



## THE FAULT DIAGNOSIS INVERSE PROBLEM: ACO AND ACO-d

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**Abstract.** *The automatic early detection, isolation, and localization of faults is of high interest in industrial systems, for improving reliability and safety. This process is characterized as fault diagnosis (FDI) and some problems related with robustness to external disturbances, sensitivity to incipient faults and processing time are still considered as limitations for many of the current FDI methods. This work is focused on the formulation of the fault diagnosis by an inverse problem methodology. The FDI problem is formulated as an optimization problem and takes results from the diagnosis area for acquiring prior information. The optimization problem is solved by the stochastic algorithm Ant Colony Optimization (ACO) and its modified version fuzzy-ACO (ACO-d). The proposed approach is tested using simulated data of the Inverted-Pendulum system which is recognized as a benchmark for control and diagnosis. With the purpose of analyzing the advantages of such approach, some experiments, with data corrupted with noise, are considered. The influence of ACO parameters are also taken in consideration. The results obtained show the suitability of the approach and also indicate that the parameters values allowing a greater exploration of the search space yields a better diagnosis. The ACO-d algorithm enables better diagnosis than ACO.*

**Keywords.** *Ant Colony Optimization, fault diagnosis, inverse problem, processing time, robustness, structural detectability, structural separability*

### 1 INTRODUCTION

The automatic early detection, isolation and localization of faults that have an effect on industrial systems are of high interest in order to improve reliability, safety and efficiency (Isermann, 2005). This process is called Fault Diagnosis or Fault Detection and Isolation (FDI) (Simani et al., 2002).

The increasing complexity of the systems causes an increase in the probability of failure. As a consequence, the FDI gain more importance and many methods for that purpose have been developed since the early 1970s (Isermann, 2005; Simani et al., 2002).

The FDI methods should guarantee the fast detection of the fault while rejecting false alarms attributable to noise, external disturbances and spurious signal. The first characteristic is named sensitivity and the second one is called robustness. An adequate balance of these properties is the key for the practical applicability of the FDI methods (Isermann, 2005; Simani et al., 2002) and it is still considered as a limitation of the current FDI methods (Simani et al., 2002; Simani & Patton, 2008).

The methods for Fault Diagnosis are separated in three general groups: those which do not use a model of the process, those which do use a qualitative model of the process and those that are based on a quantitative model (Angeli & Chatzinikolaou, 2004; Venkatasubramanian et al., 2003a,b,c).

The model-based approaches using the quantitative analytical model allow a deep insight into the process behavior (Isermann, 2005) and can be brought down to a few basic types such as: the parity space; observer approach; the fault detection filter approach and the parameter identification or estimation approach. The parameter estimation approach is based on the diagnosis of the faults via estimation of the parameters of the mathematical model (Frank, 1990; Isermann, 1984, 2005; Patton et al., 2000) and it is required the knowledge of the relationship between such parameters and the physical coefficients of the system, as well the influence of the faults in these coefficients (Frank, 1990; Isermann, 2005).

In the particular case of the parameter estimation based methods, there is an additional inconvenient: the high processing time makes them almost unfeasible for most online applications (Frank, 1990; Isermann, 1993).

Despite the fact that the FDI problem is an inverse problem: based on an observed behavior of the system, the causes (faults) that produced this effect should be determined, this approach has not been intensively used. Just some recent incursions have been reported in that sense (Witzak, 2006; Yang et al., 2007). With the aim of developing new and viable FDI methods and taking into account this similitude, this work presents the formulation of the fault diagnosis as an inverse problem which is written as an optimization problem and solved with stochastic algorithms. Some results related with detectability and separability are applied in order to obtain more prior information of the inverse problem and as a consequence improving the quality and reliability of the diagnosis.

The stochastic algorithm Ant Colony Optimization (ACO) and its modified version ACO-fuzzy (ACO-d) have been applied for obtaining the solution of the optimization problem. This selection is based on the adaptable and robust performance of ACO in other optimization problems as well as the better performance reported for ACO-d in order to avoid local optima based on a more intensive exploration of the search space (Becceneri et al., 2008).

Our proposal is illustrated using simulated data of the Inverted-Pendulum System (IPS) which is widely recognized as a benchmark for control and diagnosis problems. With the purpose of analyzing the advantages of such approach, mainly with respect to robustness, sensitivity and processing time, some experiments with noisy data are considered. With the aim of analyzing the influence of some ACO and ACO-d parameters in the quality of the diagnosis, different sets of values for such parameters are taken in consideration.

