Iterated Least-Squares Performance Evaluation with Real Data for Real Time Ambiguity Resolution

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Abstract: The Global Positioning System (GPS) is a satellite-based navigation system which allows the user to determine position and time with high precision. However, phase measurements has an inherent difficulty, which is the ambiguity determination in number of signal wavelengths. Once ambiguities are resolved, positioning can reach sub-meter accuracies. The purpose of this work is to estimate the position coordinates of a GPS receiver using double differenced phase positioning techniques and evaluate accuracy of the solution. The performance of ambiguity resolution, with LSAST and LAMBDA methods, and an iterated least-squares estimator are evaluated and positioning errors are shown with half meter level for static and less than meter level for kinematic positioning.

Key-Words: Ambiguity resolution, Relative positioning, LAMBDA method, LSAST method, Least-squares.

1 Introduction

The Global Positioning System (GPS) is a satellite-based navigation system which allows the user to determine position and time with high precision. GPS measurements are subject to several error sources. The combined effects of these errors in the propagation signal cause a degradation in precision of positioning. However, using phase measurements, it is possible in certain cases to increase positioning accuracy up to 100 times, if compared with positioning using code pseudorange [8].

However, phase measurements has an inherent difficulty, which is the ambiguity determination in wavelength number of signal. While signal phase changes from epoch to epoch can be measured with high accuracy, cycles integer number along propagation path (integer ambiguity) remains unknown. Once the ambiguities are solved, phase measurements can be used as very precise pseudorange measurements. Therefore, ambiguity resolution is a fundamental issue for sub-meter positioning.

Sub-meter accuracy is required in many applications. For aircraft navigation, high accuracy is required for landing, especially for automatic landings. GPS antennas and receivers can also be mounted on a vehicle or spacecraft so that position and attitude information of the vehicle can be derived [3]. Precise kinematic differential GPS will also be useful in navigating agricultural vehicles, playing a role in the distribution of work, navigation of the harvesters, and the guidance of tractors. Vehicle control flow can also be improved [4]. Therefore, the objective of this work is to compare two methods of ambiguity resolution, LSAST and LAMBDA, in order to achieve sub-meter accuracies in real time.

In LSAST method (Least Squares Ambiguity Search Technique), ambiguity parameters are divided into two groups: primary ambiguities (typically three double difference ambiguities), and the secondary ambiguities. Only the primary ambiguities are fully searched. For each set of the primary ambiguities, there is a unique set of secondary ambiguities. Therefore, the search dimension is smaller and the computation time is significantly shorter than the full search approach. LAMBDA method (Least-squares AMBiguity Decorrelation Adjustment) is a procedure to estimate ambiguities based on double difference models. This method uses a decorrelating transformation followed by a integer search, reducing computational time because it is not necessary a search through whole space. The estimation is carried out in three steps: float solution, integer solution, and correction of position from resolved integer ambiguities.

In this work, positioning tests using double differenced carrier phase measurements were carried out. LSAST and LAMBDA methods were used in ambiguity resolution process. Two tests were executed: static and kinematic. Static test data were collected by two dual frequency Trimble R8 GPS receivers, and the kinematic test data were collected by two Ashtech Z12 GPS receivers. The positioning errors in these tests are shown to remain with magnitude less than half a meter in static case, and with magnitude less than a meter in kinematic case, using an iterated least-squares estimator for both tests. The results were compared to a known receiver position (in static test) or a reference trajectory (in kinematic test). An off-line adjustment, which leads to smaller errors, is also made for comparison.

