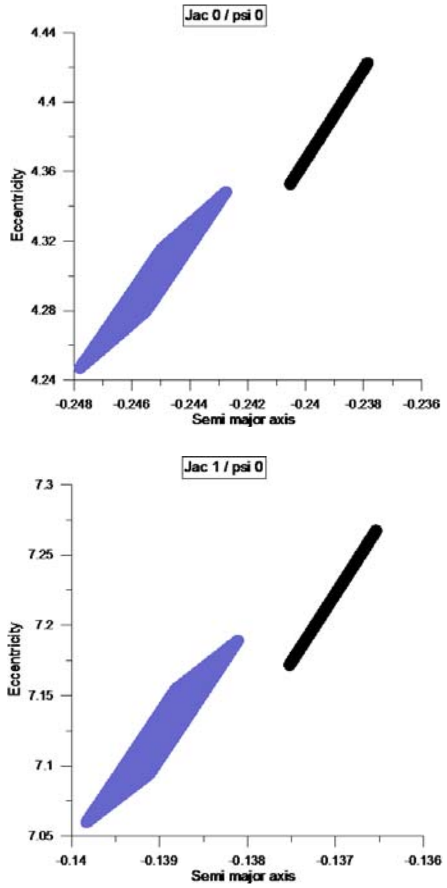

Fig. 2 - Semi major axis and eccentricity for the particles after the close approach.

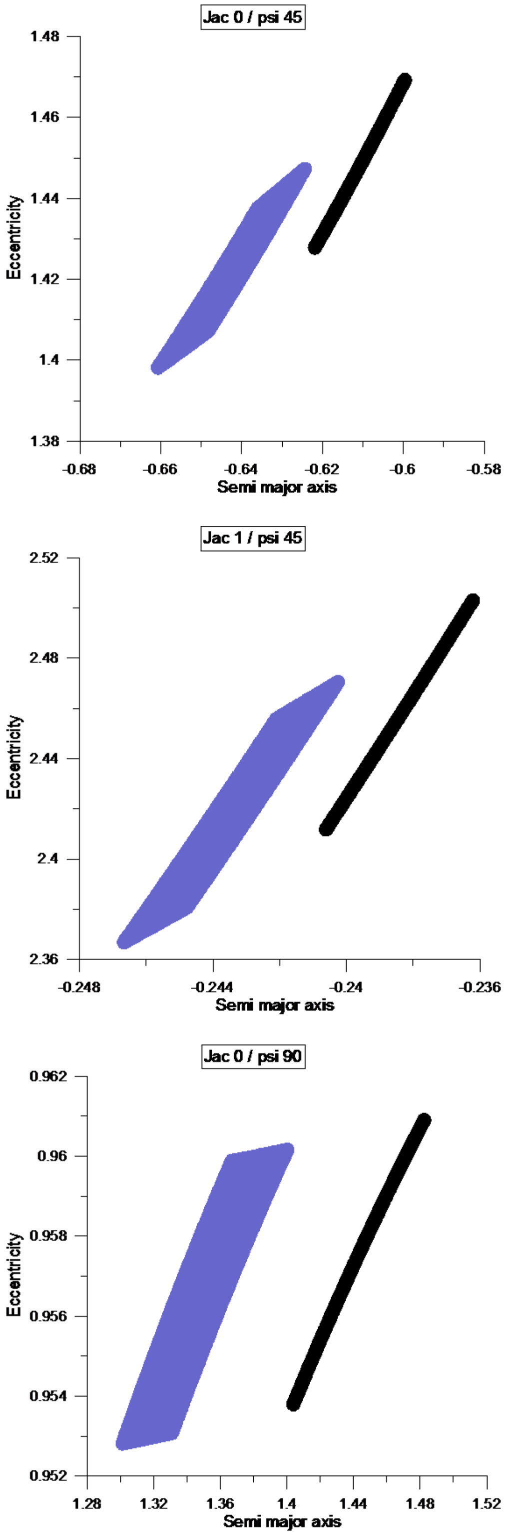

Fig. 2 - Semi major axis and eccentricity for the particles after the close approach (cont.).

