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MULTI-PARTICLE COLLISION ALGORITHM WITH HOOKE-JEEVES FOR SOLVING A STRUCTURAL DAMAGE DETECTION PROBLEM

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Summary: *A structural damage detection problem is tackled using a recent metaheuristic algorithm (Multi-Particle Collision Algorithm – MPCA) associated with a deterministic approach – the Hooke-Jeeves approach. The inverse problem of damage identification is formulated as an optimization problem assuming the displacement time history as experimental data. The objective function is defined by the square difference between the measured displacement and the displacement computed by using the forward model. The proposed hybrid approach consists of the MPCA metaheuristic coupled with the local search method of Hooke-Jeeves. A parallel version of MPCA technique using Message Passing Interface (MPI) is implemented since it reduces the computation time.*

Three different structures were considered for testing the methodology: damped spring-mass system, truss, and beam structure models. Finite element method was used for structure modeling with a different number of degrees of freedom (DOF).

Experimental data was created in silico (synthetic experimental data). Time-invariant damages were assumed to generate the synthetic displacement data. Experiments with noiseless and noisy data were carried out. Level of noise of two and five percent were considered. Good estimations of damage location and quantification have been achieved.

1. INTRODUCTION

Monitoring structural integrity and damages identification is an interesting application in the field of system identification. Modal parameters (notable frequencies, mode shape, and modal damping) in those systems are function of the physical properties of the structure (such as mass, damping and stiffness). Changes in physical properties caused by damages on the structure, such as cracks or loosening of connections, will cause detectable changes in these modal properties.

Since the damage identification problem can be described as an inverse problem, its solution is usually unstable. Small random errors, as perturbation or noise on the measurements, can cause large oscillations on the solution.

Many papers handle this class of problems. A review of damage identification techniques in structural systems, based on vibration response, was published by Doebling et al.[1]. In [2, 3], an approach for solving the damage identification problem using a hybrid method where the Genetic Algorithm is used coupled with the Conjugate Gradient method is presented. In [4], the authors deal with a structural damage detection problem using displacement measurements as experimental data, and the problem is solved using two different methodologies: the conjugate gradient method with the adjoint equation and an artificial neural network. In [5], the damage identification is performed using the measurements of natural vibration frequency and eigenvalues of a real structure.

Metaheuristic algorithms are powerful methods from the Artificial Intelligence field. They can solve optimization problems with a complexity that other classic optimization algorithms usually can not. During recent years, metaheuristics algorithms have been used to solve real-world optimization problems. There are a sort of metaheuristics which are inspired on nature processes, such as evolutionary (Genetic Algorithms, Differential Evolution, Evolution Strategies), social cooperation in animals (Particle Swarm Optimization, Ant Colony Optimization, Artificial Bee Optimization) and physical processes (Simulated Annealing).

Multi-Particle Collision Algorithm (MPCA) [6] is a metaheuristic algorithm based on the physics in the nuclear reactor, with particles (neutrons) traveling in the reactor. Two phenomena can be pointed out during the travel trajectory: absorption and scattering. The latter process is the mechanism to scape of local minima. Three principal functions in the algorithm control all the process: perturbation, exploration and scattering. Particles in the whole population behave cooperatively, i.e., after a number of objective function evaluations, defined by the user, the best particle is over-copied for all other particles, through a blackboard strategy. This algorithm has been used successfully in the solution of many optimization problems such as fault diagnosis [7], automatic configuration of neural networks applied to different problems such as atmospheric temperature profile identification [8], data assimilation [9] and climate prediction [10].

In this paper, MPCA is hybridized with the pattern search method of Hooke-Jeeves (HJ) [13]. This is a simple and efficient local optimization method that consists in exploratory and pattern moves. It uses a sequence of exploratory movements on a base point, considering a certain radius. If, during the exploration, a better solution is computed, the base point changes for this better point. Otherwise, the exploration radius is changed for a smaller value.