2 Iterated least-squares with orthogonal transformations

In non-linear least-squares method, the cost function is euclidean weighted norm by a matrix $\mathbf{W}$, and with *a priori* information $\hat{\mathbf{x}}_0$ and $\mathbf{P}_0$:

$$\begin{aligned} J &= \|\delta\mathbf{y} - \mathbf{H}\delta\mathbf{x}\|^2_{\mathbf{W}} + \|\delta\hat{\mathbf{x}}_0 - \delta\mathbf{x}\|^2_{\mathbf{P}_0^{-1}} \\ &= (\delta\mathbf{y} - \mathbf{H}\delta\mathbf{x})^T\mathbf{W}(\delta\mathbf{y} - \delta\mathbf{H}\mathbf{x}) \\ &\quad + (\delta\hat{\mathbf{x}}_0 - \delta\mathbf{x})^T\mathbf{P}_0^{-1}(\delta\hat{\mathbf{x}}_0 - \delta\mathbf{x}) \end{aligned} \tag{1}$$

where $\mathbf{H}$ is observation design matrix and $\mathbf{y}$ is measurements vector. The minimization of cost function gives:

$$\begin{aligned} \delta\hat{\mathbf{x}} &= \hat{\mathbf{P}}(\mathbf{P}_0^{-1}\delta\mathbf{x}_0 + \mathbf{H}^T\mathbf{W}\delta\mathbf{y}) \\ \hat{\mathbf{P}} &= (\mathbf{P}_0^{-1} + \mathbf{H}^T\mathbf{W}\mathbf{H})^{-1} \end{aligned} \tag{2}$$

where $\hat{\mathbf{x}} = \bar{\mathbf{x}} + \delta\hat{\mathbf{x}}$ is the final estimate, and $\hat{\mathbf{P}}$ is the covariance matrix.

The normal equations solution must invert a $n \times n$ matrix, where n is the number of states in $\hat{\mathbf{x}}$. These inversions are a potential numerical error source, especially when the matrix is almost singular. However, literature contains several works which intend to increase numerical performance of least-squares [7; 1]. In this work, a matrix triangularization technique is used in matrix $\mathbf{H}$. Eq. (1) can be rewritten as:

$$\begin{aligned} J &= \|\mathbf{W}^{1/2}(\delta\mathbf{y} - \mathbf{H}\delta\mathbf{x})\|^2 + \|\mathbf{S}_0^{1/2}(\delta\mathbf{x}_0 - \delta\mathbf{x})\|^2 \\ &= \left\| \begin{bmatrix} \mathbf{S}_0^{1/2}\delta\mathbf{x}_0 \\ \mathbf{W}^{1/2}\delta\mathbf{y} \end{bmatrix} - \begin{bmatrix} \mathbf{S}_0^{1/2} \\ \mathbf{W}^{1/2}\mathbf{H} \end{bmatrix} \delta\mathbf{x} \right\|^2 \end{aligned} \tag{3}$$

where $\mathbf{S}_0 = \mathbf{P}_0^{-1}$.

As $\mathbf{H}$ is $m \times n$, with $m > n$, be $\mathbf{T}$ an orthogonal matrix $m \times m$ which triangularizes $\mathbf{H}$:

$$\begin{aligned} \mathbf{T}\mathbf{H} &= \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{0} \end{bmatrix} \begin{matrix} \leftarrow \quad n \times n \\ \leftarrow (m-n) \times n \end{matrix} \\ \mathbf{T}\mathbf{y} &= \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \begin{matrix} \leftarrow \quad n \times 1 \\ \leftarrow (m-n) \times 1 \end{matrix} \end{aligned} \tag{4}$$

where $\mathbf{H}_1$ is triangular superior (result from triangularization), and m is the number of measurements.

As multiplication by orthogonal matrix does not change the norm, Eq. (3) can be given by:

$$\begin{aligned} J &= \left\| \mathbf{T} \begin{bmatrix} \mathbf{S}_0^{1/2}\delta\mathbf{x}_0 \\ \mathbf{W}^{1/2}\delta\mathbf{y} \end{bmatrix} - \mathbf{T} \begin{bmatrix} \mathbf{S}_0^{1/2} \\ \mathbf{W}^{1/2}\mathbf{H} \end{bmatrix} \delta\mathbf{x} \right\|^2 \\ &= \left\| \begin{bmatrix} \delta\mathbf{y}_1 \\ \delta\mathbf{y}_2 \end{bmatrix} - \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{0} \end{bmatrix} \delta\mathbf{x} \right\|^2 \end{aligned} \tag{5}$$

