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THE FAULT DIAGNOSIS INVERSE PROBLEM: ACO AND ACO-d

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Abstract. *The automatic early detection, isolation, and localization of faults is of high interest in industrial systems, for improving reliability and safety. This process is characterized as fault diagnosis (FDI) and some problems related with robustness to external disturbances, sensitivity to incipient faults and processing time are still considered as limitations for many of the current FDI methods. This work is focused on the formulation of the fault diagnosis by an inverse problem methodology. The FDI problem is formulated as an optimization problem and takes results from the diagnosis area for acquiring prior information. The optimization problem is solved by the stochastic algorithm Ant Colony Optimization (ACO) and its modified version fuzzy-ACO (ACO-d). The proposed approach is tested using simulated data of the Inverted-Pendulum system which is recognized as a benchmarck for control and diagnosis. With the purpose of analyzing the advantages of such approach, some experiments, with data corrupted with noise, are considered. The influence of ACO parameters are also taken in consideration. The results obtained show the suitability of the approach and also indicate that the parameters values allowing a greater exploration of the search space yields a better diagnosis. The ACO-d algorithm enables better diagnosis than ACO.*

Keywords. *Ant Colony Optimization, fault diagnosis, inverse problem, processing time, robustness, structural detectability, structural separability*

1 INTRODUCTION

The automatic early detection, isolation and localization of faults that have an effect on industrial systems are of high interest in order to improve reliability, safety and efficiency (Isermann, 2005). This process is called Fault Diagnosis or Fault Detection and Isolation (FDI) (Simani et al., 2002).

The increasing complexity of the systems causes an increase in the probability of failure. As a consequence, the FDI gain more importance and many methods for that purpose have been developed since the early 1970s (Isermann, 2005; Simani et al., 2002).

The FDI methods should guarantee the fast detection of the fault while rejecting false alarms attributable to noise, external disturbances and spurious signal. The first characteristic is named sensitivity and the second one is called robustness. An adequate balance of these properties is the key for the practical applicability of the FDI methods (Isermann, 2005; Simani et al., 2002) and it is still considered as a limitation of the currents FDI methods (Simani et al., 2002; Simani & Patton, 2008).

The methods for Fault Diagnosis are separated in three general groups: those which do not use a model of the process, those which do use a qualitative model of the process and those that are based on a quantitative model (Angeli & Chatzinikolaou, 2004; Venkatasubramanian et al., 2003a,b,c).

The model-based approaches using the quantitative analytical model allow a deep insight into the process behavior (Isermann, 2005) and can be brought down to a few basic types such as: the parity space; observer approach; the fault detection filter approach and the parameter identification or estimation approach. The parameter estimation approach is based on the diagnosis of the faults via estimation of the parameters of the mathematical model (Frank, 1990; Isermann, 1984, 2005; Patton et al., 2000) and it is required the knowlegde of the relationship between such parameters and the physical coefficients of the system, as well the influence of the faults in these coefficients (Frank, 1990; Isermann, 2005).

In the particular case of the parameter estimation based methods, there is an additional inconvenient: the high processing time makes them almost unfeasible for most online applications (Frank, 1990; Isermann, 1993).

Despite the fact that the FDI problem is an inverse problem: based on an observed behavior of the system, the causes (faults) that produced this effect should be determined, this approach has not been intensively used. Just some recent incursions have been reported in that sense (Witczak, 2006; Yang et al., 2007). With the aim of developing new and viable FDI methods and taking into account this similitude, this work presents the formulation of the fault diagnosis as an inverse problem which is written as an optimization problem and solved with stochastic algorithms. Some results related with detectability and separability are applied in order to obtain more prior information of the inverse problem and as a consequence improving the quality and reliability of the diagnosis.

The stochastic algorithm Ant Colony Optimization (ACO) and its modified version ACO-fuzzy (ACO-d) have been applied for obtaining the solution of the optimization problem. This selection is based on the adaptable and robust performance of ACO in other optimization problems as well as the better performance reported for ACO-d in order to avoid local optima based on a more intensive exploration of the search space (Becceneri et al., 2008).

Our proposal is illustrated using simulated data of the Inverted-Pendulum System (IPS) which is widely recognized as a benchmark for control and diagnosis problems. With the purpose of analyzing the advantages of such approach, mainly with respect to robustness, sensitivity and processing time, some experiments with noisy data are considered. With the aim of analyzing the influence of some ACO and ACO-d parameters in the quality of the diagnosis, different sets of values for such parameters are taken in consideration.

The main contributions of this paper may be summarized as follows: the study of a new approach for the development of robust and sensitive FDI methods based on direct fault estimation and its formulation as an inverse problem; the combination of this approach with some results for diagnosis based on the structure of the model; and a comparison between the direct fault estimation with the stochastic algorithm ACO and is modified version ACO-d. The viability of the proposal is demonstrated by diagnosing simulated fault data of the IPS.

This work is organized as follows. In the second section the modeling of faults and the model-based FDI methods with an inverse problem formulation are introduced. After that, the third section details the study case, IPS, and the simulation results. The following section describes the application of some reported results for getting prior information of the system. The fifth section provides a brief description of the algorithms ACO and ACO-d. The Results section shows the application of the methodology proposed to the solution of the study case and the results obtained. Finally some concluding remarks are presented.