The remainder of this article presents, in Section 2., the forward problem defined by the dynamic equation of motion of a N degrees of freedom (N-DOF) structural system. Section 3. presents the inverse problem of identifying damages in structural systems. Section 4. presents the hybrid method of MPCA-HJ. In Section 5., three cases of simples structures are used to evaluate the proposed method. Finally, in Section 6., the final remarks are presented.

2. FORWARD PROBLEM

The general mathematical formulation of a forced vibration system is given by Eq. (1), with initial conditions given by Eq. (2).

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t)\,; \tag{1}$$

$$\begin{aligned} \mathbf{x}(0) &= \mathbf{x}_0\,, \\ \dot{\mathbf{x}}(0) &= \dot{\mathbf{x}}_0\,, \end{aligned} \tag{2}$$

where $\mathbf{M}$ is the system mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{F}$ is the external forces vector, $\mathbf{x}$ represents the displacements vector.

A numerical solution with the Newmark method [11] was implemented. The direct problem calculates the system displacement $\mathbf{x}$, knowing $\mathbf{M}, \mathbf{K}, \mathbf{C}, \mathbf{F}$ and assuming initial conditions $\mathbf{x}(0)$ and $\dot{\mathbf{x}}(0)$.

The systems considered in this work (a damped spring-mass system with 10-DOF, a truss with 12-DOF and a beam with 20-DOF) are detailed in Section 5..

3. INVERSE PROBLEM

The inverse problem of localization and quantification of damages on structures is solved as an optimization problem, through the minimization of the objective function defined in Eq. (3).

$$J(\mathbf{K}) = \sum_{i=0}^{N} \left[\mathbf{x}_i(t) - \hat{\mathbf{x}}_i(t)\right]^2\,, \tag{3}$$

where $\mathbf{x}_i(t)$ and $\hat{\mathbf{x}}_i(t)$ are the computed and measured displacements at time t, respectively. This problem will be solved using the hybrid method of MPCA-HJ, which is described in the next section.

4. HYBRID METHOD: MULTI-PARTICLES COLLISION ALGORITHM-HOOKE JEEVES

In the solution of the inverse problem, MPCA looks for good solutions within the entire search space. After a predefined number of function evaluations, the MPCA execution is interrupted. Then, HJ method is applied, taking the best solution found with MPCA as input parameter. Thus, HJ tries to improve the solution by performing local search for a low number of evaluations. If the final solution of HJ phase is better than that provided at the final of MPCA phase, it is taken as the current solution. MPCA and HJ are described in the two next subsections.

4.1 MULTI-PARTICLES COLLISION ALGORITHM

The MPCA algorithm consist in a set of particles (candidate solutions) that travels inside a nuclear reactor, which are perturbed, and then, depending on their fitness, are absorbed or scattered. The algorithm create random initial particles. The Perturbation function performs a random variation for each particle within a defined range, creating new particles. If a new particle is better that the old one, then an intensification (or local search) is made by means of the Exploration function, generating small perturbations on the particle. If the new particle was not accepted (was worse than the old particle) then a Metropolis scheme is activated (by means of the Scattering function): the particle is replaced (with a defined probability) by a new random solution, or a series of small perturbations are performed [6, 12]. After a number of function evaluations, the best particle is shared among all the particles in the set, using a Blackboard strategy, implementing a mechanism of cooperation. The MPCA pseudo-algorithm is shown in Algorithm 1 (assume $N_{\mathbf{processors}}$ = 1).

4.2 HOOKE-JEEVES

The direct search method of Hooke-Jeeves [13] consists of the application of two movements, repeatedly: the exploratory move and the pattern move. In a D-dimensional problem, a candidate solution is denoted as a vector s, of length D. An objective function f measures the fitness of the solution.