The main contributions of this paper may be summarized as follows: the study of a new approach for the development of robust and sensitive FDI methods based on direct fault estimation and its formulation as an inverse problem; the combination of this approach with some results for diagnosis based on the structure of the model; and a comparison between the direct fault estimation with the stochastic algorithm ACO and its modified version ACO-d. The viability of the proposal is demonstrated by diagnosing simulated fault data of the IPS.

This work is organized as follows. In the second section the modeling of faults and the model-based FDI methods with an inverse problem formulation are introduced. After that, the third section details the study case, IPS, and the simulation results. The following section describes the application of some reported results for getting prior information of the system. The fifth section provides a brief description of the algorithms ACO and ACO-d. The Results section shows the application of the methodology proposed to the solution of the study case and the results obtained. Finally some concluding remarks are presented.

## 2 MODELING FAULTS AND FORMULATION OF THE FDI INVERSE PROBLEM

Let's consider the following process model

$$\begin{aligned}\dot{\mathbf{x}}(t) &= f(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\theta}) \\ \mathbf{y}(t) &= g(\mathbf{x}(t), \boldsymbol{\theta})\end{aligned}\quad (1)$$

that represents as close as possible the physical laws which govern the process behavior (Isermann, 2005). The vector of state variables is represented by  $\mathbf{x}(t) \in \mathbb{R}^n$ . The measurable input signals  $\mathbf{u}(t) \in \mathbb{R}^m$  and output signals  $\mathbf{y}(t) \in \mathbb{R}^l$  can be directly obtained by the use of physical sensors;  $\boldsymbol{\theta}_p \in \mathbb{R}^j$  is the process parameters vector and determines the model parameters vector  $\boldsymbol{\theta} = [\boldsymbol{\theta}_p]^T$ .

The components of the process parameters vector are identified with the components of the physical process coefficients vector  $\boldsymbol{\rho} \in \mathbb{R}^r$ , and in general  $r \neq j$ . The variations of these coefficients are generally related with faults. The estimations of the vector  $\boldsymbol{\theta}_p$  will allow to detect the faults once the relationships between  $\boldsymbol{\theta}_p - \boldsymbol{\rho}$  and  $\boldsymbol{\rho}$  - faults are established (Isermann, 1984). This divides the diagnosis into two steps, the first one considers the estimations of the parameters vector  $\boldsymbol{\theta}_p$ , permitting the detection; and the second includes the determination of the faults based on the mentioned relationships. If  $j \leq r$  the relationship between process parameters and physical coefficients will be not one to one and as consequence some faults will be not separable (Isermann, 1984, 2005).

For estimating  $\boldsymbol{\theta}_p$ , two main approaches have been considered: minimize the equation error or minimize the output error. The first one permits the use of the least squares estimator and it is also necessary the use of the derivatives of the input and output data vector as well the use of filters for improving the numerical properties. In the second case numerical optimization is necessary, and the resulting high computational time brings difficulties in the applications for real on-line processes (Isermann, 2005). Some applications of evolutionary algorithms and neural networks have been reported in that sense (Witczak, 2006; Yang et al., 2007).

In order to avoid the described problem of the FDI based on the parameters estimation we have considered the model that also includes the faults. In this case the model in Eq. (1) considers that the influence of the faults is absolutely represented by the fault parameters vector  $\boldsymbol{\theta}_f$  being  $\boldsymbol{\theta} = [\boldsymbol{\theta}_p \ \boldsymbol{\theta}_f]^T$ . This vector  $\boldsymbol{\theta}_f$  contains the information regarding magnitude of each fault that can affect the system. That is the reason why the estimations of the vector  $\boldsymbol{\theta}_f$  will allow diagnosing directly the system.

The modelation of faults in a state-space representation of a Linear Time Invariants Systems (LTI), (Ding, 2008), permits to incorporate additive and multiplicative faults, while allows modelling the faults in the three main parts of the system: actuator, process and sensors. Let's  $f_a \in \mathbb{R}^s$ ,  $f_p \in \mathbb{R}^q$ ,  $f_s \in \mathbb{R}^p$  be the vectors containing the additive faults in the actuator, process and sensors respectively. Making  $\boldsymbol{\theta}_f = [f_a \ f_p \ f_s]$  and introducing it in the state-space LTI model of the system we obtain

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}_f\boldsymbol{\theta}_f \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{F}_f\boldsymbol{\theta}_f\end{aligned}\quad (2)$$

The matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are known with appropriate dimensions, and the matrices  $\mathbf{E}_f$ ,  $\mathbf{F}_f$  are:

$$\mathbf{E}_f = [\mathbf{B} \ \mathbf{E} \ p_{n \times q} \ \mathbf{0}_{n \times p}] \quad \mathbf{F}_f = [\mathbf{0}_{l \times (s+q)} \ I_{l \times p}] \quad (3)$$

where the matrix  $\mathbf{E}p$  represents the influence of the process faults and  $I$  represents the identical matrix.

