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Massive Gravitons and their Possible Observational Signatures

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Abstract. We study here three different theories of gravity with massive gravitons, namely, the modified Fierz-Pauli model, bimetric theory, and "massive gravity". In particular, we show that the vector gravitational-wave polarization modes, in the modified Fierz-Pauli theory, can give rise to an unusual vector Sachs-Wolfe effect leaving a signature in the quadrupole form on the cosmic microwave background (CMB) radiation polarization. As an additional result, we numerically solve the Boltzmann equations for the massive tensor modes obtaining the correlation function C_l^{TT}. This result shows that massive gravitons can leave a signature on the spectrum of CMB for low multipoles.

Keywords: gravitational waves, modified theories of gravity, cosmic microwave background radiation polarization
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INTRODUCTION

An accurate statistical analysis of the anisotropies of the CMB using the seven-years WMAP data [1] has shown that the most favored cosmological model to fit the data is the flat ΛCDM model, which includes not only the very well known baryonic matter, but the mysterious dark matter and dark energy (DE) as well. However, the inclusion of dark energy is not simple. Ordinary matter, either baryonic or dark, cannot accelerate the Universe as shown by recent observations. Thus, is the Universe accelerating due to a repulsive gravity effect produced by dark energy, or does General Relativity (GR) break down on cosmological scales? Many attempts have been made addressing these two possible cases, either by introducing new features into GR, or by modifying it.

The simplest possibility for modifying GR is the introduction of a mass for the graviton. It was performed for the first time by M. Fierz and W. Pauli [2], where they considered a linearized field theory of spin-two massive particles. On the other hand, [3] presented an improvement of the earlier model pushed forward by M. Fierz and W. Pauli. It is constructed by adding a mass term to the linearized Einstein-Hilbert action through the prescriptions of field theory. We shall call this model "modified Fierz-Pauli" henceforth. Another step forward to a massive theory of gravity was taken by [4]. The author introduces a Lorentz-violating massive gravity model in which discontinuity, ghosts, and low strong coupling scale are absent. A more general Lorentz-violating gravitational theory is discussed in [5], showing that there is a number of different regions in the mass parameter space of this theory in which it can be described by a consistent low-energy effective theory without instabilities.

In the present work we shall consider these three different theories of gravitation with massive gravitons. In particular, we show that the vector gravitational-wave polarization modes, in the modified Fierz-Pauli theory, can give rise to an unusual vector Sachs-Wolfe effect leaving a signature in the quadrupole form on the cosmic microwave background radiation polarization. Based upon these results we then qualitatively show that vector mode signatures, if they do exist, could clearly be distinguished on the CMB polarization from the usual tensor mode signatures [7].

POLARIZATION STATES FOR GRAVITATIONAL WAVES

The polarization states for a GW in a arbitrary metric theory of gravitation are given by the independent modes of the Riemann tensor. In general they are represented by the set $\{\Psi_2, \Psi_3, \bar{\Psi}_3, \Psi_4, \bar{\Psi}_4, \Phi_{22}\}$ and they are so-called Newman-Penrose (NP) amplitudes [8, 9]. They play the role of definite helicity states $s = (0, \pm 1, \pm 2)$ under rotations around the z axis in a nearly Lorentz coordinate frame. In particular, the two real NP amplitudes (Ψ_2, Φ_{22}) correspond to the state $s = 0$ (which defines the scalar modes), whereas the complex NP amplitudes $(\Psi_3, \bar{\Psi}_3)$ correspond to $s = \pm 1$ (vector modes), and $(\Psi_4, \bar{\Psi}_4)$ to $s = \pm 2$ (tensor modes). Defining the "driving-force matrix" as $S_{ij}(t) := R_{i0j0}(u)$, where t is