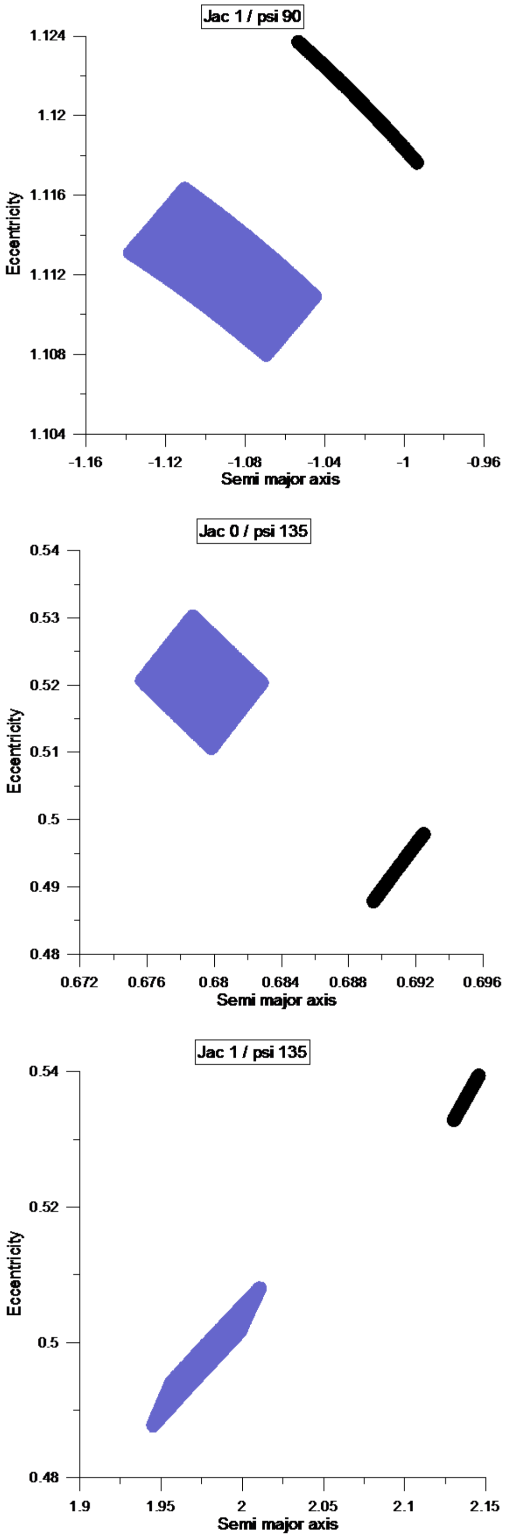

Fig. 2 - Semi major axis and eccentricity for the particles after the close approach (cont.).