This kind of faults modelling has been widely used for the residual generation in other FDI model based methods such as parity space and observer approach (Ding, 2008; Frank, 1990; Simani et al., 2002), but not in the case of the methods based on parameters estimation.

Considering the process parameters vector  $\boldsymbol{\theta}_p$  to be constant, the FDI inverse problem can be established as estimating the vector  $\boldsymbol{\theta}_f$ . It can be obtained from the solution of the parameter estimation inverse problem that can be formulated as

a minimization problem:

$$\begin{aligned} \min \quad & F(\hat{\theta}_f) = \sum_{t=1}^{N_s} [\mathbf{y}_t(\theta_f) - \hat{\mathbf{y}}_t(\hat{\theta}_f)]^2 \\ \text{s.a} \quad & \theta_{f(\min)} \leq \hat{\theta}_f \leq \theta_{f(\max)} \end{aligned} \quad (4)$$

where  $N_s$  is the number of sampling instants,  $\hat{\mathbf{y}}_t(\hat{\theta}_f)$  is the estimated vector output at each time instant  $t$ , and it is obtained from the model given by the system of equations (2);  $\mathbf{y}_t(\theta_f)$  is the output vector measured by the sensors at the same instant  $t$  (Isermann, 2005).

For the solution of the optimization problem that was specified in Eq. (4), even in a noisy environment and for both linear or non linear problems, stochastic algorithms can be applied. In the present work the ACO and ACO-d are implemented.

The idea behind the application of ACO and ACO-d is to perform a robust diagnosis of the system, via direct fault estimation, with an acceptable computational effort, which makes it feasible for the on-line diagnosis and also avoiding to divide the diagnosis in two steps as required by the usual FDI parameter estimation methods.

### 3 STUDY CASE: INVERTED-PENDULUM SYSTEM (IPS)

This system is considered as a benchmark for control and diagnosis. It is formed by an inverted pendulum mounted on a motor-driven car. The objective is to keep the beam aligned with the vertical position. Here it has been considered only the two -dimensional problem where the pendulum moves only in the plane of the paper, see Fig. 1.

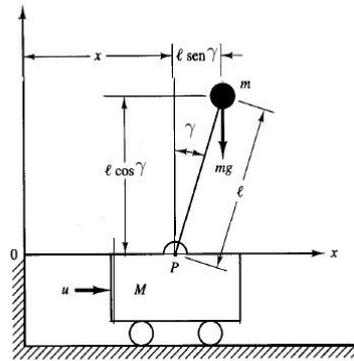


Figure 1: Inverted- Pendulum System

The mathematical model of the IPS has been widely studied, see (Ding, 2008). The system is described by a state-space representation of a linear time invariant system, affected by additive faults, see the system of equations (2). The state vector is  $\mathbf{x} = [\gamma \ \dot{\gamma} \ x \ \dot{x}]^T$ , where  $\gamma$  and  $\dot{\gamma}$  are the angle of the pendulum with respect to the vertical position and the angular velocity respectively;  $x$  and  $\dot{x}$  are the position and the velocity of the car respectively. The outputs of the system are  $\mathbf{y} = [\gamma \ x]^T$  and the input  $\mathbf{u}(t) = F$  is the control force applied to the car. The relationship between each element of the fault vector  $\theta_f = [f_1 \ f_2 \ f_3]^T$  and the faults of the system is one to one:  $f_1$  causes undesired movement of the car taking place in the actuator, this kind of fault is represented by an additive fault affecting the system input  $F$ ;  $f_2$  represents a fault in the sensor of  $\gamma$  and  $f_3$  identifies faults in the sensor that measures  $x$ . The matrices  $A$ ,  $B$ ,  $C$ ,  $E_f$ ,  $F_f$  are known and with appropriate dimensions:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{m+M}{Ml}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M}g & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ -\frac{1}{M} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E_f = [B \ 0_{4 \times 2}] \quad F_f = [0_{1 \times 2} \ I_{2 \times 2}]$$