2 MODELING FAULTS AND FORMULATION OF THE FDI INVERSE PROBLEM

Let's consider the following process model

$$\begin{aligned} \dot{\mathbf{x}}(t) &= f(\mathbf{x}(t), \mathbf{u}(t), \theta) \\ \mathbf{y}(t) &= g(\mathbf{x}(t), \theta) \end{aligned} \tag{1}$$

that represents as close as possible the physical laws which govern the process behavior (Isermann, 2005). The vector of state variables is represented by $\mathbf{x}(t) \in \mathbb{R}^n$. The measurable input signals $\mathbf{u}(t) \in \mathbb{R}^m$ and output signals $\mathbf{y}(t) \in \mathbb{R}^l$ can be directly obtained by the use of physical sensors; $\theta_p \in \mathbb{R}^j$ is the process parameters vector and determines the model parameters vector $\theta = [\theta_p]^t$.

The components of the process parameters vector are identified with the components of the physical process coefficients vector $\rho \in \mathbb{R}^r$, and in general $r \neq j$. The variations of these coefficients are generally related with faults. The estimations of the vector θ_p will allow to detect the faults once the relationships between $\theta_p - \rho$ and $\rho -$ faults are established (Isermann, 1984). This divides the diagnosis into two steps, the first one considers the estimations of the parameters vector θ_p, permitting the detection; and the second includes the determination of the faults based on the mentioned relationships. If $j \leq r$ the relationship between process parameters and physical coefficients will be not one to one and as consequence some faults will be not separable (Isermann, 1984, 2005).

For estimating θ_p, two main approaches have been considered: minimize the equation error or minimize the output error. The first one permits the use of the least squares estimator and it is also necessary the use of the derivatives of the input and output data vector as well the use of filters for improving the numerical properties. In the second case numerical optimization is necessary, and the resulting high computational time brings difficulties in the applications for real on-line processes (Isermann, 2005). Some applications of evolutionary algorithms and neural networks have been reported in that sense (Witczak, 2006; Yang et al., 2007).

In order to avoid the described problem of the FDI based on the parameters estimation we have considered the model that also includes the faults. In this case the model in Eq. (1) considers that the influence of the faults is absolutely represented by the fault parameters vector θ_f being $\theta = [\theta_p \;\; \theta_f]^t$. This vector θ_f contains the information regarding magnitude of each fault that can affect the system. That is the reason why the estimations of the vector θ_f will allow diagnosing directly the system.

The modelation of faults in a state-space representation of a Linear Time Invariants Systems (LTI), (Ding, 2008), permits to incorporate additive and multiplicative faults, while allows modelling the faults in the three main parts of the system: actuator, process and sensors. Let's $f_a \in \mathbb{R}^s$, $f_p \in \mathbb{R}^q$, $f_s \in \mathbb{R}^p$ be the vectors containing the additive faults in the actuator, process and sensors respectively. Making $\theta_f = [f_a \; f_p \; f_s]$ and introducing it in the state-space LTI model of the system we obtain

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t) + E_f \theta_f \\ \mathbf{y}(t) &= C\mathbf{x}(t) + F_f \theta_f \end{aligned} \tag{2}$$

The matrices A, B and C are known with appropiate dimensions, and the matrices E_f, F_f are:

$$E_f = [B \; Ep_{n\times q} \; 0_{n\times p}] \qquad F_f = [0_{l\times(s+q)} \; I_{l\times p}] \tag{3}$$

where the matrix Ep represents the influence of the process faults and I represents the identical matrix.

This kind of faults modelling has been widely used for the residual generation in other FDI model based methods such as parity space and observer approach (Ding, 2008; Frank, 1990; Simani et al., 2002), but not in the case of the methods based on parameters estimation.

Considering the process parameters vector θ_p to be constant, the FDI inverse problem can be established as estimating the vector θ_f. It can be obtained from the solution of the parameter estimation inverse problem that can be formulated as

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a minimization problem:

$$\begin{array}{ll} \min & F(\hat{\theta}_f) = \sum_{t=1}^{N_s} \left[\mathbf{y}_t(\theta_f) - \hat{\mathbf{y}}_t(\hat{\theta}_f)\right]^2 \\ \text{s.a} & \theta_{f(min)} \leq \hat{\theta}_f \leq \theta_{f(max)} \end{array} \tag{4}$$

where N_s is the number of sampling instants, $\hat{\mathbf{y}}_t(\hat{\theta}_f)$ is the estimated vector output at each time instant t, and it is obtained from the model given by the system of equations (2); $\mathbf{y}_t(\theta_f)$ is the output vector measured by the sensors at the same instant t (Isermann, 2005).

For the solution of the optimization problem that was specified in Eq. (4), even in a noisy environment and for both linear or non linear problems, stochastic algorithms can be applied. In the present work the ACO and ACO-d are implemented.

The idea behind the application of ACO and ACO-d is to perform a robust diagnosis of the system, via direct fault estimation, with an acceptable computational effort, which makes it feasible for the on-line diagnosis and also avoiding to divide the diagnosis in two steps as required by the usual FDI parameter estimation methods.

3 STUDY CASE: INVERTED-PENDULUM SYSTEM (IPS)

This system is considered as a benchmark for control and diagnosis. It is formed by an inverted pendulum mounted on a motor-driven car. The objective is to keep the beam aligned with the vertical position. Here it has been considered only the two -dimensional problem where the pendulum moves only in the plane of the paper, see Fig. 1.