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the proper time and $u = t - z/c$ represents a null "retarded time" as measured by an ideal detector in the coordinate system $\{t, x^i\}$, we have

$$S = \begin{pmatrix} -\frac{1}{2}(p_4+p_6) & \frac{1}{2}p_5 & -2p_2 \\ \frac{1}{2}p_5 & \frac{1}{2}(p_4-p_6) & 2p_3 \\ -2p_2 & 2p_3 & -6p_1 \end{pmatrix} = \sum_{r=1}^{6} p_r(\hat{z},t)\, E_r(\hat{z}), \tag{1}$$

where $E_r(\hat{z})$ are the *basis polarization matrices* [7, 10]. We represent the NP amplitudes as

$$p_1(\hat{z},t) = \Psi_2(u), \quad p_2(\hat{z},t) = \mathrm{Re}\,\Psi_3(u), \quad p_3(\hat{z},t) = \mathrm{Im}\,\Psi_3(u), \tag{2}$$

$$p_4(\hat{z},t) = \mathrm{Re}\,\Psi_4(u), \quad p_5(\hat{z},t) = \mathrm{Im}\,\Psi_4(u), \quad p_6(\hat{z},t) = \Phi_{22}(u). \tag{3}$$

Therefore, the polarization of a GW in an arbitrary metric theory of gravity can be fully described by the basis polarization matrices $E_r(\hat{z})$. However, due to the tensorial character of the space-time metric it is convenient to cast the polarization basis into a tensor; hence, along with its spatial components, given by $(E_r)_{ij}(\hat{z})$, there are the 00 and $0i$ components, which are zero by the very definition of the "full driving-force matrix" S, so that

$$S_{00}(t) = R_{0000}(u) = 0, \quad S_{0i}(t) = R_{0i00}(u) = 0. \tag{4}$$

Hence, the *polarization tensor* assumes the form

$$\begin{aligned}
\varepsilon^1 &= \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1 \end{pmatrix}, \ \varepsilon^2 = \begin{pmatrix} 0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0 \end{pmatrix}, \ \varepsilon^3 = \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0 \end{pmatrix} \\
\varepsilon^4 &= \begin{pmatrix} 0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0 \end{pmatrix}, \ \varepsilon^5 = \begin{pmatrix} 0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0 \end{pmatrix}, \ \varepsilon^6 = \begin{pmatrix} 0&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0 \end{pmatrix}.
\end{aligned} \tag{5}$$

GRAVITATION WITH MASSIVE GRAVITONS

The modified Fierz-Pauli model

As discussed by [7], the Einstein equations with the modified Fierz-Pauli action produce, in the absence of sources, $\left(\partial^\mu \partial_\mu + m^2\right) h_{ij} = 0$, which is clearly a Klein-Gordon equation for a wave propagating in the direction $\hat{\mathbf{k}}$. Due to the oscillatory character of the wave equation we may expand the tensor field h_{ij} into the Fourier modes as follows,

$$h_{ij}(x) = \int_{-\infty}^{\infty} \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \tilde{h}_{ij}(\mathbf{k})\, e^{-i\mathbf{k}\cdot\mathbf{r}} \quad \text{where} \quad \tilde{h}_{ij}(\mathbf{k}) = \sum_{r=1}^{6} \varepsilon^r_{ij}(\mathbf{k}) \tilde{h}^r(\mathbf{k}). \tag{6}$$

In particular, for the transverse-traceless (TT) component of the tensor perturbation to the metric, corresponding to the Ψ_4 mode with $r = 4,5$, we write $\tilde{h}^{\perp}_{ij}(\mathbf{k}) = \varepsilon^4_{ij}(\mathbf{k})\tilde{h}^4(\mathbf{k}) + \varepsilon^5_{ij}(\mathbf{k})\tilde{h}^5(\mathbf{k})$. The tensor $h^{\perp}_{ij}$ encompasses then both transverse polarization modes "+" and "×" characteristic of GR. We write the extension of its definition to the Ψ_3 modes (associated with $r = 2,3$) as $\tilde{h}^{\parallel}_{ij}(\mathbf{k}) = \varepsilon^2_{ij}(k)\tilde{h}^2(\mathbf{k}) + \varepsilon^3_{ij}(\mathbf{k})\tilde{h}^3(\mathbf{k})$, which corresponds to a longitudinal polarization state. In this paper we do not consider the GW scalar polarization modes Ψ_2 and Φ_{22}, since they couple to the δg_{00} scalar component in the metric perturbation on cosmological scales, and then do not produce "handedness" to excite the CMB B-polarization mode.