Considering the system with the characteristics  $M = 2$  kg,  $m = 0.1$  kg and  $l = 0.5$  m, the following matrices are obtained:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E_f = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \end{bmatrix} \quad F_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Considering the nature of the faults and the properties of the IPS, then the elements of  $\theta_f$  have the following restric-

tions:

$$\begin{aligned} \theta_{f_1} \in \mathbb{R} : & -0.5 \leq \theta_{f_1} \leq 0.5 & \text{N} \\ \theta_{f_2} \in \mathbb{R} : & 0 \leq \theta_{f_2} \leq 0.01 & \text{rad} \\ \theta_{f_3} \in \mathbb{R} : & 0 \leq \theta_{f_3} \leq 0.02 & \text{m} \end{aligned}$$

In order to make a direct diagnosis of the system we must obtain estimates for  $\theta_f$ . In that sense the inverse problem of FDI that was formulated in Eq.(4) should be solved.

### 3.1 Data simulation

The behavior of the system was simulated for free of faults and under different faulty situations. The direct problem given by the system of equations in (2), was numerically solved with the fourth order Runge Kutta method. In Figs. 2 and 3 are shown two different situations that were simulated.

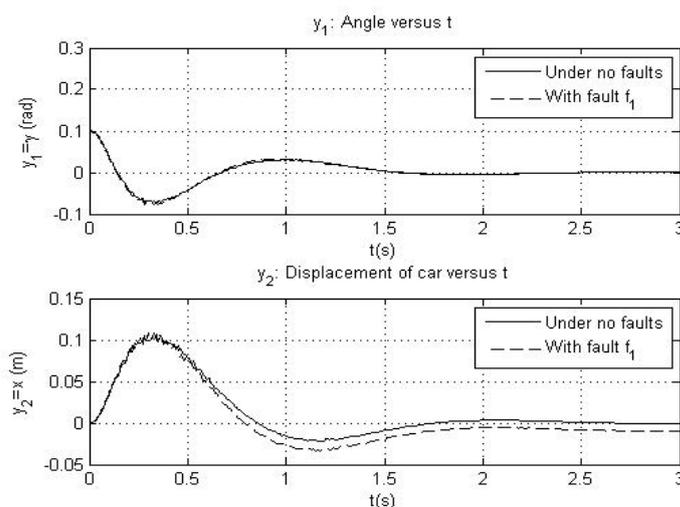


Figure 2: Simulation with no faults and fault  $f_1$ , corrupted with 5 % level noise

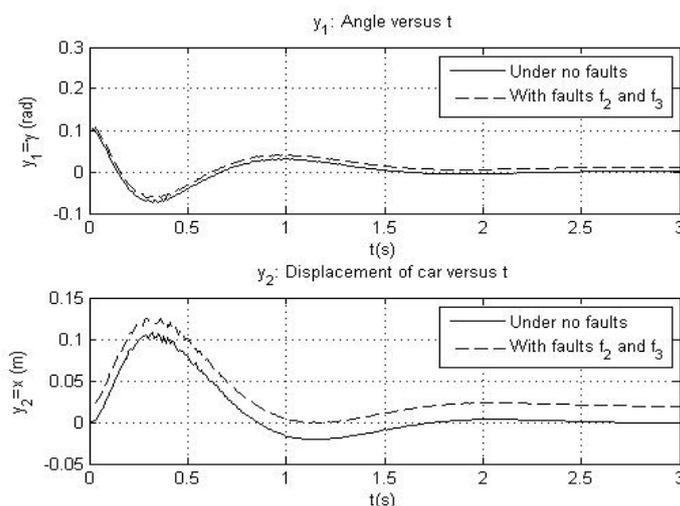


Figure 3: Simulation with no faults and faults  $f_2$  and  $f_3$ , corrupted with 5 % level noise

## 4 INVESTIGATION OF THE PROBLEM STRUCTURE

The diagnosis includes the detectability, isolability and causes of the faults. In the case of analytical model-based FDI methods, we can distinguish between the influence of the model in these characteristics, as well as the way that the method used for diagnosing permits dealing with the balance between sensitivity and robustness (Ding, 2008).

For obtaining some prior information about the uniqueness, or not, of the set of fault parameters values that can justify the observed behavior of the system, some results related with sensor placement for faults detectability and separability are applied (Aslund & Frisk, 2008; Krysander & Frisk, 2008).