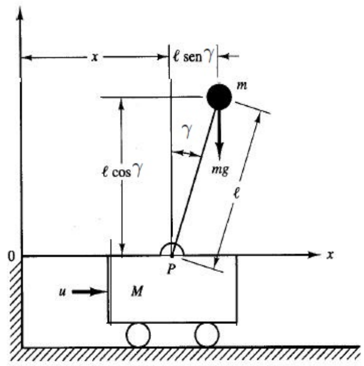

Figure 1: Inverted- Pendulum System

The mathematical model of the IPS has been widely studied, see (Ding, 2008). The system is described by a state-space representation of a linear time invariant system, affected by additive faults, see the system of equations (2). The state vector is $\mathbf{x} = [\gamma\, \dot{\gamma}\, x\, \dot{x}]^t$, where γ and $\dot{\gamma}$ are the angle of the pendulum with respect to the vertical position and the angular velocity respectively; x and $\dot{x}$ are the position and the velocity of the car respectively. The outputs of the system are $\mathbf{y} = [\gamma\, x]^t$ and the input $\mathbf{u}(t) = F$ is the control force applied to the car. The relationship between each element of the fault vector $\theta_f = [f_1\ f_2\ f_3]^t$ and the faults of the system is one to one: f_1 causes undesired movement of the car taking place in the actuator, this kind of fault is represented by an additive fault affecting the system input F; f_2 represents a fault in the sensor of γ and f_3 identifies faults in the sensor that measures x. The matrices A, B, C, E_f, F_f are known and with appropiate dimensions:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{m+M}{Ml} g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M} g & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ -\frac{1}{M} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E_f = [B\ 0_{4\times 2}] \quad F_f = [0_{1\times 2}\ I_{2\times 2}]$$

Considering the system with the characteristics $M = 2$ kg, $m = 0.1$ kg and $l = 0.5$ m, the following matrices are obtained:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E_f = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \end{bmatrix} \quad F_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Considering the nature of the faults and the properties of the IPS, then the elements of θ_f have the following restric-

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tions:

$$\begin{array}{lll} \theta_{f1} \in \mathbb{R}: & -0.5 \leq \theta_{f1} \leq 0.5 & \text{N} \\ \theta_{f2} \in \mathbb{R}: & 0 \leq \theta_{f2} \leq 0.01 & \text{rad} \\ \theta_{f3} \in \mathbb{R}: & 0 \leq \theta_{f3} \leq 0.02 & \text{m} \end{array}$$

In order to make a direct diagnosis of the system we must obtain estimates for θ_f. In that sense the inverse problem of FDI that was formulated in Eq.(4) should be solved.

3.1 Data simulation

The behavior of the system was simulated for free of faults and under different faulty situations. The direct problem given by the system of equations in (2), was numerically solved with the fourth order Runge Kutta method. In Figs. 2 and 3 are shown two different situations that were simulated.

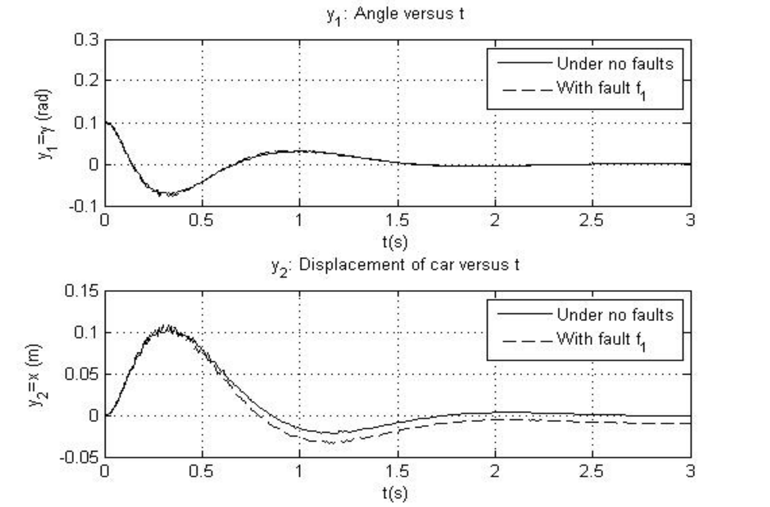

Figure 2: Simulation with no faults and fault f_1, corrupted with 5 % level noise

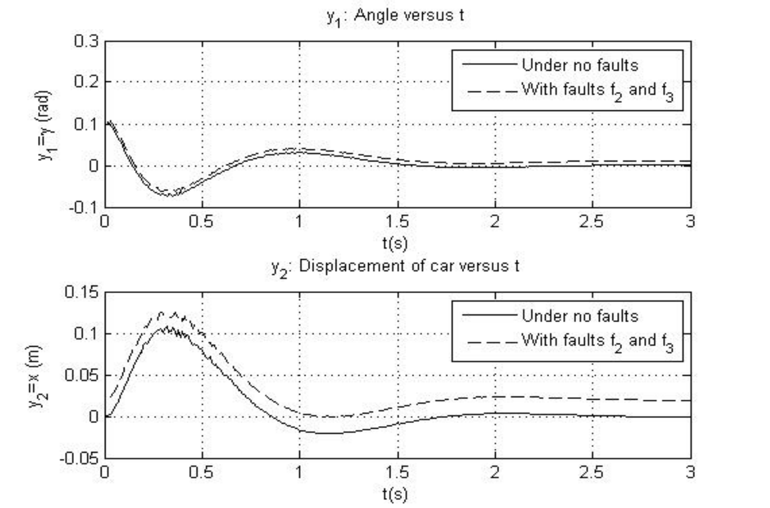

Figure 3: Simulation with no faults and faults f_2 and f_3, corrupted with 5 % level noise