In the exploratory move (see Algorithm 2), the matrix $\mathbf{V}(D\times D)$ stores the search directions. To change a solution in an exploratory move, one column of $\mathbf{V}$ (named $\mathbf{v}_d$) scaled by a step size Δ, is added to s. In the simplest case, $\mathbf{V}$ is the identity matrix, so adding $\Delta\mathbf{v}_d$ to s implies to change the d^{th} element of the solution vector s. This process is made over all the decision variables (dimensions of the problem). A new solution is accepted if it is better than the original s. An exploratory move will return an improved solution s^n, if it was successful.

A pattern move s^n_* is computed adding to s^n a search direction $(\mathrm{s}^n - \mathrm{s}^c)$. If x^n_* is better than s^c then it will replace the latter, else s^n will become the new s^c. If no improvement is found for s^c, the step size Δ is reduced in ρ times.

As stopping criteria, a minimum step size Δ_{min} and a maximum number of function evaluations $N_{hj_{max}}$ were defined. Algorithm 2 shows the pseudo-code of the method.

5. EXPERIMENTAL RESULTS

In this work, three simple structures have been considered: a 10-DOF damped spring-mass system, a 3-bay truss structure with 12-DOF, and a 20-DOF beam structure. Damages were assumed throughout the structure, by means of reducing values of stiffness of some elements.

Experimental data were created *in silico* and obtained from the exact solution of the direct problem as it is (noiseless), or adding a random error to it (noisy data), as shown in Eq. (4).

$$\hat{\mathbf{x}}_i(t) = \mathbf{x}_i(t) + \mathcal{N}(0, \sigma) \tag{4}$$

Algorithm 1 Multi-Particle Collision Algorithm

```
for i ← 1, N_processors do                                   ▷ Initial set of particles
    NFE_i = 0, lastUpdate_i = 0
    for j ← 1, N_particles do
        currentP_{i,j} = RandomSolution
        NFE_i = NFE_i + 1
    end for
end for
for i ← 1, N_processors do                                   ▷ Initial blackboard
    bestP_i = UpdateBlackboard(currentP_{i,−})
end for
while NFE < NFE_max do                                       ▷ Stopping criteria
    for i ← 1, N_processors do
        for j ← 1, N_particles do
            newP_{i,j} = Perturbation(currentP.Solution_{i,j})
            if newP_{i,j}.Fitness < currentP_{i,j}.Fitness then
                currentP_{i,j} = newP_{i,j}
                currentP_{i,j} = Exploration(currentP_{i,j})
            else
                currentP_{i,j} = Scattering(currentP_{i,j}, newP_{i,j}, bestP_i)
            end if
            if currentP_{i,j}.Fitness < bestP_i.Fitness then
                bestP_i = currentP_{i,j}
            end if
        end for
    end for
    if NFE_i - lastUpdate_i > N_blackboard then
        for i ← 1, N_processors do                           ▷ Blackboard
            bestP_i = UpdateBlackboard(currentP_{i,−})
            lastUpdate_i = NFE_i
        end for
    end if
end while
for i ← 1, N_processors do                                   ▷ Final blackboard
    bestP_i = UpdateBlackboard(currentP_{i,−})
end for
return bestP_1
```

where $\mathcal{N}(0, \sigma)$ is a normal distribution with mean 0 and standard deviation σ. In these experiments, noises with $\sigma = 0.02$ and $\sigma = 0.05$ are used. The hybrid approach MPCA-HJ was configured with the control parameters presented in Table 1.

5.1 Damped Spring-mass system with 10-DOF

A damped spring-mass system with 10-DOF is shown in Figure 1. Structural parameters for the undamaged configuration are assumed as $M_i = 10.0\ kg$, $K_i = 2 \times 10^5\ N/m$ and the