The detectability of a fault indicates if the effect of the fault in the system can be monitored. The isolability of a fault is related with the separability of the fault from the other faults that may be eventually affecting the system. Some recent papers have shown that some information concerning this topic can be extracted from the structural representation of the model (Aslund & Frisk, 2008; Krysander & Frisk, 2008). These results are based on the description of the model as a bipartite graph and its Dulmage- Mendelsohn Decomposition (Krysander & Frisk, 2008).

Let's denote  $E = \{e_1, e_2, \dots, e_h\}$ , with  $h \geq (n+m)$ , the set of equations in the model of the system, and  $V = \{x_1, x_2, \dots, x_k\}$ , with  $k \geq n$ , the set of variables. Let's also assume that each fault  $f_i$  only affects one equation  $e_{f_i}$ . Let's construct the biadjacency matrix  $M$  of the bipartite graph  $G = (E, V)$  that represents the structural information of the model formed by the equations of  $E$  and the variables of  $V$ , whose elements are:

$$m_{ij} = \begin{cases} 1 & \text{if } x_j \vee \dot{x}_j \in e_i \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Let's be  $M^+$  the overdetermined part of the model  $M$ , the fault  $f_i$  is structurally detectable if there exists an observation that is consistent with fault mode  $f_i$  and inconsistent with the non fault mode, in other words the fault  $f_i$  can violate a monitorable part of the model:  $e_{f_i} \in M^+$ .

On the other hand a fault  $f_i$ , isolable from  $f_j$ , can violate a monitorable equation in the model describing the behavior of the process having fault  $f_j$ . This motivates the definition that  $f_i$  is structurally separable from  $f_j$  in a model  $M$  if  $e_{f_i} \in (M \setminus e_{f_j})^+$ .

The Dulmage-Mendelsohn decomposition (Krysander & Frisk, 2008) allows partitioning the model  $M$  in three parts,  $M_0$  the structurally undetermined part,  $\bigcup_{i=1}^n M_i$  the just-determined part and  $M^+$  the structurally overdetermined part. Not all the parts may be present in a given model. In graph theory terms the Dulmage-Mendelsohn decomposition finds a maximum-size matching in the bipartite graph of  $M$ . This has been applied for determining the detectability and isolability properties of a model, as well the minimum number of sensors to be placed in order to achieve the detectability and isolability requirements (Aslund & Frisk, 2008).

These results will be adapted and applied for the determination of some information about the structure of the system to be diagnosed. This information will permit to decide when the diagnosis can satisfy detectability and isolability requirements, which leads to the possibility of studying the more appropriate method for satisfying robustness and sensitivity conditions. This will be exemplified with the IPS.

Let's consider the model of the IPS when no sensors are added, therefore only a fault  $f_1 = \theta_{f_1}$  is affecting the system. The model of the system in this case is:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 20.601 x_2 - u - \theta_{f_1} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -0.4905 x_1 + 0.5 u + 0.5 \theta_{f_1} \end{aligned} \quad (6)$$

With the aim of satisfying the requirement that a fault affects only one equation, the new variable  $x_5$  and the new equation  $e_5 : x_5 = \theta_{f_1}$  are introduced. The equation affected by the fault  $f_1$  is  $e_{f_1} = e_5$ . The biadjacency matrix of the system has the form:

$$\begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \quad (7)$$

From matrix (7), that represents the model  $M$  described by the equations in (6), is obtained its Dulmage- Mendelsohn decomposition. This is made with the function `dmperm` of MATLAB. The result is shown in matrix (8), and it can be seen that  $\bigcup_{i=1}^n M_i = M$ , in other words, the actuator fault  $f_1$  can not be detected without sensors.

$$\begin{matrix} & x_3 & x_4 & x_1 & x_2 & x_5 \\ \begin{matrix} e_3 \\ e_4 \\ e_1 \\ e_2 \\ e_5 \end{matrix} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \quad (8)$$

The Dulmage- Mendelsohn decomposition also gives the order between the connected or strongly connected components of the graph with the biadjacency matrix  $M$ . Such order permits to know what variables should be measured in order to obtain that a certain equation  $e_h$  forms part of  $M^+$ . In this case as a consequence of this order, the order between the variables is:

$$x_3, e_3 > x_4, e_4 > x_1, e_1 > x_2, e_2 > x_5, e_5 \quad (9)$$

This order indicates that the measurements of any of the variables  $x_1, x_2, \dots, x_5$  make the equation  $e_{f_1} = e_5$  be part of  $M^+$ , in other words, make the fault  $f_1$  be structurally detectable. Let's introduce a sensor for measuring the variable  $x_3$ , the position of the car. This is described in the model by adding a new equation  $e_6 : y_2 = x_3$ . The relationship expressed in (9) indicates that the monitorable part of the new model  $M \cup e_6$  is  $(M \cup e_6)^+ = M \cup e_6$ , then  $e_{f_1} \in (M \cup e_6)^+$  and as consequence,  $f_1$  is detectable.