Therefore, cost function becomes:

$$J = \|\delta\mathbf{y}_1 - \mathbf{H}_1\mathbf{x}\|^2 + \|\delta\mathbf{y}_2\|^2 \tag{6}$$

and whose minimum is $J = \|\delta\mathbf{y}_2\|^2$. The solution obtained by described method is the least-squares solution. In Eq. (6), the solution $\delta\hat{\mathbf{x}} = \mathbf{H}_1^{-1}\delta\mathbf{y}_1$ does not need explicit inverse of $\mathbf{H}_1$, because this matrix is triangular superior. The solution $\delta\hat{\mathbf{x}}$ is obtained by backwards substitution. In this work, the Householder transformation is used. This technique triangularizes matrix by succession of orthogonal transformations, which are numerically efficient.

This method is iterative, once current estimative $\hat{\mathbf{x}}$ can be used as a new reference:

$$\hat{\mathbf{x}} = \bar{\mathbf{x}} + \delta\hat{\mathbf{x}}, \quad \hat{\mathbf{x}} \rightarrow \hat{\mathbf{x}}_0$$

3 LSAST Method

LSAST method (Least Squares Ambiguity Search Technique) was proposed in [2]. This method involves a modified sequential least-squares technique, in which ambiguity parameters are divided into two groups: primary ambiguities (typically three double difference ambiguities), and the secondary ambiguities. Only the primary ambiguities are fully searched. For each set of the primary ambiguities, there is a unique set of secondary ambiguities. Therefore, the search dimension is smaller and the computation time is significantly shorter than the full search approach. The choice of primary group measurements is based on GDOP value.

3.1 Potential Solutions

Equations for the three double difference primary group solution is:

$$\begin{bmatrix} \phi^1 + N^1 \\ \phi^2 + N^2 \\ \phi^3 + N^3 \end{bmatrix} = \begin{bmatrix} C_i^1 & C_j^1 & C_k^1 \\ C_i^2 & C_j^2 & C_k^2 \\ C_i^3 & C_j^3 & C_k^3 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} \tag{7}$$

where ϕ is phase double difference, N is ambiguity, C represents the direction cosines to the satellites, δx,

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δy and δz are estimated baseline correction, subscripts i, j and k designate x, y and z directions and superscripts designate the satellites.

Rewriting Eq. (7) in matrix form gives:

$$M_p = B_p X_p \tag{8}$$

where M is the measurement vector, B is direction cosines matrix, X is the solution vector and subscript p designates primary group.

The solution for X_p is:

$$X_p = B_p^{-1} M_p \tag{9}$$

For all potential solutions corresponding to all different choices of M due to different combination of ambiguity N, the value of B_p^{-1} does not change. This allows the potential solutions to arise from a combination of three basis vectors:

$$M_1^T = [1 \;\; 0 \;\; 0]\,,\; M_2^T = [0 \;\; 1 \;\; 0]\,,\; M_3^T = [0 \;\; 0 \;\; 1] \tag{10}$$

Using vectors from Eq. (10) in Eq. (9), the solutions are:

$$X_1 = B_p^{-1} M_1,\; X_2 = B_p^{-1} M_2,\; X_3 = B_p^{-1} M_3 \tag{11}$$

Thus, the general measurement vector is:

$$M_p^T = \begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix} \tag{12}$$

With solutions from Eq. (11), the solution for the measurement given by Eq. (12) is:

$$X_p = \alpha X_1 + \beta X_2 + \gamma X_3 \tag{13}$$

Making α, β e γ values vary in loops, it is possible to generate a set of potential solutions covering an extended volume of space.

3.2 Eliminating incorrect potential solutions

In order to eliminate unnecessary storing information, the secondary group could be used to test the potential solutions as they are formed in the loop. Those which do not agree with the additional measurements could be eliminated.

Firstly, the innovation vectors for the secondary group are calculated:

$$Y_s = M_s - B_s X_p \tag{14}$$

where Y is the innovation vector and subscript s refers to secondary group. The innovations corresponding to primary group are zero.

$$\Delta X = (B_c B_c^T)^{-1} B_s^T Y_s = C Y_s \tag{15}$$

where subscript c refers to complete set of double differences.

