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MULTI-OBJECTIVE OPTIMIZATION OF INTERPLANETARY TRAJECTORIES WITH SEQUENTIAL GRAVITY ASSISTED MANEUVERS

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Abstract: *This work is a study of optimization of interplanetary trajectories using gravity assisted maneuvers on different planets on the same mission. This maneuver consists in using the gravity of a planet to gain or lose energy, velocity and angular momentum, minimizing fuel consumption. The trajectory is divided into three parts, where the first and the last one are inside the sphere of influence of a planet, and the intermediate part is a heliocentric phase. This methodology is called Patched Conics. When the optimization of more than one objective is desired, considering that the objectives are conflicting, a multi-objective method is needed. The optimization problem is solved using a methodology based on the Non Inferiority Criterion (Pareto, 1909) and the Smallest Loss Criterion (Rocco et al. 2003), where all objectives are considered simultaneously, without reducing the problem to the case of optimizing a single objective as occurs in most methods found in literature. For this purpose a sequence of gravity assisted maneuvers on some planets were tested with different combinations of dates for the maneuvers, launch and arriving windows.*

Key words: multi-objective optimization, interplanetary trajectories, gravity assisted maneuvers.

1. OPTIMIZATION PROBLEM

The optimization problem consists in minimizing or maximizing an objective. Computational algorithms are used to solve this problem and find the best solution. An optimization problem can have more than one objective to be optimized. In this case the problem is considered a multi-objective problem.

According to Cohon (1978), the static optimization of problems with one objective can be defined in the following way:

$$\begin{aligned} &\text{Maximize} \quad Z(\boldsymbol{x}) \ \text{ with relation to } \ \boldsymbol{x} \in \boldsymbol{R}^n \\ &\text{Subject to} \quad g_i(\boldsymbol{x}) \le 0 \qquad i = 1, 2, \dots, m \\ &\qquad\qquad\quad \boldsymbol{x} \ge 0 \\ &\text{Given} \qquad Z(\cdot),\ g_i(\cdot) \end{aligned} \tag{1}$$

or

$$\begin{aligned} &\text{Maximize} \quad Z(\boldsymbol{x}) \ \text{ with relation to } \ \boldsymbol{x} \in \boldsymbol{R}^n \\ &\text{Subject to} \quad \boldsymbol{x} \in \boldsymbol{F}_d \\ &\text{Given} \qquad Z(.),\ \boldsymbol{F}_d \end{aligned} \tag{2}$$

$\boldsymbol{F}_d$ is the feasible area of the decision space, defined by:

$$\boldsymbol{F}_d = \{\boldsymbol{x} \in \boldsymbol{R}^n \mid g_i(\boldsymbol{x}) \le 0, i = 1, 2, \dots, m; \boldsymbol{x} \ge 0\} \tag{3}$$

The multi-objective problem can be defined by:

Maximize $\mathbf{Z}(\boldsymbol{x}) = [\mathrm{Z}_1(\boldsymbol{x}), \mathrm{Z}_2(\boldsymbol{x}), \ldots, \mathrm{Z}_p(\boldsymbol{x})]$ (4)

Subject to $\boldsymbol{x} \in \boldsymbol{F}_d$

Therefore, in this case, the objective function, is a vector with dimension p.

In problems of one-dimensional optimization (when we have one objective), the possible solutions ($\boldsymbol{x} \in \boldsymbol{F}_d$) can be compared by means of the objective function, that is, given two solutions $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ we can compare $Z(\boldsymbol{x}^1)$ with $Z(\boldsymbol{x}^2)$ and determine the optimal solution so that $\boldsymbol{x} \in \boldsymbol{F}_d$ doesn't exist such that $Z(\boldsymbol{x}) > Z(\boldsymbol{x}^*)$. In problems of multi-dimensional optimization (multi-objective problem), in general, it is not possible to compare all the possible solutions because the comparison on the basis of one objective can be contradicted with the comparison based on another objective. Namely, supposing that:

$$\mathbf{Z}(\boldsymbol{x}^1) = [\mathrm{Z}_1(\boldsymbol{x}^1), \mathrm{Z}_2(\boldsymbol{x}^1)]$$
$$\mathbf{Z}(\boldsymbol{x}^2) = [\mathrm{Z}_1(\boldsymbol{x}^2), \mathrm{Z}_2(\boldsymbol{x}^2)] \quad (5)$$