The bimetric model and the Massive Gravity

The bimetric model proposed by Visser in [6] combines into a single theory a dynamical space-time metric with a nondynamical metric which allows the introduction of a mass for the graviton even in the strong-field limit. However, as was shown by de Paula *et. al.* [11], in the weak-field limit the bimetric model is absolutely equivalent to the modified Fierz-Pauli model as discussed above. Due to this property we henceforth consider the modified Fierz-Pauli solely.

In the case of massive gravity the underlying ideas are a bit different. The space-time metric in the weak field is $g_{\alpha\beta} = \eta_{\alpha\beta} + \delta g_{\alpha\beta}$, where $\delta g_{\alpha\beta}$ plays the role of a perturbation to the Minkowski background metric, and assumes the form: $\delta g_{00} = 2\varphi, \quad \delta g_{0i} = S_i - \partial_i B, \quad \delta g_{ij} = -\chi_{ij} - \partial_i F_j - \partial_j F_i + 2\left(\psi\delta_{ij} - \partial_i\partial_j E\right)$, where φ, ψ, B, E are scalar fields, F_i and S_i are vector fields, and χ_{ij} is a tensor field. The vector and tensor fields satisfy the well known constraints $\chi_{ij},{}^{j} = 0$, $\chi^{i}{}_{i} = 0$, and $F^{i}{}_{,i} = S^{i}{}_{,i} = 0$ necessary to match the number of independent fields to the ten independent components of the metric $\delta g_{\alpha\beta}$. In particular, we can obtain after some calculations the following equation $\left(\partial^\mu\partial_\mu + m_2^2\right)\chi_{ij} = 0$ (see, for details, [4, 7]). Here, m_2 is the mass parameter of the "massive gravity". See that this equation is exactly the same equation for the TT metric contribution of the modified Fierz-Pauli model representing the "genuine" GW polarization modes for massive gravity. The Ψ_3 content of the modified Fierz-Pauli model plays the role of extra GW longitudinal polarization modes; however, in "massive gravity", the vector fields F_i and S_i evolve as massive spin-one particles with transverse polarization, and have nothing to do with extra GW polarization states.

The cosmological perturbations

For cosmological perturbations associated with massive gravitons, the metric $\delta g_{\alpha\beta}$ is decomposed exactly in the same way as we did above but, in this case, we multiply all these components by the square of the scale factor of the universe $a(\eta)$ (η is the conformal time). Since we are interested in the CMB polarization induced by GWs, we focus only on the TT part of metric perturbation for the massive gravity. However in the case of the modified Fierz-Pauli model the same metric decomposition cannot be performed due to the extra polarization modes. We instead introduce

$$\delta g_{\alpha\beta} = a(\eta)^2 \begin{pmatrix} 2\phi & X_i - Q_{,i} \\ X_i - Q_{,i} & -h_{ij} \end{pmatrix}, \tag{7}$$

where ϕ and Q are scalar fields, X_i is a divergenceless vector field, and h_{ij} is the cosmological version of the tensor given by the solution to equation (6), carrying the correspondent six polarization modes spanned in the NP formalism. The two scalar fields, plus the two components of the transverse vector field and the six modes of the tensor field give exactly the required ten degrees of freedom. With these considerations, we can write the following dynamical equations for the modified Fierz-Pauli model (TT and longitudinal components):

$$h^{\perp\prime\prime}{}_{ij} - \nabla^2 h^{\perp}{}_{ij} + 2\mathscr{H} h^{\perp\prime}_{ij} + a^2 m^2 h^{\perp}_{ij} = 0, \qquad h^{\parallel\prime\prime}{}_{ij} - \nabla^2 h^{\parallel}{}_{ij} + 2\mathscr{H} h^{\parallel\prime}_{ij} + a^2 m^2 h^{\parallel}_{ij} = 0, \tag{8}$$

and for the tensor field χ_{ij} of the "massive gravity" the Einstein equations produce

$$\chi''{}_{ij} - \nabla^2 \chi_{ij} + 2\mathscr{H}\chi'_{ij} + a^2 m_2^2 \chi_{ij} = 0. \tag{9}$$

Thus, the Fierz-Pauli modified model and massive gravity give rise to the same results for the TT polarization modes of the tensor perturbations as can be seen from equations (8), whereas for vector perturbations the situation changes drastically. In the modified Fierz-Pauli model the vector modes of GW polarization obey the same equation as the TT modes so that they really may contribute to the polarization of CMB as we have shown in [10]. However, for the field χ_{ij} in massive gravity, the vector perturbations behave exactly as in GR, which means that they decay too fast after the inflationary phase and do not leave any signature on CMB polarization.