4 INVESTIGATION OF THE PROBLEM STRUCTURE

The diagnosis includes the detectability, isolability and causes of the faults. In the case of analytical model- based FDI methods, we can distinguish between the influence of the model in these characteristics, as well the way that the method used for diagnosing permits dealing with the balance between sensitivity and robustness (Ding, 2008).

For obtaining some prior information about the uniqueness, or not, of the set of fault parameters values that can justify the observed behavior of the system, some results related with sensor placement for faults detectability and separability are applied (Aslund & Frisk, 2008; Krysander & Frisk, 2008).

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The detectability of a fault indicates if the effect of the fault in the system can be monitored. The isolability of a fault is related with the separability of the fault from the other faults that may be eventually affecting the system. Some recent papers have shown that some information concerning this topic can be be extracted from the structural representation of the model (Aslund & Frisk, 2008; Krysander & Frisk, 2008). These results are based on the description of the model as a bipartite graph and its Dulmage- Mendelsohn Decomposition (Krysander & Frisk, 2008).

Let's denote $E=\{e_1,e_2,...e_h\}$, with $h\geq(n+m)$, the set of equations in the model of the system, and $V=\{x_1,x_2..x_k\}$, with $k\geq n$, the set of variables. Let's also asume that each fault f_l only affects one equation e_{f_l}. Let's construct the biadjacency matrix M of the bipartite graph $G=(E,V)$ that represents the structural information of the model formed by the equations of E and the variables of V, whose elements are:

$$m_{ij}=\begin{cases}1 & \text{if} \quad x_j\vee\dot{x}_j\in e_i\\ 0 & \text{otherwise}\end{cases} \tag{5}$$

Let' s be M^+ the overdetermined part of the model M, the fault f_l is structurally detectable if there exists an observation that is consistent with fault mode f_l and inconsistent with the non fault mode, in other words the fault f_l can violate a monitorable part of the model: $e_{fl}\in M^+$.

On the other hand a fault f_l, isolable from f_j, can violate a monitorable equation in the model describing the behavior of the process having fault f_j. This motivates the definition that f_l is structurally separable from f_j in a model M if $e_{fl}\in(M\backslash e_{fj})^+$.

The Dulmage-Mendelsohn decomposition (Krysander & Frisk, 2008) allows partitionating the model M in three parts, M_0 the structurally undetermined part, $\bigcup_{i=1}^{n}M_i$ the just-determined part and M^+ the structurally overdeterminaded part. Not all the parts may be present in a given model. In graph theory terms the Dulmage-Mendelsohn decomposition finds a maximum-size matching in the bipartite graph of M. This has been applied for determining the detectability and isolability properties of a model, as well the minimum number of sensors to be placed in order to achieve the detectability and isolability requeriments (Aslund & Frisk, 2008).

These results will be adapted and applied for the determination of some information about the structure of the system to be diagnosed. This information will permit to decide when the diagnosis can satisfy detectability and isolability requirements, which leads to the possibility of studying the more appropriate method for satisfying robustness and sensitivity conditions. This will be exemplified with the IPS.

Let's consider the model of the IPS when no sensors are added, therefore only a fault $f_1=\theta_{f1}$ is affecting the system. The model of the system in this case is:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= 20.601\,x_2-u-\theta_{f1}\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= -0.4905\,x_1+0.5\,u+0.5\,\theta_{f1}\end{aligned} \tag{6}$$

With the aim of satisfying the requirement that a fault affects only one equation, the new variable x_5 and the new equation $e_5: x_5=\theta_{f1}$ are introduced. The equation affected by the fault f_1 is $e_{f1}=e_5$. The biadjacency matrix of the system has the form:

$$\begin{array}{c} \\ e_1\\ e_2\\ e_3\\ e_4\\ e_5\end{array}\begin{array}{c} \begin{matrix}x_1 & x_2 & x_3 & x_4 & x_5\end{matrix}\\ \begin{pmatrix}1&1&0&0&0\\1&1&0&0&1\\0&0&1&1&0\\1&0&0&1&1\\0&0&0&0&1\end{pmatrix}\end{array} \tag{7}$$

From matrix (7), that represents the model M described by the equations in (6), is obtained its Dulmage- Mendelsohn decomposition. This is made with the function dmperm of MATLAB. The result is shown in matrix (8), and it can be seen that $\bigcup_{i=1}^{n}M_i=M$, in other words, the actuator fault f_1 can not be detected without sensors.

$$\begin{array}{c} \\ e_3\\ e_4\\ e_1\\ e_2\\ e_5\end{array}\begin{array}{c} \begin{matrix}x_3 & x_4 & x_1 & x_2 & x_5\end{matrix}\\ \begin{pmatrix}1&1&0&0&0\\0&1&1&0&0\\0&0&1&1&0\\0&0&1&1&1\\0&0&0&0&1\end{pmatrix}\end{array} \tag{8}$$