$\boldsymbol{x}^1$ is better than $\boldsymbol{x}^2$ if and only if:

$$\mathrm{Z}_1(\boldsymbol{x}^1) > \mathrm{Z}_1(\boldsymbol{x}^2) \text{ and } \mathrm{Z}_2(\boldsymbol{x}^1) \geq \mathrm{Z}_2(\boldsymbol{x}^2) \quad (6)$$

or

$$\mathrm{Z}_1(\boldsymbol{x}^1) \geq \mathrm{Z}_1(\boldsymbol{x}^2) \text{ and } \mathrm{Z}_2(\boldsymbol{x}^1) > \mathrm{Z}_2(\boldsymbol{x}^2) \quad (7)$$

If $\mathrm{Z}_1(\boldsymbol{x}^1) > \mathrm{Z}_1(\boldsymbol{x}^2)$ and $\mathrm{Z}_2(\boldsymbol{x}^1) < \mathrm{Z}_2(\boldsymbol{x}^2)$ we cannot conclude anything regarding $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$, that is, $\boldsymbol{x}^1$ e $\boldsymbol{x}^2$ cannot be compared.

2. MULTI-OBJECTIVE METHODS

Multi-objective problems usually have objectives that are conflicting. The classic methods are not recommended on these cases, because when one objective is optimized the others are no longer optimal. Multi-objective optimization methods can be used, leading to a sub optimal solution for each of the objectives separately. Most of the studies involving multi-objective optimization were developed in areas like economy, sociology and psychology.

There are numerous approaches to solve a multi-objective problem. The best known are: The Goal Programming Method; The Surrogate Worth Tradeoff Method; The Constraint Method; The Weighting Method; Non Inferiority.

The Pareto Method, or Non-inferiority Method, considers that any candidate that belongs to a group of possible solutions could be chosen as the solution for the multi-objective problem. Therefore, the solution choice would be made by a specialist capable of analyzing the gains and losses for each candidate of solution.

To select the group of Pareto's solutions, it's necessary to use an algorithm that will systematically make a comparison between the candidates. A solution *x* can only be considered optimal for a certain group of objectives, if there is not a solution *y* better in all objectives. The solution *x* is called non-inferior or non-dominated. Then, a solution *x* is non-dominated in case there is not a solution *y*, such that:

$$Z(y) \geq Z(x) \quad (8)$$

or

$$Z_k(y) \geq Z_k(x),\ k=1, 2,..., p \quad (9)$$

If *y* exists, *x* is considered a dominated solution or inferior, and then it can't be an optimal Pareto's solution. A solution dominates the other if it's better in all objectives. It can be said that any pair of solutions in the group of the non-dominated solutions must be non-dominated one over another. And any dominated solution must be dominated by at least one solution of the group of the non-dominated solutions. The curve that contains the non-dominated solutions in known as Pareto frontier.

According to Kuhn-Tucker (1951), if *x* is a non-dominated solution, then there must be multipliers $u_i \geq 0,\ i= 1, 2,..., m$ and $w_k \geq 0,\ k= 1, 2,..., p$ such that:

$$X \in F_d \quad (10)$$

$U_i g_i(x)=0,\ i=1, 2,..., m$ (3.14)

$$\sum_{k=1}^{p} w_k \nabla Z_k(x) - \sum_{i=1}^{m} u_i \nabla g_i(x) = 0 \qquad (11)$$

The first and third condition of Kuhn-Tucker are necessary conditions so that x is non-dominated. They are also sufficient in case that $Z_k(x)$ is concave for $k= 1, 2,.., p$, F_d is convex and $w_k > 0$ for every k.

2.1 The Smallest Loss Criterion

On the Pareto method the solution for the multi-objective problem with conflicting objectives can be chosen within a set of solutions considered non-dominated solutions or non-inferior. However, in this case we have a group of possibilities but not a final solution, since a solution must be chosen within this group. In any solution that is chosen there would be an objective being prioritized, then it would be necessary to stipulate weights for each objective, and this would be one more process of optimization. Using a specialist makes the solution of the problem individual for that one specific specialist, making it difficult to reproduce the results by another specialist. It would be convenient to use a methodology capable of finding a solution that covers all the objectives simultaneously.