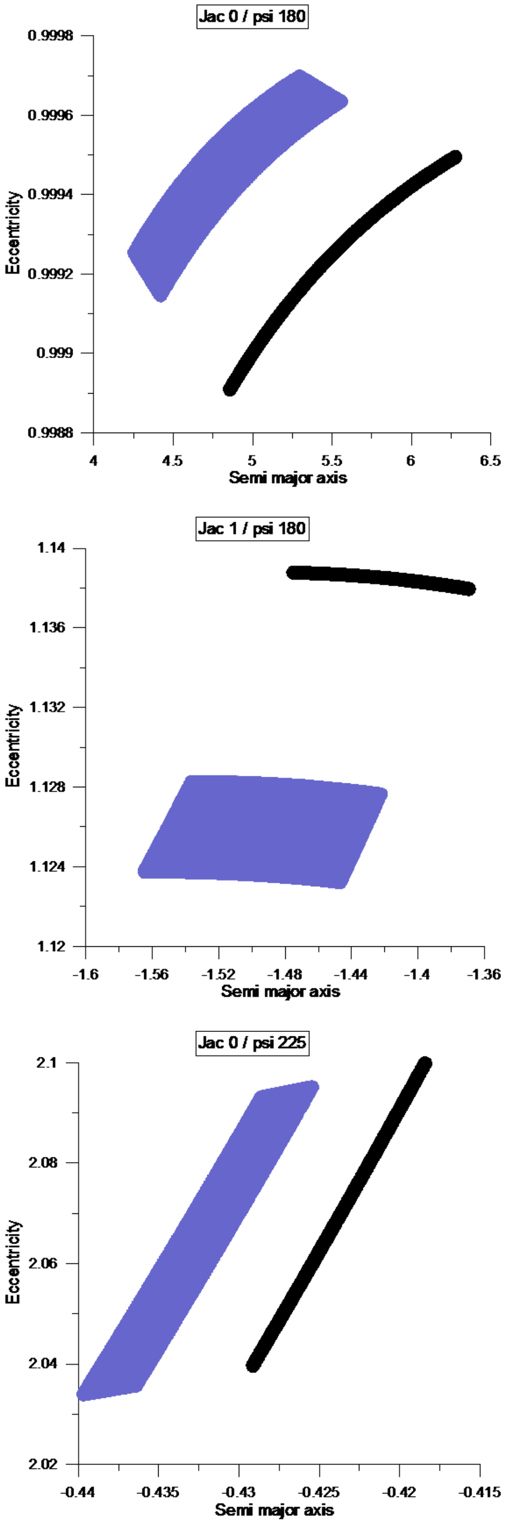

Fig. 2 - Semi major axis and eccentricity for the particles after the close approach (cont.).

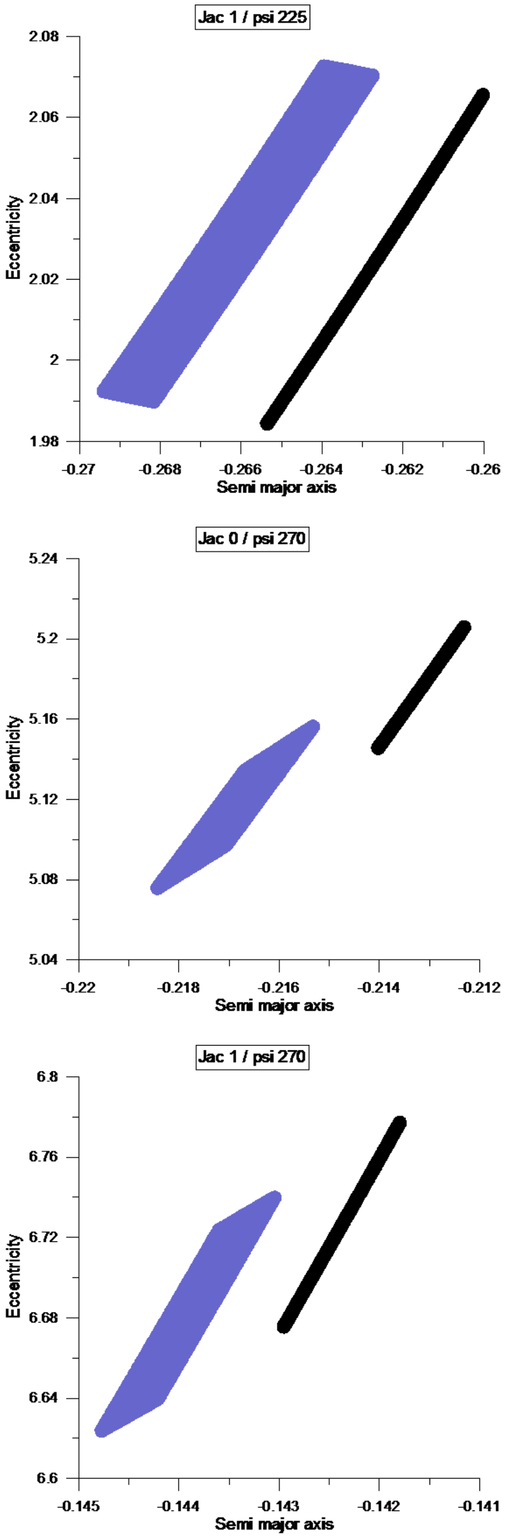

Fig. 2 - Semi major axis and eccentricity for the particles after the close approach (cont.).