But now let's suppose that the sensor of  $x_3$  can be affected by an additive fault  $f_3$ , then the equation  $e_6 : y_2 = x_3 + f_3$  is affected by this fault. The fault  $f_3$  is detectable with no more sensors because of  $e_6 = e_{f_3} \in (M \cup e_6)^+$ . The other question is related with the separability of the faults  $f_1$  and  $f_3$ . Based on the definition of faults structurally separable,  $f_1$  will be separated from  $f_3$  if  $e_{f_1} \in ((M \cup e_6) \setminus e_{f_3})^+$  which produces the same problem as described in (7-9), which means that another sensor to measure any of the variables is necessary in order to separate  $f_1$  from  $f_3$ . Let's add such a sensor measuring the variable  $x_1$ , the angle  $\gamma$ , with equation  $e_7 : y_1 = x_1$ , now  $e_{f_1} \in (M \cup e_7)^+ = e_1 \cup e_2 \cup e_5 \cup e_7$ , see the Dulmage- Mendelsohn decomposition of the matrix  $M \cup e_7$  in matrix (10),

$$\begin{matrix} & x_3 & x_4 & x_1 & x_2 & x_5 \\ e_3 & \left( \begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right) & & & & \\ e_4 & & & & & & \\ e_1 & & & & & & \\ e_2 & & & & & & \\ e_7 & & & & & & \end{matrix} \quad (10)$$

It is observed that with the two added sensors it is possible to achieve detectability and separability of the faults  $f_1$  and  $f_3$ . It means that the estimation of those faults will be possible. Assuming that the new sensor  $y = x_1$  can also be affected by an additive fault  $f_2$  we have  $e_{f_2} = e_7 : y_1 = x_1 + f_2$ . This fault is detectable as a consequence of  $e_{f_2} \in (M \cup e_6 \cup e_7)^+$ , but know the cases for analyzing separability have been increased with the appearing of five new situations:  $f_1$  separable from  $f_2$ ;  $f_2$  separable from  $f_3$ ,  $f_1$  separable from  $f_2$  and  $f_3$  simultaneously,  $f_2$  separable from  $f_1$  and  $f_3$  simultaneously and  $f_3$  separable from  $f_1$  and  $f_2$  simultaneously.

From the previous analysis it can be seen that  $f_1$  is separable from  $f_2$ , and  $f_2$  separable from  $f_3$ . For situations in which only two faults are affecting the system, the faults can be separated, which means that the faults can be estimated. For the cases when the three faults are affecting the system we can not achieve separability with the considered sensors. In all the cases new sensors need to be added, which, considering that the sensor can be affected by faults, means that the problem of separability can not be solved adding more sensors if their measurements are not free of faults.

Let's show the case when considering the separability between the fault  $f_1$  and the faults  $f_2, f_3$  affecting the system at the same time. By the definition of structurally separable,  $f_1$  is separable from  $f_2, f_3$  if  $e_{f_1} \in ((M \cup \{e_6, e_7\}) \setminus \{e_{f_2}, e_{f_3}\})^+$ . This means that we have returned to the initial problem,  $M = (M \cup \{e_6, e_7\}) \setminus \{e_{f_2}, e_{f_3}\}$ , and its analysis was presented in matrix (8) which indicated that  $M^+ = \emptyset$  and  $f_1$  is not separable from  $f_2, f_3$ . New sensors can be added for obtaining this separability which can introduce new faults and the problem has no solution when the sensor is not free of faults.

After this analysis we can expect that the diagnosis of the IPS based on fault estimation will be better for situations with single or only two faults affecting the system, since for the three faults affecting the system at the same time the diagnosis can be incorrect. This indicates that we can invest in algorithms for estimating the faults in a robust an sensitive way only for the first two situations.

## 5 FDI WITH ANT COLONY OPTIMIZATION

Ant Colony Optimization (ACO) was initially proposed for integer programming problems (Dorigo & Blum, 2005) but recently it has been successfully extended and adapted to continuous optimization problems (Silva-Neto & Becceneri, (Eds.; Socha & Dorigo, 2008). A good feature of this algorithm is that its parameters can be manipulated in order to achieve a more exploitation or exploration driven structure which allows an efficient hybridization with other algorithms. ACO is inspired on the behavior of ants seeking a path between their colony and a food source. This behavior is due to the deposition and evaporation of a substance, the pheromone.