The specialist choice is based on particular criterion that will lead to a solution that can't be called optimal, since according to Pareto, there is a group of non-dominated solutions with the same degree of optimality. Therefore, can be said that any solution of that group could be chosen randomly, eliminating the need of a specialist.

There are some multi-objectives problems where all of the candidates for solution are non-dominated. In this case there's no use in determining a group of non-dominated solutions. There is still the need of finding a solution somehow. Therefore, it's necessary to apply another method.

The Smallest Loss Criterion (Rocco et al. 2000; 2001; 2003; Rocco, 2002) was elaborated in order to find one final solution that attends all objectives simultaneously in the best possible way, without the need of prioritizing any of them. Some applications of this method can be found on Rocco et al. (2005a), Rocco et al. (2005b) and Rocco et al. (2005c).

The solution for a problem with n conflicting objectives, where the goal is to optimize equally and simultaneously the objectives, must be the one that results on the smallest loss for each of the objectives, since there is no solution capable of optimizing the n objectives individually.

A way to obtain the smallest loss solution for a multi-objective problem would be to find the barycentre of a normalized n-dimensional figure, where on each vertex would be the optimal solution of each objective isolated. On a problem with three objectives, for example, the smallest loss solution would be at the center of a normalized triangle. So, for a problem with n objectives, the smallest loss solution would be at the center of this normalized n-dimensional figure. Figure *1*shows the example of a three conflicting objectives:

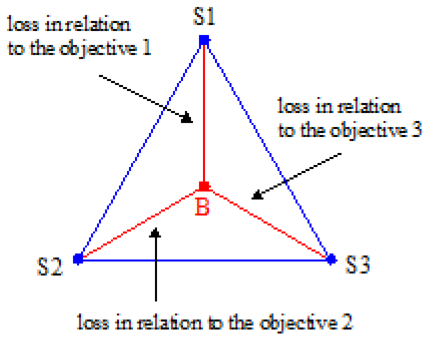

Fig 1: Smallest loss for three conflicting objectives

On this example, *S1*, *S2* e *S3* are the optimal solutions for each one of the objectives separately. *B* is the barycentre of the figure where each objective would have the smallest loss considering all the objectives together. Therefore, the distance between *S1* and *B* represents the smallest loss for the objective 1, the distance between *S2* and *B* for the objective 2 and the distance between *S3* and *B* the smallest loss for the objective 3. Then, according to figure *1*, the best solution for the multi-objective problem, considering all the objectives equally, would be at the center of the triangle.

3. GRAVITY ASSITED MANEUVERS

The smaller the ΔV, which is the change in velocity, the smaller will be the fuel cost. The gravity assisted maneuvers, or swing-by, can be used to reduce fuel consumption on interplanetary missions. It also can help to reduce the duration of a mission.

The first time this maneuver was applied on a real mission was in 1974, when was launched the Mariner 10 probe, with swing-by on Venus and Mercury. It's possible to make a sequential swing-by on different bodies on the same mission, so that the velocity gain can be even higher. On the Voyager mission, for example, the swing-by was made on Jupiter, Saturn, Uranus and Neptune, gaining more energy on each planet until there was enough energy to leave the solar system (Kohlhase & Penzo, 1977). In 2006 the New Horizon spacecraft was launched with the purpose of studying Pluto and its moons (Guo & Farquhar, 2006). On February of 2007 New Horizons made a swing-by on Jupiter and will arrive in Pluto on 2015.

The swing-by maneuvers can also be used for other purposes: on the Ulisses mission, in 1985, this maneuver was used to modify the orbital plane inclination of the probe; the use of consecutive swing-bys on the moon to obtain the geometry of the orbits desired, like satellites used to study the solar phenomenon.

4. SIMULATION OF INTERPLANETARY TRAJECTORIES

To obtain the minimum fuel consumption the gravity assist maneuvers were applied on Earth, Venus, Jupiter, Saturn and Neptune. An interplanetary trajectory program (Sukhanov, 2004) was used to simulate the mission with minimum fuel consumption based on the *patched conic* method. The program was written in Fortran language and can be used to generate launch and swing-by windows; optimum transfer trajectories for each day of the launch window, with the ΔV, mission duration, trajectory parameters; generate trajectories with restrictions such as total time of transfer; generate graphs for the trajectories that were obtained.