The Dulmage- Mendelsohn decomposition also gives the order between the connected or strongly connected components of the graph with the biadjacency matrix M. Such order permits to know what variables should be measured in order to obtain that a certain equation e_h forms part of M^+. In this case as a consequence of this order, the order between the variables is:

$$x_3,e_3>x_4,e_4>x_1,e_1>x_2,e_2>x_5,e_5 \tag{9}$$

This order indicates that the measurements of any of the variables $x_1, x_2, ...x_5$ make the equation $e_{f_1} = e_5$ be part of M^+, in other words, make the fault f_1 be structurally detectable. Let's introduce a sensor for measuring the variable x_3, the position of the car. This is described in the model by adding a new equation $e_6 : y_2 = x_3$. The relationship expressed in (9) indicates that the monitorable part of the new model $M \bigcup e_6$ is $(M \bigcup e_6)^+ = M \bigcup e_6$, then $e_{f1} \in (M \bigcup e_6)^+$ and as consequence, f_1 is detectable.

But now let's suppose that the sensor of x_3 can be affected by an additive fault f_3, then the equation $e_6 : y_2 = x_3 + f_3$ is affected by this fault. The fault f_3 is detectable with no more sensors because of $e_6 = e_{f3} \in (M \bigcup e_6)^+$. The other question is related with the separability of the faults f_1 and f_3. Based on the definition of faults structurally separable, f_1 will be separated from f_3 if $e_{f1} \in ((M \bigcup e_6) \backslash e_{f3})^+$ which produces the same problem as described in (7-9), which means that another sensor to measure any of the variables is necessary in order to separate f_1 from f_3. Let's add such a sensor measuring the variable x_1, the angle γ, with equation $e_7 : y_1 = x_1$, now $e_{f1} \in (M \bigcup e_7)^+ = e_1 \bigcup e_2 \bigcup e_5 \bigcup e_7$, see the Dulmage- Mendelsohn decomposition of the matrix $M \bigcup e_7$ in matrix (10),

$$\begin{array}{c} \\ e_3 \\ e_4 \\ e_1 \\ e_2 \\ e_7 \end{array} \begin{array}{c} \begin{array}{ccccc} x_3 & x_4 & x_1 & x_2 & x_5 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array} \tag{10}$$

It is observed that with the two added sensors it is possible to achieve detectability and separability of the faults f_1 and f_3. It means that the estimation of those faults will be possible. Assuming that the new sensor $y = x_1$ can also be affected by an additive fault f_2 we have $e_{f2} = e_7 : y_1 = x_1 + f_2$. This fault is detectable as a consequence of $e_{f2} \in (M \bigcup e_6 \bigcup e_7)^+$, but know the cases for analyzing separability have been increased with the appearing of five new situations: f_1 separable from f_2; f_2 separable from f_3, f_1 separable from f_2 and f_3 simultaneously, f_2 separable from f_1 and f_3 simultaneously and f_3 separable from f_1 and f_2 simultaneously.

From the previous analysis it can seen that f_1 is separable from f_2, and f_2 separable from f_3. For situations in which only two faults are affecting the system, the faults can be separated, which means that the faults can be estimated. For the cases when the three faults are affecting the system we can not achieve separability with the considered sensors. In all the cases new sensors need to be added, which, considering that the sensor can be affected by faults, means that the problem of separability can not be solved adding more sensors if their measurements are not free of faults.

Let'a show the case when considering the separability between the fault f_1 and the faults f_2, f_3 affecting the system at the same time. By the definition of structurally separable, f_1 is separable from f_2, f_3 if $e_{f_1} \in ((M \bigcup \{e_6, e_7\}) \setminus \{e_{f3}, e_{f3}\})^+$. This means that the we have returned to the initial problem, $M = (M \bigcup \{e_6, e_7\}) \setminus \{e_{f3}, e_{f3}\}$, and its analysis was presented in matrix (8) which indicated that $M^+ = \phi$ and f_1 is not separable from f_2, f_3. New sensors can be added for obtaining this separability which can introduce new faults and the problem has no solution when the sensor is not free of faults.

After this analysis we can expect that the diagnosis of the IPS based on fault estimation will be better for situations with single or only two faults affecting the system, since for the three faults affecting the system at the same time the diagnosis can be incorrect. This indicates that we can invest in algorithms for estimating the faults in a robust an sensitive way only for the first two situations.

5 FDI WITH ANT COLONY OPTIMIZATION

Ant Colony Optimization (ACO) was initially proposed for integer programming problems (Dorigo & Blum, 2005) but recently it has been successfully extended and adapted to continuous optimization problems (Silva-Neto & Becceneri, (Eds.; Socha & Dorigo, 2008). A good feature of this algorithm is that its parameters can be manipulated in order to achieve a more exploitation or exploration driven structure which allows an efficient hybridization with other algorithms. ACO is inspired on the behavior of ants seeking a path between their colony and a food source. This behavior is due to the deposition and evaporation of a substance, the pheromone.