### 5.1 Description of the algorithm

For the continuous case the idea of the ACO is to mimic this behavior with simulated ants which are identified with feasible solutions (Dorigo & Blum, 2005; Silva-Neto & Becceneri, (Eds.; Socha & Dorigo, 2008). The first step is to divide the feasible interval of each variable of the problem in  $k$  possible values  $x_n^k$ . For each iteration a family of  $Z$  new ants is generated based on the information obtained from the previous ants and based on a selection mechanism. The information of the previous ants is saved on the pheromone accumulative probability matrix  $PC$  (the matrix has dimensions  $n \times k$  where  $n$  is the number of variables in the problem) whose elements are

$$pc_{ij}(t) = \frac{\sum_{l=1}^j f_{il}(t)}{\sum_{l=1}^k f_{il}(t)} \quad (11)$$

and it is updated at each iteration;  $f_{ij}$  are the elements of the pheromone matrix  $\mathbb{F}$  and express the pheromone level of the discrete  $j^{th}$  value of the  $i^{th}$  variable. This matrix is updated in each iteration based on an evaporation factor  $C_{evap}$  and an incremental factor  $C_{inc}$ :

$$f_{ij}(t+1) = (1 - C_{evap})f_{ij}(t) + \delta_{i,j,best} C_{inc} f_{ij}(t) \quad (12)$$

where

$$\delta_{ij,best} = \begin{cases} 1 & \text{if } x_i^j = x_i^{best} \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

The scheme for generating the new colony of ants considers a parameter  $q_0$  and a family of  $n$  random numbers  $q_1^{rand}, q_2^{rand}, \dots, q_n^{rand}$  for the  $z^{th}$  ant to be generated. For each variable  $x_n^{(z)}$  that will be part of the  $z^{th}$  ant is set the following generation mechanism:

$$x_n^{(z)} = \begin{cases} x_n^{\bar{m}} & \text{if } q_n^{rand} < q_0 \\ x_n^{\hat{m}} & \text{if } q_n^{rand} \geq q_0 \end{cases} \quad (14)$$

where

$$\bar{m} : f_{n\bar{m}} \geq f_{nm} \forall m = 1, 2, \dots, k \quad (15)$$

and

$$\hat{m} : (pc_{n\hat{m}} > q_n^{rand}) \wedge (pc_{n\hat{m}} \leq pc_{nm}) \forall m \geq \hat{m} \quad (16)$$

The control parameter  $q_0$  allows controlling the level of randomness during the ant generation. This fact determines, together  $Z$  and  $k$ , the level of exploitation or exploration of ACO (Dorigo & Blum, 2005; Silva-Neto & Becceneri, (Eds.: Socha & Dorigo, 2008). The general scheme of the algorithm is presented in Fig. 4.

---

**Data:**  $C_{evap}, C_{inc}, q_0, k, Z, Itr_{max}$

Generate a random initial pheromone matrix  $\mathbb{F}$  with the condition that all  $f_{ij}$  are the same;

Compute the matrix  $PC$  with Eq. (11);

Generate the random initial ants with Eqs. (14-16) and update  $X^{best}$ ;

**for**  $l = 1$  **to**  $l = Itr_{max}$  **do**

    Update  $F$  with Eq. (12)

    Update  $PC$  with Eq. (11)

**for**  $z = 1$  **to**  $z = Z$  **do**

        Generate a new ant with Eqs. (14-16)

        Update  $X^{best}$ ;

**end**

    Verify stopping criterio;

**end**

---

Figure 4: Algorithm ACO

---

## 5.2 Fuzzy- Ant Colony Optimization

The Fuzzy- Ant Colony Optimization intends to mimic a more realistic behavior of the pheromone deposit: the pheromone is an exhale odor substance and its deposit will not only affect the path where it was deposited but also those nearby paths. The idea behind the ACO-d is to simulate that kind of pheromone dispersion which will allow a more efficient exploration of the search space (Becceneri et al., 2008).

The difference between the ACO and ACO-d is based on the way the pheromone matrix is updated. In the ACO-d a fuzzy rule is used, and the amount of pheromone to be deposited on each path is proportional to the distance to the best one (Becceneri et al., 2008).