4.1. Patched Conics Methodology

The methodology *patched conics* (Broucke, 1988) divides the trajectory into parts. The first part is the planetocentric one, inside the sphere of influence of the origin planet; the second is the heliocentric part, where the spacecraft is traveling from one planet to another; the third part is a planetocentric part, inside the sphere of influence of the destination planet.

Considering the variables: M_1 as being a massive body in the center of the cartesian system, M_2 a smaller body, that could be a planet or a satellite of M_1; M_3 a body with an infinitesimal mass around M_1, going towards M_2. The relative velocities are: V_2, is the velocity related to M_1; V_∞^- e V_∞^+ are the velocity vectors of the spacecraft related to M_2 before and after the encounter respectively; And the angles: δ, which is half of the angle between V_∞^- e V_∞^+ ; ψ, the angle between the periapsis line and the M_1-M_2 line. The variable r_p is the distance of maximum approximation when there is the encounter of M_2 and M_3.

The expression for δ can be obtained using the theory of hyperbolic orbits, given by:

$$\mathrm{sen}(\delta) = \frac{1}{1 + \frac{r_p V_\infty^2}{\mu_2}}, \tag{12}$$

$\mu_2 = M_2 = Gm_2$, and G being the gravitational constant.

The spacecraft enters the sphere of influence of M_2 after leaving the Keplerian orbit around M_1, and from that on the effects of M_1 can be neglected. The velocities of the spacecraft before and after the encounter with M_2 are given by equations *(13)* and *(14)* respectively:

$$\vec{V}_\infty^- = \vec{V}_i - \vec{V}_2 \tag{13}$$

$$\vec{V}_\infty^+ = \vec{V}_o - \vec{V}_2 \tag{14}$$

The difference between the inertial velocity before and after the swing-by is obtained using the equations *(13)* and *(14):*

$$\vec{V}_\infty^+ - \vec{V}_\infty^- = \Delta\vec{V} = \vec{V}_o - \vec{V}_i \tag{15}$$

According to the vector diagram, the magnitude of the velocity variation is given by:

$$\Delta V = \left|\Delta \vec{V}\right| = 2\left|\vec{V}_\infty\right| \sin(\delta) = 2V_\infty \sin(\delta) \tag{16}$$

The components X and Y of the velocity increasing are:

$$\Delta \dot{X} = -2V_\infty \sin(\delta)\cos(\psi) \tag{17}$$

$$\Delta \dot{Y} = -2V_\infty \sin(\delta)\sin(\psi) \tag{18}$$

The angular momentum C is given by:

$$C = X\dot{Y} - Y\dot{X} \tag{19}$$

Therefore, the angular momentum variation *ΔC* is:

$$\Delta C = X\left(\Delta \dot{Y}\right) + (\Delta X)\dot{Y} - Y\left(\Delta \dot{X}\right) - (\Delta Y)\dot{X} \tag{20}$$

Considering that the encounter is instantaneous, in other words, that ΔX = ΔY = 0, t = 0 e Y = 0, the angular momentum variation will be:

$$\Delta C = X\Delta \dot{Y} \tag{21}$$

Resulting in:

$$\omega \Delta C = -2V_2 V_\infty \operatorname{sen}(\delta)\operatorname{sen}(\psi) \tag{22}$$

It's possible to obtain the energy variation subtracting the energy after the swig-by E_+ from the energy before $E\_$:

$$\Delta E = E_+ - E_- = \frac{1}{2}\left[\left(\dot{X} + \Delta \dot{X}\right)^2 + \left(\dot{Y} + \Delta \dot{Y}\right)^2\right] - \frac{1}{2}\left(\dot{X}^2 + \dot{Y}^2\right) \tag{23}$$

Figure 2 shows the variables on a swing-by maneuver (Prado, 2001):

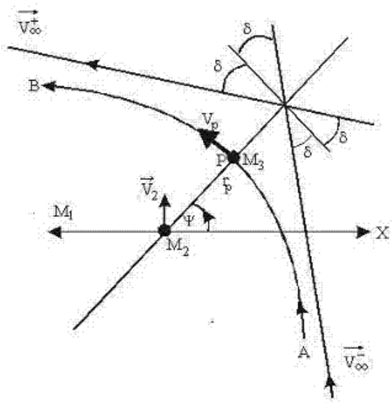

Figure 2: Swing-by maneuver

5. RESULTS

The multi-objective problem was solved considering three objectives: fuel consumption (*ΔV*); the duration of the mission; the waiting time on Earth for launch. The reference day for the waiting time for launch is January 1st of 2012. The normalization was obtained using the maximum value of each objective.