THE SACHS-WOLFE EFFECT

As discussed above, the modified Fierz-Pauli and massive gravity are equivalent with respect to the GW TT tensor modes, but only the vector modes of the first model may give rise to relevant contributions to CMB polarization. The cosmological perturbations imprint a signature on the photon angular pattern, the so-called Sachs-Wolfe (SW) effect, which can be understood as the shift of photon frequency along the line of sight. Thus, we can get the following geodesic equations [7]:

$$\Psi_3: \ \frac{1}{\nu_0}\frac{d\nu_0}{d\eta} \propto -\frac{\partial h^{2,3}}{\partial\eta} Y_{2,\pm1}(\mu,\varphi), \quad \text{and} \quad \Psi_4: \ \frac{1}{\nu_0}\frac{d\nu_0}{d\eta} \propto -\frac{1}{2}\frac{\partial h^{4,5}}{\partial\eta} Y_{2,\pm2}(\mu,\varphi), \tag{10}$$

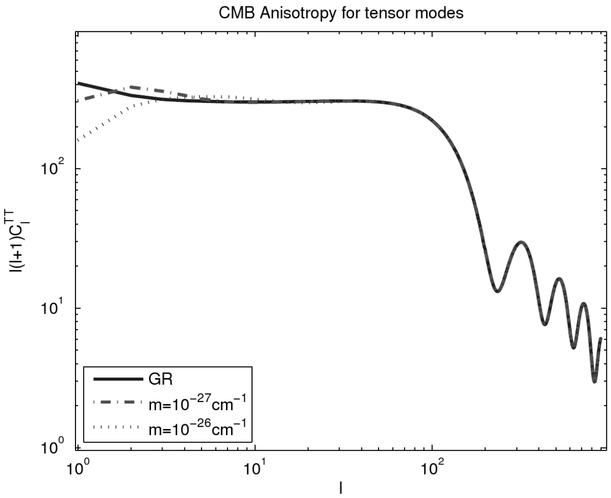

FIGURE 1. The correlation functions C_l^{TT} for general relativity and the massive gravity. Note that massive gravitons leave a signature on the spectrum of CMB for low multipoles.

where $\nu_0 = \nu a(\eta)$ and $Y_{lm}(\mu, \varphi)$ are the usual spherical harmonics. The results show us that the GW imprint upon the photon angular distribution is in the form of a quadrupole, with $m = \pm 2$ for the Ψ_4 modes ($r = 4, 5$, which coincides with GR), and with $m = \pm 1$ for the Ψ_3 modes ($r = 2, 3$).

Then, massive gravitons with the Ψ_4 polarization modes give rise to the usual tensor SW effect in GR, whereas the Ψ_3 modes do not yield its well-known vector version in GR. This happens because Ψ_3 modes arise as GW longitudinal states of polarization, and not as a massive vector fields as in GR or massive gravity. Since the experiments in the Planck satellite will improve the WMAP7 results for the E and B-modes, we may expect that such future measurements might decide whether nontrivial GW signatures for Ψ_3-modes appear or not in the CMB polarization spectrum [7, 10]. In this case, we conclude that CMB polarization measurements may be decisive to test alternative theories of gravitation - in particular, the massive model as we discussed here. As a second result of this work, we numerically solve the Boltzmann equations for the modes Ψ_4 (see [10] for details) evaluating the correlation functions for the massive tensor modes. In Figure 1 we can observe a clear signature of massive gravitons in low multipolos of the CMB. The figure shows distinct signatures for massless and massive gravitons. Therefore, for the range of masses selected $\sim 10^{-63} - 10^{-64}$ g, massive tensor modes leave a clear signature on low multipoles < 30.

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