5.1 Description of the algorithm

For the continuous case the idea of the ACO is to mimic this behavior with simulated ants which are identified with feasible solutions (Dorigo & Blum, 2005; Silva-Neto & Becceneri, (Eds.; Socha & Dorigo, 2008). The first step is to divide the feasible interval of each variable of the problem in k possible values x_n^k. For each iteration a family of Z new ants is generated based on the information obtained from the previous ants and based on a selection mechanism. The information of the previous ants is saved on the pheromone accumulative probability matrix PC (the matrix has dimensions $n \times k$ where n is the number of variables in the problem) whose elements are

$$pc_{ij}(t) = \frac{\sum_{l=1}^{j} f_{il}(t)}{\sum_{l=1}^{k} f_{il}(t)} \tag{11}$$

and it is updated at each iteration; f_{ij} are the elements of the pheromone matrix $\mathbb{F}$ and express the pheromone level of the discrete j^{th} value of the i^{th} variable . This matrix is updated in each iteration based on an evaporation factor C_{evap} and an incremental factor C_{inc}:

$$f_{ij}(t+1) = (1 - C_{evap}) f_{ij}(t) + \delta_{ij,best} C_{inc} f_{ij}(t) \tag{12}$$

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where

$$\delta_{ij,best} = \begin{cases} 1 & \text{if} \quad x_i^j = x_i^{best} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

The scheme for generating the new colony of ants considers a parameter q_0 and a family of n random numbers q_1^{rand}, q_2^{rand},..., q_n^{rand} for the z^{th} ant to be generated. For each variable $x_n^{(z)}$ that will be part of the z^{th} ant is set the following generation mechanism:

$$x_n^{(z)} = \begin{cases} x_n^{\bar{m}} & \text{if} \quad q_n^{rand} < q_0 \\ x_n^{\hat{m}} & \text{if} \quad q_n^{rand} \geq q_0 \end{cases} \tag{14}$$

where

$$\bar{m} : \quad f_{n\bar{m}} \geq f_{nm} \, \forall \, m = 1, 2, \ldots, k \tag{15}$$

and

$$\hat{m} : \quad \left(pc_{n\hat{m}} > q_n^{rand} \right) \wedge \left(pc_{n\hat{m}} \leq pc_{nm} \right) \forall m \geq \hat{m} \tag{16}$$

The control parameter q_0 allows controlling the level of randomness during the ant generation. This fact determines, together Z and k, the level of exploitation or exploration of ACO (Dorigo & Blum, 2005; Silva-Neto & Becceneri, (Eds.; Socha & Dorigo, 2008). The general scheme of the algorithm is presented in Fig. 4.

Data: C_{evap}, C_{inc}, q_0, k, Z, Itr_{max}
Generate a random initial pheromone matrix $\mathbb{F}$ with the condition that all f_{ij} are the same;
Compute the matrix PC with Eq. (11);
Generate the random initial ants with Eqs. (14-16) and update X^{best};
for $l = 1$ *to* $l = Itr_{max}$ **do**
  Update F with Eq. (12)
  Update PC with Eq. (11)
  for $z = 1$ *to* $z = Z$ **do**
    Generate a new ant with Eqs. (14-16)
    Update X^{best};
  end
  Verify stopping criterio;
end

Figure 4: Algorithm ACO

5.2 Fuzzy- Ant Colony Optimization

The Fuzzy- Ant Colony Optimization intends to mimic a more realistic behavior of the pheromone deposit: the pheromone is an exhale odor substance and its deposit will not only affect the path where it was deposited but also those nearby paths. The idea behind the ACO-d is to simulate that kind of pheromone dispersion which will allow a more efficient exploration of the serach space (Becceneri et al., 2008).

The difference between the ACO and ACO-d is based on the way the pheromone matrix is updated. In the ACO-d a fuzzy rule is used, and the amount of pheromone to be deposited on each path is proportional to the distance to the best one (Becceneri et al., 2008).

In (Becceneri et al., 2008) this scheme is applied to the traveling salesman problem, in the present work we have adapted the ACO-d to the continuous problem. Therefore we have considered a new parameter C_{dis} that indicates the coefficient of dispersion. The pheromone deposition considers the scheme described in (12) and includes a deposit (dispersion) of the pheromone in the solutions nearby to the best one X^{best}. This deposit is inversely proportional to the distance to X^{best}.

For deciding the maximum number of neighbors of X^{best} that receive pheromone at each iteration we have adopted a scheme in which each component x_n^{best} has a maximum number of neighbors for receiving pheromone, let's call such set of neighbors $V[x_n^{best}]$ and let's define it as:

$$V[x_n^{best}] = \left\{ x_n^m : \, d(x_n^{best}, x_n^m) < d_{max}, \; 0 < m \leq k \right\} \tag{17}$$

The distance d_{max} is computed taking the average of the half of all the possible distances between values x_n^m and x_n^r with $m, r = 1, 2, ...k$. Based on the structure of the search space with the average

$$d_{max} = \frac{h + 2h + 3h + ... + \left[\frac{k}{2}\right] h}{\left[\frac{k}{2}\right]} \tag{18}$$

where $h = \frac{b-a}{h}$ with $x_n \in (a,b)$, and $[x]$ represents the nearest integer from x. Working with Eq. (18) and applying the expression for the sum of the first n integers can be obtained:

$$d_{max} = h \frac{\left[\frac{k}{2}\right] + 1}{2} \tag{19}$$

Noting that $d(x_n^m, x_n^{m+1}) = h$ we reformulate equation (17) as

$$V[x_n^{best}] = \left\{ x_n^m : d(x_n^{best}, x_n^m) < \frac{\left\lfloor\frac{k}{2}\right\rfloor + 1}{2},\ 0 < m \leq k \right\} \tag{20}$$

making $x_n^m = a + hm$ and $x_n^{best} = a + h\bar{m}$ we can also reformulate (20) as

$$V[x_n^{best}] = \left\{ x_n^m : \bar{m} - \frac{\left\lfloor\frac{k}{2}\right\rfloor + 1}{2} < m < \bar{m} + \frac{\left\lfloor\frac{k}{2}\right\rfloor + 1}{2},\ 0 < m \leq k \right\} \tag{21}$$