Three different possibilities were considered:

Case I- using only the extreme non-dominated candidates: on this case, to obtain the barycentre of the figure, are considered only the three extreme non-dominated candidates. These candidates are those in which at least one of the objectives is optimal, in other words, one candidate has the objective *a* optimal, the second candidate has the objective *b*, and the third the objective *c* optimal. Therefore, the solution for the multi-objective problem is found using the barycentre of the extreme non-dominated candidates.

Case II – using all the candidates: on this case the barycentre of the figure is obtained considering all the candidates, and then the search for the smallest loss solution is made

Case III – using the utopian solution: the utopian solution would be the ideal solution for the problem, where each of the three objectives is optimal. However, because the objectives are conflicting, this solution is not possible. Then on this case, the optimal solution is the closest one to the utopian solution.

A simulation was obtained using multiple swing-bys on different planets in the same trajectory. The sequence of the swing-by is: Earth, Venus, Earth, Jupiter, Saturn and Neptune.

Table 1 shows the launch window and swing-by window for each planet:

Table 1: Launch and swing-by windows.

Planet	**Launch and swing-by window (dd/mm/yyyy)**
Earth	01/03/2012 - 01/04/2012
Venus	01/09/2012 - 01/10/2012
Earth	01/08/2013 - 01/09/2013
Jupiter	01/01/2016 - 01/01/2017
Saturn	01/01/2018 - 01/01/2019
Neptun	01/01/2024 - 01/01/2025

Figure 3 shows the sequence of the trajectory considered on this work:

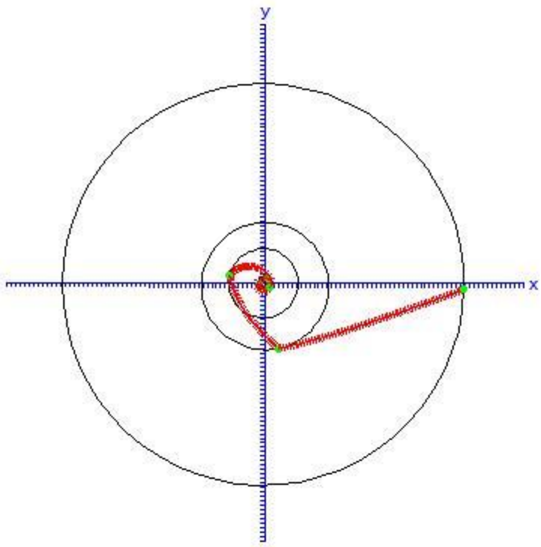

Figure 3: Sequential swing-by.

Figure 4 shows all the 30 candidates for the optimal solution considering the three objectives to be optimized, where *an* is the fuel consumption, *bn* the duration of the mission and *cn* the waiting time for launch. Figure 5, 6 and 7 shows for cases I, II and III, respectively: all 30 candidates, the utopian solution, the three extreme non-dominated candidates, the barycentre of all candidates, and the smallest loss solution chosen according to each case. Figure 8 shows the normalized distances for each case, where the best candidate for the optimization problem is the one with the shortest distance from the barycentre. Table 2 shows the values of the Δv, duration of the mission, and waiting time for launch for each candidate. The solution for each case, according to the Smallest Loss Criterion, is shown on table 2.