Algorithm 2 Hooke-Jeeves

Choose $x_c, \Delta, \Delta_{min}$
while $\Delta > \Delta_{min}$ **do**
  $x^n = \text{EXPLORATORY}(x^c, \Delta)$
  if $f(x^n) < f(x^c)$ **then**
    $x^n_* = x^n + (x^n - x^c)$
    if $f(x^n_*) < f(x^n)$ **then**
      $x^c = x^n_*$
    else
      $x^c = x^n$
    end if
  else
    $\Delta = \Delta * \rho$
  end if
end while
return x^c

function EXPLORATORY(x^c, Δ)
  $x = x^c$
  for $d \leftarrow 1, D$ **do**
    $f = f(x)$
    if $f(x + \Delta v_d) < f(x)$ **then**
      $x^n = x + \Delta v_d$
    else if $f(x - \Delta v_d) < f(x)$ **then**
      $x^n = x - \Delta v_d$
    end if
  end for
  return x^n
end function

MPCA	$N_{particles}$	10
	NFE_{max}	10000
	$N_{blackboard}$	1000
	IL	0.7
	SL	1.1
HJ	Δ_{min}	1×10^{-10}
	ρ	0.8
	$N_{hj_{max}}$	10000

Table 1: Control Parameters for MPCA-HJ

damping coefficients are assumed proportional to the undamaged stiffness $C_i = 5.0 \times 10^{-3}\ K_i$, $i = 1, \ldots, 10$. All external forces $F_i = 5\ N$ are assumed constants. For the experiments, the numerical integration was performed assuming $t_f = 5\ s$ with a time step $\Delta t = 5 \times 10^{-3}\ s$. For this system, the following damage configuration was assumed: 10% on the 1^{st} spring, 25% on the 3^{rd}, 15% on the 4^{th}, 5% on the 5^{th}, 30% on the 7^{th}, 20% on the 8^{th}, and 10% on the 10^{th}. All the others elements have been assumed as undamaged.

Figure 2 shows the results for the estimated damage factor. Lighter gray shows the exact damage factor. Table 2 presents the relative error percent between the real and estimated values of the stiffness. Almost perfect damage estimation were obtained for noiseless data. Good results were obtained using 2% and 5% noisy experimental data.