The scheme to lay down the pheromone is ready with the following expression for the $x_n^m \in V[x_n^{best}]$,

$$f_{nm}(t+1) = f_{nm}(t+1) + \frac{C_{dis}}{\bar{m} - m} \tag{22}$$

5.3 Implementation

The variants of ACO were based on the different values for the parameters q_0. The parameter q_0 permits to establish the level of randomness in the selection of the discret value of the variable (Silva-Neto & Becceneri, (Eds.), determining the trend of the search. The values $q_0 = 0.15$, $q_0 = 0.55$ and $q_0 = 0.85$ indicate a more exploration driven procedure, a balance between exploration and exploitation, and a more exploitation of the search space, respectively. All the variants are based on the algorithm of Fig. 4 and Tab. 1 shows the values for the parameters of the algorithm in each variant. The number of ants was set in $Z = 30$. For the case of ACO-d the same parameters values were considered and the value for $C_{dis} = 0.10$ in all the cases.

Table 1: Values for the parameters in ACO and ACO-d

	k	q_o	**Z**	C_{dis}
ACO-1, ACO-d1	63	0.15	30	0.10
ACO-2, ACO-d2	63	0.55	30	0.10
ACO-3, ACO-d3	63	0.85	30	0.10

The stopping criterion is satisfies one of the conditions:

1. Condition 1: Maximum number of iterations $Itr_{max} = 100$.
2. Condition 2: Maximum number of iterations for which the best value of the objective function remains constant $Itr_{cte} = 20$.
3. Condition 3: Minimum value for the objective function $F(\hat{\theta}_f) < 0.001$.

6 RESULTS

With the aim to analyze the merits of the diagnosis based on faults estimation with ACO and ACO-d, two aspects have been considered: robustness and computational effort. We are also concerned with the influence of some parameters of ACO in these characteristics of the diagnosis.

With this goal in mind and with the aim of testing the conclusions derived in section 4 regarding the detectability and separability, the experiments have been divided into three parts:

- First Part: A situation of single fault is considered, and only the actuator fault f_1 can afect the system. The output of the system is corrupted with 5% level noise in order to analyze robustness. The faulty situation is the Case 1 shown in Tab. 2.
- Second Part: A situation of multiple faults is considered, the faults f_1 and f_3 are the only affecting the system. The output of the system is corrupted with 5% noise level in order to analize robustness. The faulty situation is the Case 2 shown in Tab. 2.

- Third Part: A situation of multiple faults is considered, with the faults f_1, f_2 and f_3 affecting the system. The output of the system is corrupted with 5% noise level in order to analyze robustness. The faulty situation is the Case 3 shown in Tab. 2.

All the parts considered the three sets of values for each algorithm, as shown in Tab. 1, in order to analyze their influence in the diagnosis properties.

Table 2: Faulty situations for the numerical experiments

	f_1	f_2	f_3
Case 1	0.5	-	-
Case 2	0.5	-	0.02
Case 3	0.5	0.01	0.02

Each experiment was repeated 30 times with the intention of making statistically valid the description of the results by means of computation of the arithmetic average of the parameters estimates. The abbreviations that were used in the tables and figures are: $\hat{f_l}$ and $\sigma_{\hat{f_l}}$ for the mean and variance, respectively, of the estimations for the fault f_l, $It\bar{e}r$ for the arithmetic average of the number of iterations that were achieved, and $\bar{t}$ for the arithmetic average of the computing time, in seconds. The computational effort of the algorithm is analyzed based on the number of iterations.

In Tabs. 3- 5 are shown the results of the estimations for each case considered in Tab. 2.

The results in Tab. 3 show that when only a detectable fault is considered to be affecting the system, the diagnosis is correct. The best results are for ACO-1 and ACO-d1, indicating that the major exploration of the search space provides better results in the diagnosis.

Table 3: Results of the diagnosis for the Case 1 of the Tab. 2

	$\hat{f_1}$	$\sigma_{\hat{f_1}}$	$\hat{f_2}$	$\hat{f_3}$	$\hat{t}(sec)$	$It\bar{e}r$
ACO-1	0.4903	1.2e-006	-	-	35.007	32
ACO-2	0.4755	2.1e-005	-	-	50.620	48
ACO-3	0.4606	1.5e-005	-	-	45.4218	41
ACO-d1	0.4980	1.0e-006	-	-	34.6556	30
ACO-d2	0.4795	4.2e-005	-	-	39.0320	38
ACO-d3	0.4723	3.1e-005	-	-	48.5302	47

The results in Tab. 4 show that when two separable faults are affecting the system, the diagnosis is also correct. The best results are, again, for ACO-1 and ACO-d1. The number of iterations was higher than those for the previous case.