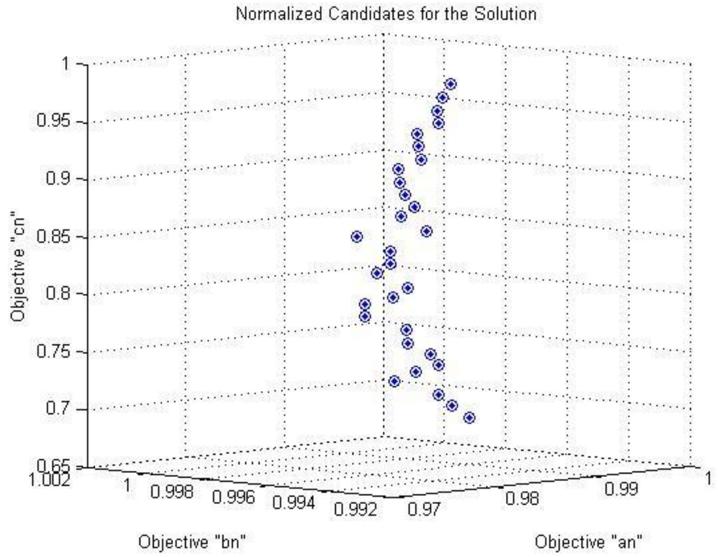

Figure 4: Candidates for solution.