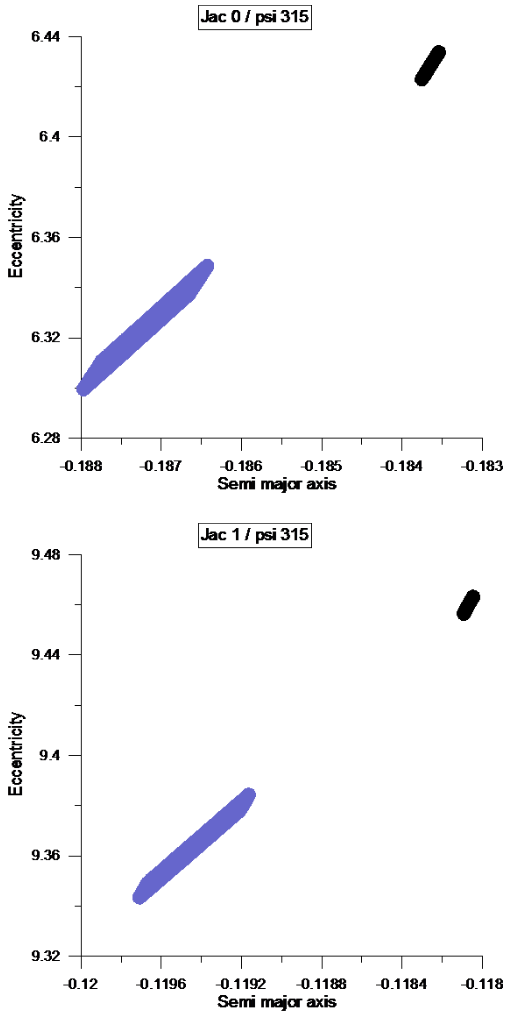

Fig. 2 - Semi major axis and eccentricity for the particles after the close approach (cont.).

There are several observations that can be made from those results. First of all it is clear that the presence of the atmospheric drag causes a larger dispersion of the particles in all the situations simulated. A figure (in black) close to a straight line becomes something near a parallelogram (in blue). Considering now every specific case, it is noticed that the maneuvers for the case where ψ = 0° is a situation where the Swing-By itself does not change the energy. The orbits of the spacecraft and the particles are all hyperbolic. The inclusion of the atmospheric drag causes an extra loss of energy, as expected. It also shows that, when the Jacobian constant increases (and so the velocity) the changes in the orbit of the spacecraft decreases. This can be explained by the fact that the spacecraft and the particles remains more time inside the atmosphere if the velocity is smaller. For the case where ψ = 45°, it is a maneuver where the Swing-By reduces the energy of the particle. The orbits of the spacecraft and the particles are also hyperbolics. The effects in the orbits are smaller when the velocity is higher, as explained before. The situation ψ = 90° represents the maneuver where the Swing-By produces the maximum loss of the energy of the particle. The orbits with Jacobian constant equal to zero represents capture trajectories, because the particle has a hyperbolic orbit before the close approach and an elliptical orbit after that. For the case where the Jacobian constant is unity, all the orbits are hyperbolic. The next situation is where ψ = 135°. It also represents Swing-Bys that produces loss of the energy of the particle. Both values of the Jacobian constant represent capture trajectories. The situation where ψ = 180° again represents a situation where the Swing-By itself does not change the orbit of the particle, as well as in the case ψ = 0°. Then comes the situation where ψ = 225°. It represents Swing-Bys that produces gain in the energy of the particle. Both values of the Jacobian constant represent escape trajectories, where the particles go from a closed trajectory to an open trajectory. The situation ψ = 270° represents the maneuvers that produces the maximum gain in the energy of the particle. The last simulations considered ψ = 315° that also represents the maneuvers with gains in the energy of the particle. All the trajectories in this case are hyperbolic orbits.

V. CONCLUSIONS

A numerical algorithm to calculate the effects of the atmosphere in a Swing-By maneuver with Jupiter is developed. It allows us to quantify expected results due to the inclusion of the drag and to make predictions for other situations. Then this algorithm was applied to a spacecraft that exploded when passing by the periapsis of its orbit around the planet. The results show several conclusions about the behavior of the particles, indicating the regions where captures and escapes occurs, as well as the effects of the atmospheric drag in the motion of the particles.

VI. ACKNOWLEDGMENTS

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VII. REFERENCES

1. C. E. Kohlhase and P. A. Penzo, Voyager Mission Description. *Space Science Reviews*, 21(2), 77-101, 1977.

2. D. V. Byrnes, and L. A. D'Amario, A Combined Halley Flyby Galileo Mission. AIAA paper 82-1462. In: AIAA/AAS Astrodynamics Conference, San Diego, CA, Aug. 1982.

3. L. A. D'Amario and D. V. Byrnes, Interplanetary Trajectory Design for the Galileo Mission. AIAA paper 83-0099. In: AIAA 21st Aerospace Sciences Meeting, Reno, NV, 10-13 Jan., 1983.

4. S. S. Weinstein, Pluto Flyby Mission Design Concepts for Very Small and Moderate Spacecraft. AIAA paper 92-4372. In: AIAA/AAS Astrodynamics Conference, Hilton Head, SC, Aug. 10-12, 1992.

5. B. L. Swenson, Neptune Atmospheric Probe Mission. AIAA paper 92-4371. In: AIAA/AAS Astrodynamics Conference, Hilton Head, SC, Aug. 10-12, 1992.