The residuals are necessary in order to quantify the quality of solutions. The residual vector R is given by:

$$R = Y_c - B_c^T \Delta X \tag{16}$$

The vector Y_c is the innovations for the secondary group Y_s, plus three zeros in elements corresponding to primary group innovations.

The estimated variance is used for measuring the quality of potential solutions:

$$q = \frac{R^T R}{m - 3} \tag{17}$$

where m is the total number of double differences. Only solutions with value of q greater than a selected threshold are retained as potential solutions.

The greater the number of double differences the higher the probability that only one solution will remain as solution which agrees with all measurement data. In addition, even when several solutions repeat as potential solutions, only the true solution will remain as the satellite geometry changes.

4 LAMBDA Method

LAMBDA method (*Least-squares AMBiguity Decorrelation Adjustment*) is a procedure for integer ambiguity estimation in carrier phase measurements. After applying a decorrelating transformation, a sequential conditional adjustment is made upon the ambiguities. As a result, integer least-squares estimates for the ambiguities are obtained. This method was introduced in [9]. [5, 6] show computational implementation aspects and ambiguity search space reducing.

These double difference observation equations are appropriate models for short baselines. The linearized equations are given by:

$$y = Aa + Bb + \epsilon \tag{18}$$

where y is observed minus computed double differences, a is integer ambiguity double difference vector, b is baseline increments vector, A and B are design matrix for ambiguity and baseline and ϵ is an unmodeled errors vector.

The LAMBDA method takes as starting point (18), using least-squares method as estimator for obtaining a and b. The minimization criterium for solving (18) is:

$$\min_{b,a} \|y - Aa - Bb\|^2_{Q_y^{-1}},\ \text{with}\ b \in \mathbb{R}^p\ \text{and}\ a \in \mathbb{Z}^n \tag{19}$$

where $\|\cdot\|^2_{Q_y^{-1}} = (\cdot)^T Q_y^{-1}(\cdot)$ and Q_y^{-1} is covariance matrix of double difference observables. The ambiguity number n is equal to the number of satellites minus one, multiplied by the used frequencies values, and the number of baseline components b is three, in case of a static receiver, or a multiple of three, in case of a moving one.

One can notice that (19) is an integer least-squares problem, because of the restriction $a \in \mathbb{Z}^n$. This problem can be solved in three steps. The first one, or float solution, consists of solving (19) with $a \in \mathbb{R}^n$ by means of a common least-squares method. As result, we have $\hat{a}$ e $\hat{b}$ as real values. The second step, or integer solution, consists of solving the minimization problem:

$$\min_a \|\hat{a} - a\|^2_{Q_{\hat{a}}^{-1}}, \quad \text{with } a \in \mathbb{Z}^n \tag{20}$$

followed by the third step, which is a correction of baseline $\hat{b}$ by difference between $\hat{a}$ and the result of minimization (20).

In fact, the second problem consists of minimizing (20), resulting in an integer estimative of ambiguity vector a. In this step is utilized the LAMBDA method [9]. The two main features of this method are: (i) ambiguity decorrelation, carried through a reparametrization (Z-transform); (ii) the actual integer ambiguity estimation.

With Z-transform, ambiguities and their covariance matrix are transformed according to:

$$z = Z^T a \quad \text{and} \quad Q_{\hat{z}} = Z^T Q_{\hat{a}} Z \tag{21}$$

Minimization itself is made upon transformed ambiguities. The minimization (20) consists of a search over grid points inside the n-dimensional ambiguity hyper-ellipsoid, defined by:

$$(\hat{z} - z)^T Q_{\hat{z}} (\hat{z} - z) \leq \chi^2 \tag{22}$$

The volume of the ellipsoid and the number of candidates can be controlled by setting the value for χ^2. Prior to the integer estimation, the ambiguities are decorrelated by application of the Z-transform: Minimization is then carried through transformed ambiguities. The output consists of $\check{z}$ together with their respective norms. Using the Z-transform, they can be transformed back to the original ambiguities:

$$\check{a} = Z^{-T} \check{z} \tag{23}$$

Estimative of $\check{b}$ and its covariance matrix $Q_{\check{b}}$ are obtained from:

$$\begin{aligned} \check{b} &= \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \check{a}) \\ Q_{\check{b}} &= Q_{\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}} \end{aligned} \tag{24}$$

The least-squares estimates $\check{b}$ and $\check{a}$ are the solution to the constrained minimization (19).