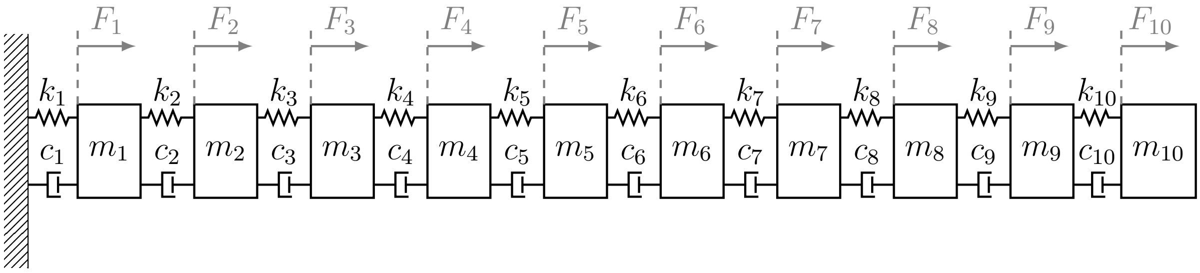

Figure 1: Spring mass system with 10-DOF

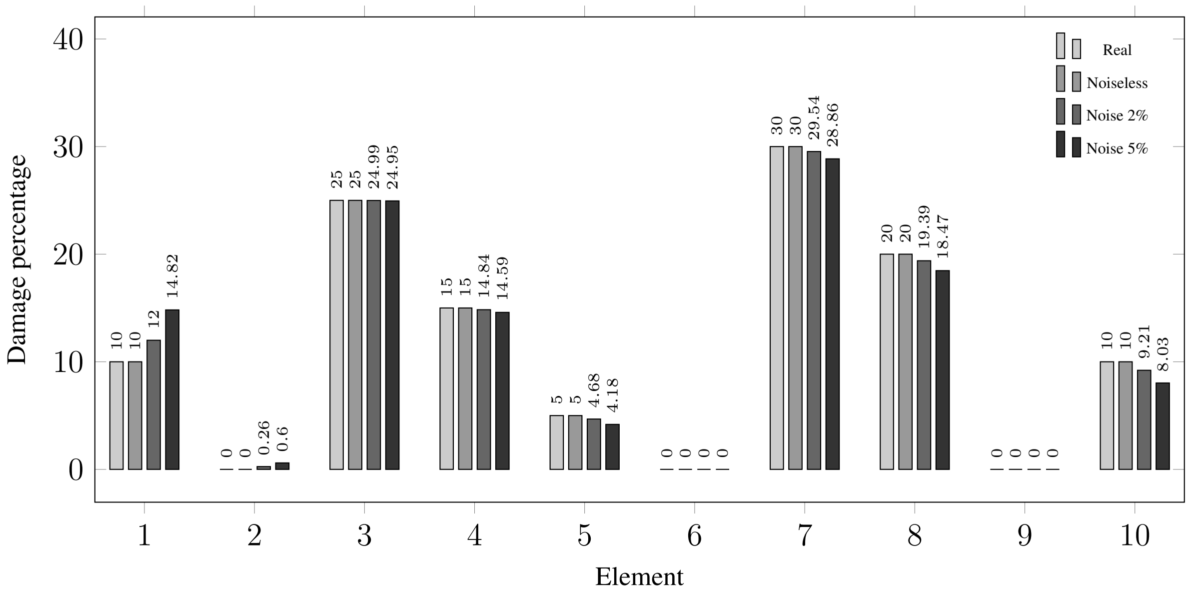

Figure 2: Estimated damages for Spring-Mass structure

Element	Noiseless	Noisy 2%	Noisy 5%
1	0.000	2.222	5.356
2	0.000	0.260	0.600
3	0.000	0.013	0.067
4	0.000	0.188	0.482
5	0.000	0.337	0.863
6	0.000	0.000	0.000
7	0.000	0.657	1.629
8	0.000	0.763	1.912
9	0.000	0.000	0.000
10	0.000	0.878	2.189

Table 2: Relative error between real and estimated stiffness in Spring-Mass structure

5.2 Truss system with 12-DOF

The truss system is composed of 12 aluminum bars ($\rho = 2700\ kg/m^3$ and $E = 70\ GPa$) with a square cross section area $A = 9 \times 10^{-4}\ m^2$ and each nondiagonal element is $l = 1.0\ m$ long. Also for the truss structure, the damping matrix is proportional to the undamaged stiffness matrix $C = 10^{-5} K_i$. Constant external forces $F = 1000\ N$ are applied to the nodes A and B in the positive diagonal direction, as shown in Figure 3. A damage configuration of 15% over the 2^{nd} element, 5% over the 4^{th}, 30% over the 7^{th}, 10% over the 10^{th} and 20% over the 12^{th} element was considered. All the others elements have been assumed as undamaged.

Figure 4 shows the results for the estimated damage factor. Lighter gray shows the exact damage factor. Table 3 presents the relative error percent between the real and estimated values of the stiffness. Almost perfect damage estimation were obtained for noiseless data. Good results were obtained using 2% noisy experimental data. Worse results were obtained using 5% noisy experimental data.

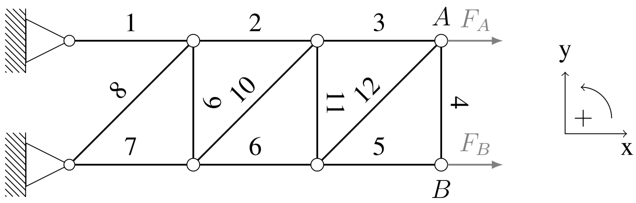

Figure 3: Three-bay truss structure

Element	Noiseless	Noisy 2%	Noisy 5%
1	0.000	2.940	3.780
2	0.000	1.118	1.235
3	0.020	0.430	2.020
4	0.116	3.968	8.947
5	0.000	0.000	0.000
6	0.000	3.730	9.010
7	0.043	0.043	0.271
8	0.010	0.000	0.000
9	0.170	6.500	0.000
10	0.011	0.967	0.722
11	0.000	0.000	0.000
12	0.013	2.238	6.200