Table 4: Results of the diagnosis for the Case 2 of the Tab. 2

	$\hat{f_1}$	$\sigma_{\hat{f_1}}$	$\hat{f_2}$	$\hat{f_3}$	$\sigma_{\hat{f_3}}$	$\hat{t}(sec)$	$It\bar{e}r$
ACO-1	0.4891	9.3e-006	-	0.0190	1.4e-008	46.154	43
ACO-2	0.4701	7.4e-005	-	0.0188	1.0e-007	59.962	57
ACO-3	0.4499	7.6e-005	-	0.0185	3.1e-007	60.005	58
ACO-d1	0.4958	8.1e-006	-	0.0198	1.2e-008	45.094	42
ACO-d2	0.4797	7.4e-005	-	0.0182	2.8e-007	57.082	55
ACO-d3	0.4555	8.0e-005	-	0.0170	9.0e-008	61.9925	60

The results in Tab. 5 show that diagnosis is not correct when the three fauls are affecting the system at the same time. These results are in agreement with the analysis of separability for the study case shown in section 4. The estimations permit to detect the faults affecting the system, but do not allow the other requirements for a diagnosis of the system. In these cases the diagnosis via faults estimation is not feasible.

The results presented in the Tabs. 3- 5 are summarize in the Fig. 5. In the Fig. 5 is shown that for the Cases 1 and 2 the diagnosis is completed, while for the Case 3 the diagnosis is not possible.

Table 5: Results of the diagnosis for the Case 3 of the Tab. 2

	$\hat{f}_1$	$\sigma_{\hat{f}_1}$	$\hat{f}_2$	$\sigma_{\hat{f}_2}$	$\hat{f}_3$	$\sigma_{\hat{f}_3}$	$\hat{t}(sec)$	$\bar{Iter}$
ACO-1	0.3892	1.1e-004	0.07	1.8e-007	0.17	2.0e-007	138.0004	98
ACO-2	0.0381	9.2e-004	0.015	9.5e-007	0.0060	1.1e-006	122.0973	85
ACO-3	-0.2707	9.2e-004	0.0001	1.8e-007	0.0115	1.3e-006	100.50	77
ACO-d1	0.0013	3.8e-004	0.0001	9.0e-007	0.0173	1.1e-007	136.865	97
ACO-d2	-0.276	9.7e-003	0.0009	9.3e-007	0.0555	3.1e-006	127.72	89
ACO-d3	-0.631	1.2e-003	0.0063	1.4e-006	0.0051	3.4e-006	109.634	70

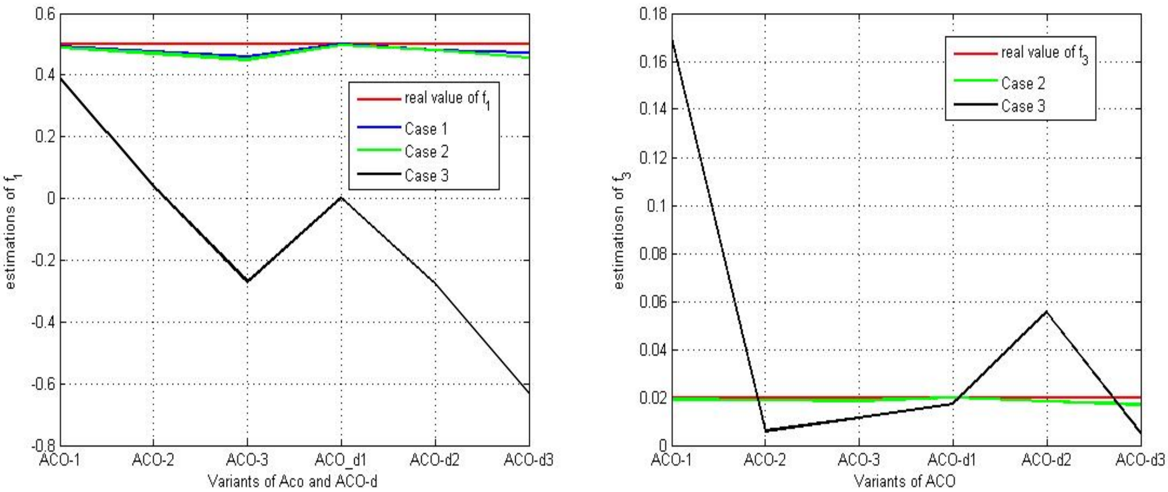

Figure 5: Comparison between the estimations of f_1 and f_3 for the faulty situations described in Tab. 2

7 CONCLUSIONS

This study indicates that the formulation and solution of an FDI based on fault estimation is feasible. The results on structural detectability and separability give a prior information for determining the limitations of the diagnosis via fault estimation ones the inverse problem is formulated.

The study of the influence of the parameters of the algorithms indicated that the best set of parameter values for ACO and ACO- d correspondes to the version that makes a major exploration of the search space, $q = 0.15$. Following that experience is evident to obtain a better diagnosis with ACO-1 than with ACO-d1. This fact is because of ACO-d conception is based on a pheromone dispersion that helps in the exploration of the search space.

The results confirm the results of the section 4: the faults in the IPS can be detected via direct fault estimation in the IPS but the diagnosis is only feasible for the situation with one or two faults at the same time. For the case with the three faults affecting the system at the same time the diagnosis is not reliable due to the lack of separability between the faults.

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