6. V. G. Szebehely, Special Orbits for the Exploration of Mars and Venus. In: AFCRL-NASA-VPI Conference on The Exploration of Mars and Venus, Virginia Polytechnic Institute, Blacksburg, VA, 1965.

7. W. M. Hollister and J. E. Prussing, Optimum Transfer to Mars Via Venus. *Astronautica Acta*, 12(2), 169-179, 1966.

8. S. A. Striepe and R. D. Braun, Effects of a Venus Swingby Periapsis Burn During an Earth-Mars Trajectory. *The Journal of the Astronautical Sciences*, 39(3), 299-312, 1991.

9. R. Carvell, Ulysses-The Sun From Above and Below. *Space*, 1, 18-55, 1986.

10. K. T. Nock and C. W. Upholf, Satellite Aided Orbit Capture. AAS/AIAA paper 79-165, 1979.

11. Prado, A. F. B. A. and Broucke, R. A.. Transfer Orbits in the Earth-Moon System Using a Regularized Model. Journal of Guidance, Control and Dynamics, 19(4):929-933, 1996.

12. Prado A.F.B.A. and Broucke, R. A. Transfer Orbits in Restricted Problem. Journal of Guidance Control and Dynamics, 18(3):593-598, 1995.

13. Marsh, S.M.; Howell, K.C.: "Double Lunar Swingby Trajectory Design". AIAA paper 88-4289.

14. D'Amario, L.A.; Byrnes, D.V.; Sacett, L.L.; Stanford, R.H.: "Optimization of Multiple Flyby Trajectories". AAS paper 79-162. In: AAS/AIAA Astrodynamics Specialists Conference, Provincetown, MA, June 1979.

15. D'Amario, L.A.; Byrnes, D.V.; Stanford, R.H.: "A New Method for Optimizing Multiple-Flyby Trajectories". *Journal of Guidance, Control, and Dynamics*, 4(6), 591-596, 1981.

16. Broucke, R.A.: "The Celestial Mechanics of Gravity Assist". AIAA paper 88-4220. In: AIAA/AAS Astrodynamics Conference, Minneapolis, MN, 15-17 Aug. 1988.

17. Prado, A.F.B.A.: "Optimal Transfer and Swing-By Orbits in the Two- and Three-Body Problems". Ph.D. Dissertation, University of Texas at Austin, Austin, Texas, Dec. 1993.

18. Prado, A. F. B. A.. Powered Swingy. Journal of Guidance, Control and Dynamics, 19(5):1142-1147, 1997.

19. Prado, A.F.B.A., Close-Approach Trajectories in the Elliptic Restricted Problem". Journal of Guidance, Control and Dynamics, 20(4):797-802, 1997.

20. Prado, A.F.B.A.. A Comparison of the Patched-Conics Approach and the Restricted Problem for Swing-bys. Advances in Space Research, 40:113-117, 2007.

21. Broucke, R.A.; Prado, A.F.B.A.: "A Classification of the Swing-By Trajectories Passing Near Jupiter". AAS 93-177. In: AAS/AIAA Space Flight Mechanics Meeting", Pasadena, California, Feb. 22-24 1993.

22. Prado, AFBA ; Broucke, Roger A . A Classification of Swing-By Trajectories Using the Moon. Applied Mechanics Reviews, 48(11), 138-142, 1995.

23. Gomes, V.M. and Prado, A.F.B.A.. Swing-by Maneuvers for a Cloud of Particles with Planets of the Solar System. WSEAS Transactions on Applied and Theoretical Mechanics, 03(11):869-878, 2008.

24. Gomes, V.M. and Prado, A.F.B.A.. A Study of the Impact of the Initial Energy in a Close Approach of a Cloud of Particles. WSEAS Transactions on Mathematics, 9:811-820, 2010.

25. Prado, A.F.B.A. . A study of the dispersion of a cloud of particles due to a close approach. Advances in Space Research, 36(3), 585-589, 2005.

26. Prado, AFBA . A Study of the Close Approach Between a Planet and a Cloud of Particles. Advances in the Astronautical Sciences, San Diego, Estados Unidos, 109(3), 1941-1958, 2002.

27. Szebehely, V.G. *Theory of Orbits*, Academic Press, New York, 1967.

28. Prado, A.F.B.A. and Broucke, R.A.. Effects of Atmospheric Drag in Swing-By Trajectory. Acta Astronautica, 36(6):285-290, 1995.