5 Results

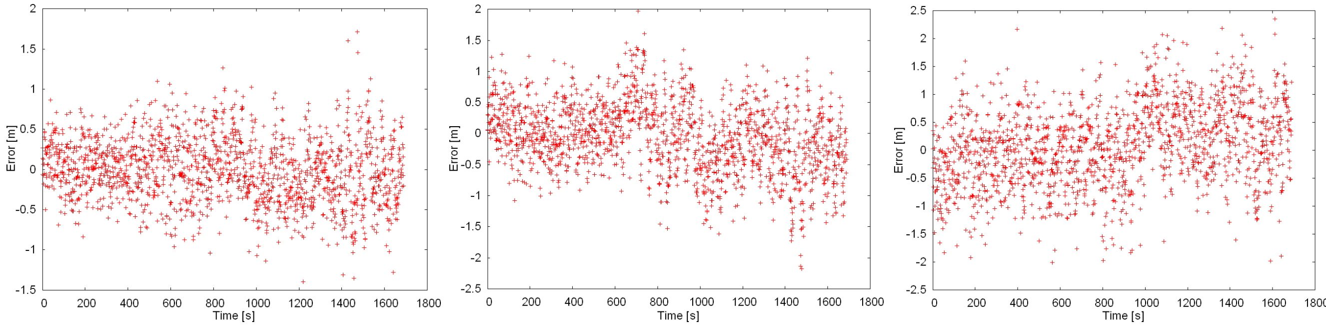
The first data set was collected by static receivers, which remain stationary at precisely known positions, to verify the quality of the proposed algorithm. The data were collected by two Trimble R8 receivers, and 1 Hz of sampling rate. Base receiver was placed on a reference landmark with coordinates N 51° 04' 45.94126", W 114° 07' 58.29947" and 1116.617 m, in ECEF coordinates of WGS-84 system, and user receiver was placed in another landmark, 2.944 m from base. The solution was attained through a iterated least-squares method with *a priori* information (section 2), processing code and carrier phase measurements. The standard deviation for code measurement was set to 1.0 m, and phase, 0.005 m.

Baseline components were calculated epoch by epoch, applying both LSAST and LAMBDA methodologies, using float ambiguity values from measurement processing in each epoch by the least-squares method. Graphics in Fig. 1 show user position error component behavior related to the base when LSAST was applied. The error statistics for each baseline component were -0.009±0.400 m on south, -0.063±0.562 m on east, and 0.080±0.709 m on vertical directions. Graphics in Fig. 2 show user position errors when using LAMBDA method, and the error statistics were 0.228±0.555 m on south, -0.015±0.387 m on east, and 0.294±1.154 m on vertical directions.

The second data set was collected by an aircraft during a flight test. These data were collected by a receiver installed on an aircraft and a fixed receiver as base. The base position coordinate are given by S 23° 13' 42.9859", W 45° 51' 23.4615" and 686.227 m, in ECEF coordinates of WGS-84 system, and sample rate was 2 Hz. For analysis purposes, the results were compared to a trajectory obtained post processing the data, which were considered precise enough for this purpose.