Table 3: Relative error between real and estimated stiffness in the Truss structure

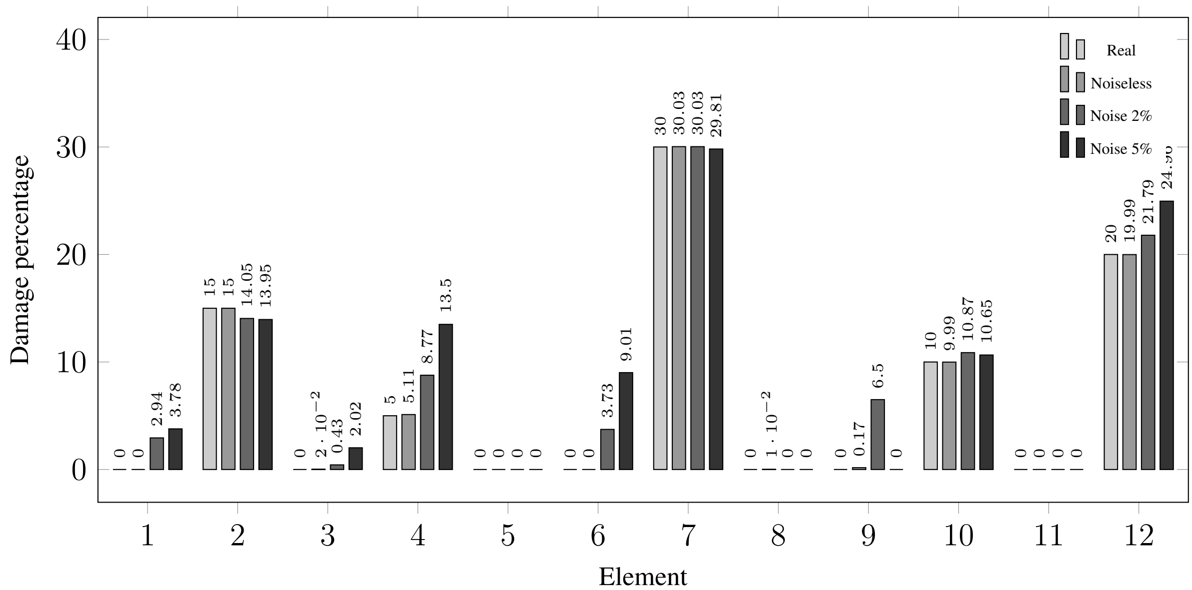

Figure 4: Estimated damages for Truss structure

5.3 Beam with 20-DOF

A beam-like structure is modeled with 10 beam finite elements and clamped at the left end, as shown in Figure 5. Each aluminum beam element ($\rho = 2700\ kg/m^3$ and $E = 70\ GPa$) has a rectangular cross section area $A = 4.5 \times 10^{-5}\ m^2$, length $l = 0.43\ m$ and inertial moment $I = 3.375 \times 10^{-11}\ m^4$. The following damage configuration was assumed: 20% damage over the 2^{nd} element; 10% over the 5^{th}; 15% over the 9^{th}; and 5% damage over the 10^{th} element. All the others elements have been assumed as undamaged.

Figure 6 shows the results for the estimated damage factor. Lighter gray shows the exact damage factor. Table 4 presents the relative error percent between the real and estimated values of the stiffness. Almost perfect damage estimation were obtained for noiseless data. Good results were obtained using 2% noisy experimental data. Worse results were obtained using 5% noisy experimental data.