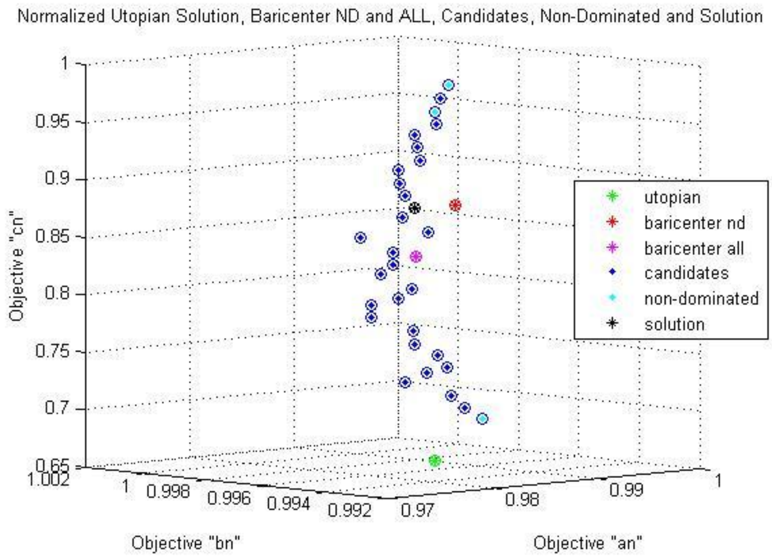

Figure 5: Case I

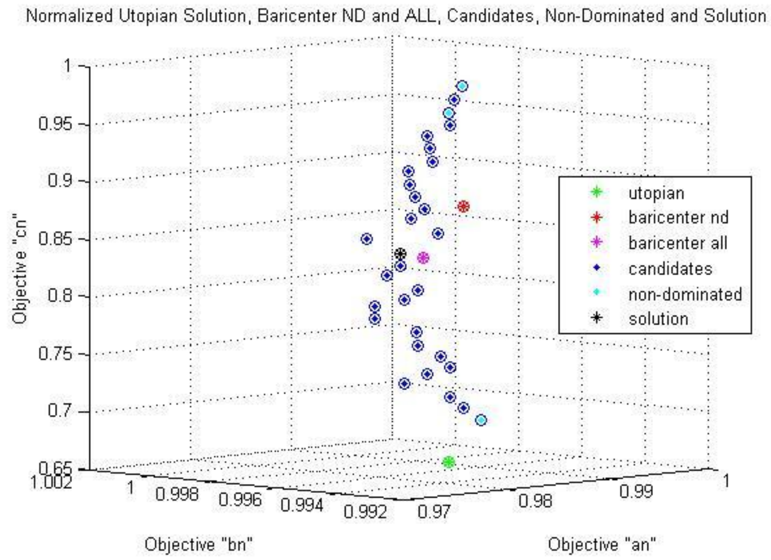

Figure 6: case II

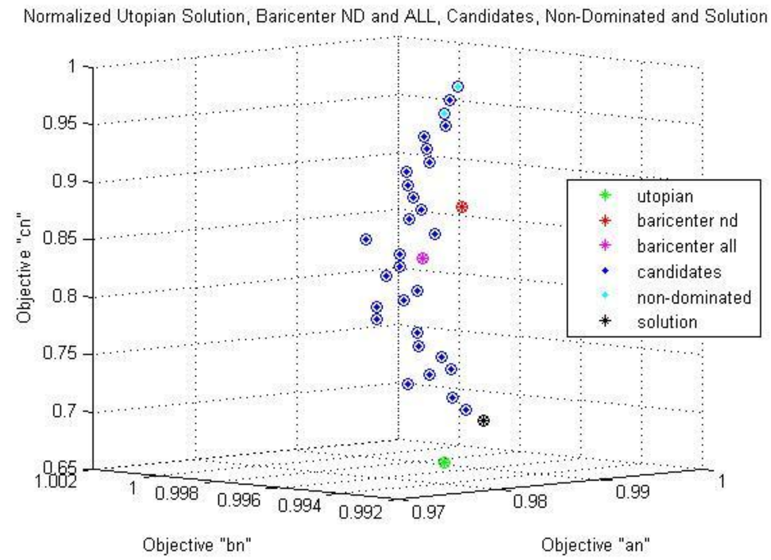

Figure 7: Case III

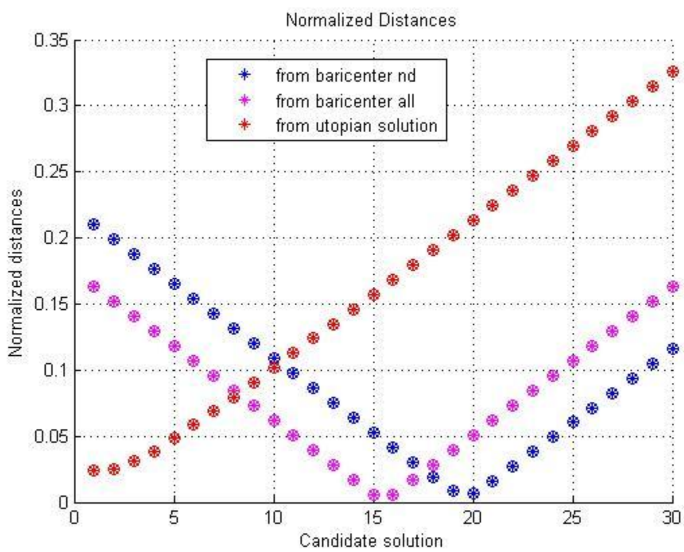

Figure 8: Normalized distances.

Table 2: Candidates for solution.

Solution	Δv(km/s)	Duration(years)	Waiting T(days)
1	7,639	12,53	60
2	7,626	12,53	61
3	7,616	12,53	62
4	7,600	12,54	63
5	7,598	12,53	64
6	7,578	12,51	65
7	7,571	12,51	66
8	7,554	12,51	67
9	7,553	12,51	68
10	7,540	12,52	69
11	7,540	12,52	70
12	7,524	12,5	71
13	7,516	12,49	72
14	7,511	12,5	73
15	7,503	12,49	74
16	7,503	12,49	75
17	7,496	12,5	76
18	7,493	12,47	77
19	7,492	12,48	78
20	7,483	12,47	79
21	7,476	12,47	80
22	7,472	12,47	81
23	7,471	12,47	82
24	7,470	12,46	83
25	7,468	12,46	84
26	7,466	12,46	85
27	7,464	12,45	86
28	7,463	12,45	87
29	7,467	12,45	88
30	7,473	12,45	89

Table 3: Solution for case I, II and III

	Solution 20 (case I)	Solution 16 (case II)	Solution 1 (case III)
Δv (km/s)	7,483	7,503	7,639
duration (years)	12,47	12,49	12,53
Waiting T (days)	79	75	60

6. CONCLUSION

A simulation was made using sequential swing-by on Earth, Venus, Jupiter, Saturn and Neptune to optimize the trajectory.

The objectives to be minimized were: fuel consumption, duration of the mission and waiting time for launch.

The multi-objective optimization program utilized to search for the optimal solutions was developed based on the smaller loss method. Different from other multi-objective methods, it is possible to find one final solution and not a group of feasible solutions (candidates to the solution for the multi-objective problem), that in most cases, leads to the necessity of considering different weights for the objectives, since it is not possible to choose only one solution that is better in all objectives. On the search for the multi-objective solution, three possibilities were considered: using only the extreme non-dominated candidates, using all the candidates, and the utopian solution. The results showed that the solution closest to the optimal one for case I is candidate 20, for case II candidate 16, and for case III candidate 1.

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