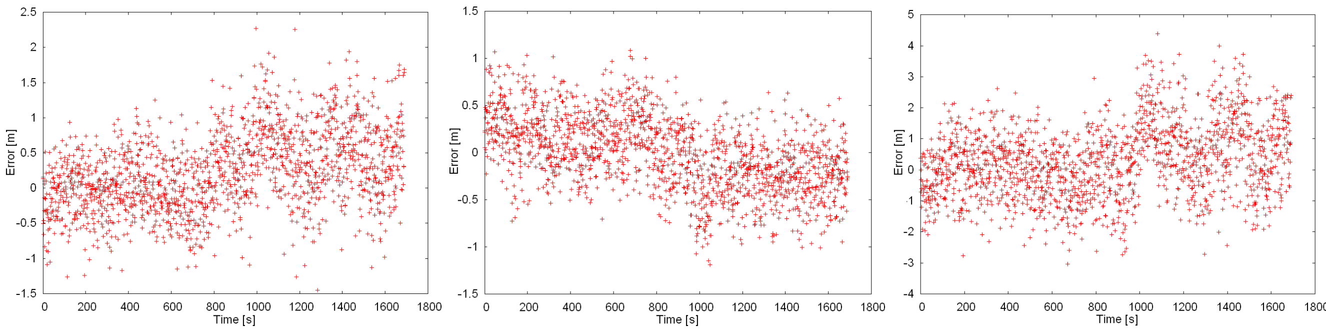
Graphics in Fig. 3 show positioning error in each direction (south, east, and vertical), compared to the reference trajectory, using LSAST method. The errors for each component were -0.399±0.735 m on south, -0.472±0.720 m on east, and -0.446±1.928 m on vertical directions. With LAMBDA method, these errors were -0.650±0.646 m on south, 0.124±0.321 m on east, and -1.553±1.174 m on vertical directions.

Compared to static, the kinematic test showed larger errors due to the aircraft motion and maneuvers, and distance to base (up to 25 km). These aspects may lead to ambiguities values within ±4 cycles



(a) (b) (c)

Figure 1: Error components using LSAST method for static data: (a) south; (b) east; (c) vertical.



(a) (b) (c)

Figure 2: Error components using LAMBDA method for static data: (a) south; (b) east; (c) vertical.

(∼0.8 m) from correct ones. It is expected, with further improvements, these values remains within ±1-2 cycles.

In order to evaluate the maximum accuracy which this method can reach in real time when all features are implemented, an off-line adjustment of ambiguities based on LAMBDA results was carried out. Graphics in Fig. 5 show error with this adjustment for each direction: -0.244±0.339 m for south, -0.015±0.183 m for east, and -0.655±0.899 m for vertical directions.

6 Conclusions and Future Works

The positioning techniques were carried out through phase measurements processing, using the LSAST and LAMBDA approach.

In static case, base receiver was placed on a landmark, and user receiver was on another landmark at 2.944 m from base, both with known positions. Measurements were processed at each epoch, through an iterated least-squares algorithm. Baseline error were, in each direction, -0.009±0.400 m on south, -0.063±0.562 m on east, and 0.080±0.709 m on vertical directions for LSAST method and 0.228±0.555 m on south, -0.015±0.387 m on east, and 0.294±1.154 m on vertical directions for LAMBDA method.

In kinematic test, user receiver was mounted on an aircraft during a test flight. The baseline error values were -0.399±0.735 m on south, -0.472±0.720 m on east, and -0.446±1.928 m on vertical directions with LSAST and -0.650±0.646 m on south, 0.124±0.321 m on east, and -1.553±1.174 m on vertical directions with LAMBDA method. These values were obtained through a iterated least-squares algorithm, in each epoch. An off-line ambiguity adjusted result showed the level of accuracy which can be attained, with the improvements to be implemented in real time. In both tests, LAMBDA is about 100 times faster than LSAST in computer processing.

This work is part of a investigation to develop a differential GPS, using carrier phase measurements in real time. Further developments consider: (i) include better dynamics in the estimation process; (ii) a better filter tunning for carrier phase measurements; (iii) add other measurements combination, such as L_2 frequency and widelane combination; (iv) validate ambiguities after resolving; (v) cycle slips detection and correction.