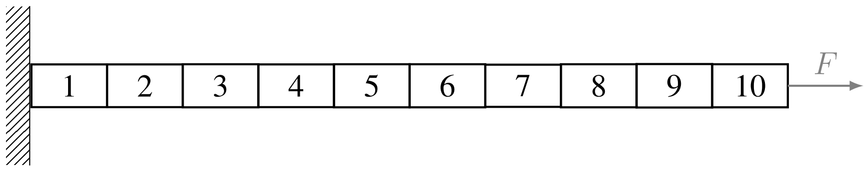

Figure 5: 10-DOF Beam structure

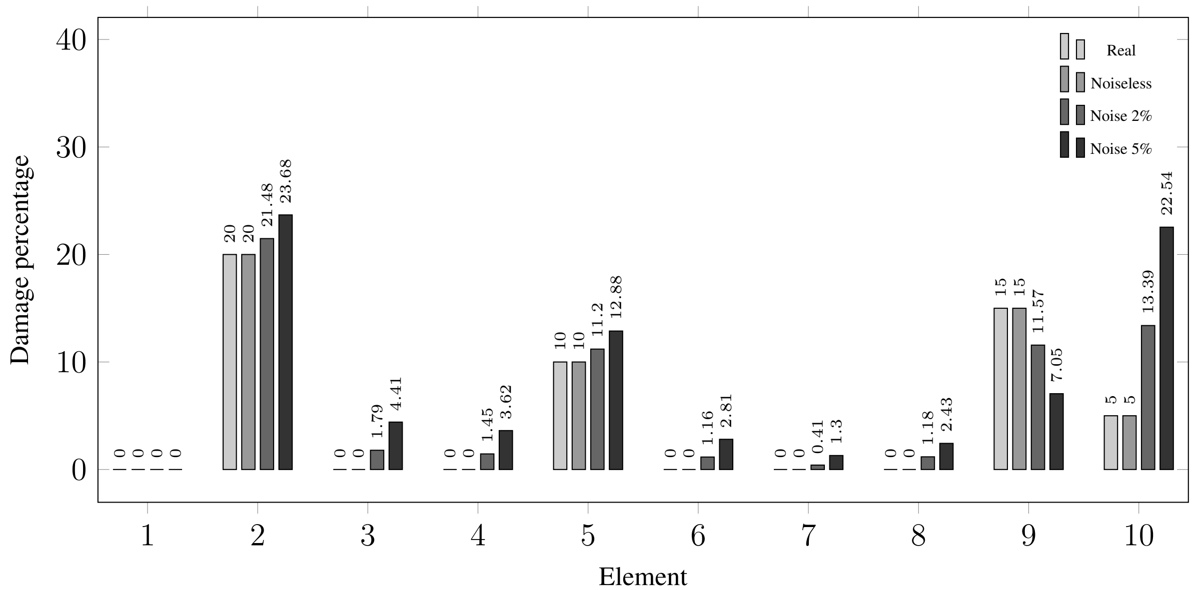

Figure 6: Estimated damages for Beam structure

Element	Noiseless	Noisy 2%	Noisy 5%
1	0.000	0.000	0.000
2	0.000	1.850	4.600
3	0.000	1.790	4.410
4	0.000	1.450	3.620
5	0.000	1.333	3.200
6	0.000	1.160	2.810
7	0.000	0.410	1.300
8	0.000	1.180	2.430
9	0.000	4.035	9.353
10	0.000	8.832	18.463

Table 4: Relative error between real and estimated stiffness in Beam structure

6. FINAL REMARKS

A hybrid approach using the MPCA metaheuristic coupled with the Hooke-Jeeves method was considered for solving the inverse problem of damage identification in structures. Three different simple structures were used to evaluate the feasibility of the approach, employing noiseless and noisy displacement measurements as experimental data. Perfect results were achieved when using noiseless experimental measures, while satisfactory results were reached when noisy experimental data are used.

It should be pointed out that the proposed methodology is able to always identify the damaged elements. Actually, a bad situation will occur if the damage was not detected. The damage quantification is more precise depending on the quality of the measurement data used in the inverse solution procedure. Nevertheless, the proposed hybrid scheme shows robustness.

In future works, the proposed hybrid method should be evaluated assuming experimental data in frequency domain and when considering more realistic structures.

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