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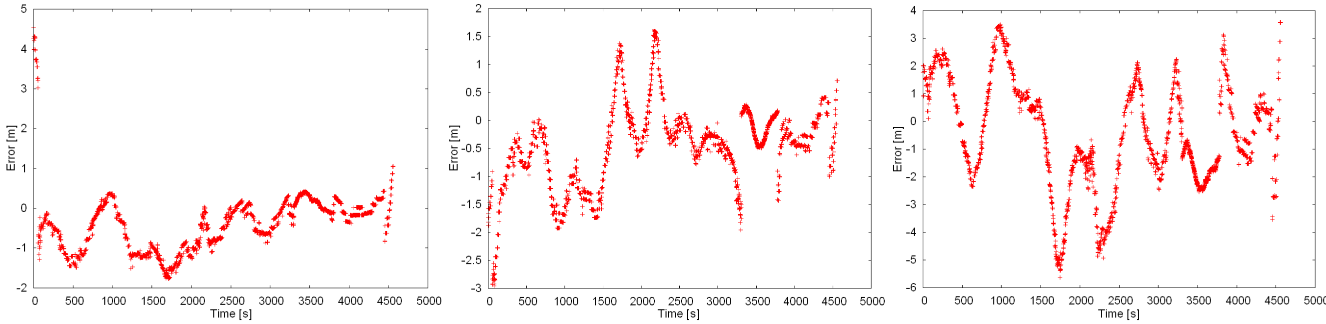
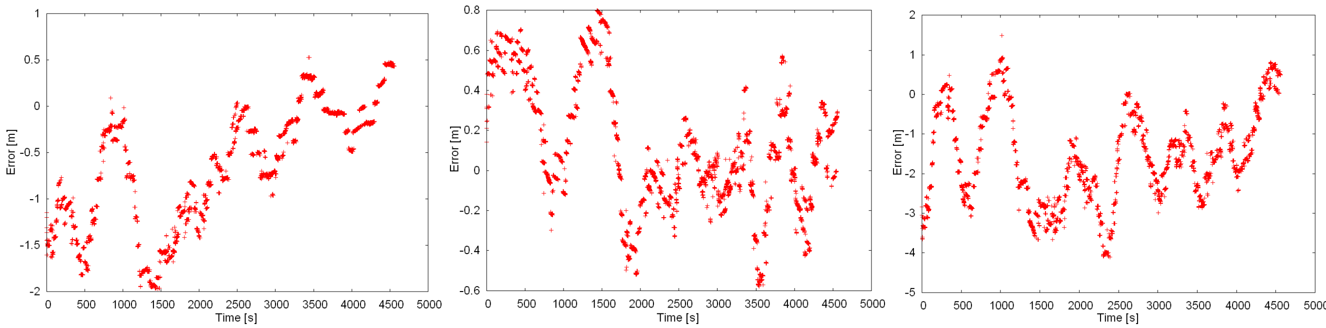


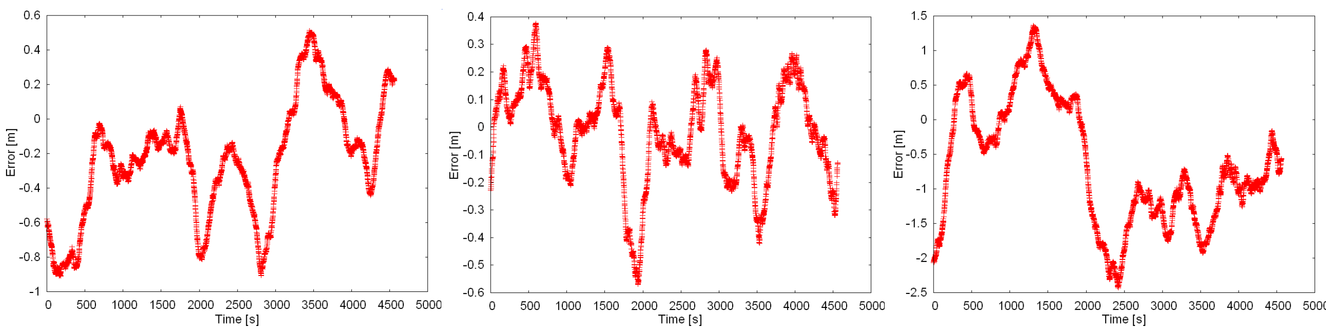
(a) (b) (c)

Figure 3: Error components using LSAST method for kinematic data: (a) south; (b) east; (c) vertical.



(a) (b) (c)

Figure 4: Error components using LAMBDA method for kinematic data: (a) south; (b) east; (c) vertical.



(a) (b) (c)

Figure 5: Error components for adjusted data: (a) south; (b) east; (c) vertical.

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