In (Becceneri et al., 2008) this scheme is applied to the traveling salesman problem, in the present work we have adapted the ACO-d to the continuous problem. Therefore we have considered a new parameter  $C_{dis}$  that indicates the coefficient of dispersion. The pheromone deposition considers the scheme described in (12) and includes a deposit (dispersion) of the pheromone in the solutions nearby to the best one  $X^{best}$ . This deposit is inversely proportional to the distance to  $X^{best}$ .

For deciding the maximum number of neighbors of  $X^{best}$  that receive pheromone at each iteration we have adopted a scheme in which each component  $x_n^{best}$  has a maximum number of neighbors for receiving pheromone, let's call such set of neighbors  $V[x_n^{best}]$  and let's define it as:

$$V[x_n^{best}] = \left\{ x_n^m : d(x_n^{best}, x_n^m) < d_{max}, 0 < m \leq k \right\} \quad (17)$$

The distance  $d_{max}$  is computed taking the average of the half of all the possible distances between values  $x_n^m$  and  $x_n^r$  with  $m, r = 1, 2, \dots, k$ . Based on the structure of the search space with the average

$$d_{max} = \frac{h + 2h + 3h + \dots + \left\lceil \frac{k}{2} \right\rceil h}{\left\lceil \frac{k}{2} \right\rceil} \quad (18)$$

where  $h = \frac{b-a}{h}$  with  $x_n \in (a, b)$ , and  $\lceil x \rceil$  represents the nearest integer from  $x$ . Working with Eq. (18) and applying the expression for the sum of the first  $n$  integers can be obtained:

$$d_{max} = h \frac{\left\lceil \frac{k}{2} \right\rceil + 1}{2} \quad (19)$$

Noting that  $d(x_n^m, x_n^{m+1}) = h$  we reformulate equation (17) as

$$V[x_n^{best}] = \left\{ x_n^m : d(x_n^{best}, x_n^m) < \frac{\left\lceil \frac{k}{2} \right\rceil + 1}{2}, 0 < m \leq k \right\} \quad (20)$$

making  $x_n^m = a + hm$  and  $x_n^{best} = a + h\bar{m}$  we can also reformulate (20) as

$$V[x_n^{best}] = \left\{ x_n^m : \bar{m} - \frac{\left\lceil \frac{k}{2} \right\rceil + 1}{2} < m < \bar{m} + \frac{\left\lceil \frac{k}{2} \right\rceil + 1}{2}, 0 < m \leq k \right\} \quad (21)$$

The scheme to lay down the pheromone is ready with the following expression for the  $x_n^m \in V[x_n^{best}]$ ,

$$f_{nm}(t+1) = f_{nm}(t) + \frac{C_{dis}}{\bar{m} - m} \quad (22)$$

### 5.3 Implementation

The variants of ACO were based on the different values for the parameters  $q_0$ . The parameter  $q_0$  permits to establish the level of randomness in the selection of the discret value of the variable (Silva-Neto & Becceneri, (Eds.), determining the trend of the search. The values  $q_0 = 0.15$ ,  $q_0 = 0.55$  and  $q_0 = 0.85$  indicate a more exploration driven procedure, a balance between exploration and exploitation, and a more exploitation of the search space, respectively. All the variants are based on the algorithm of Fig. 4 and Tab. 1 shows the values for the parameters of the algorithm in each variant. The number of ants was set in  $Z = 30$ . For the case of ACO-d the same parameters values were considered and the value for  $C_{dis} = 0.10$  in all the cases.

Table 1: Values for the parameters in ACO and ACO-d

	$k$	$q_0$	$Z$	$C_{dis}$
ACO-1, ACO-d1	63	0.15	30	0.10
ACO-2, ACO-d2	63	0.55	30	0.10
ACO-3, ACO-d3	63	0.85	30	0.10

The stopping criterion is satisfies one of the conditions:

1. Condition 1: Maximum number of iterations  $Itr_{max} = 100$ .
2. Condition 2: Maximum number of iterations for which the best value of the objective function remains constant  $Itr_{cte} = 20$ .
3. Condition 3: Minimum value for the objective function  $F(\hat{\theta}_f) < 0.001$ .

## 6 RESULTS

With the aim to analyze the merits of the diagnosis based on faults estimation with ACO and ACO-d, two aspects have been considered: robustness and computational effort. We are also concerned with the influence of some parameters of ACO in these characteristics of the diagnosis.

With this goal in mind and with the aim of testing the conclusions derived in section 4 regarding the detectability and separability, the experiments have been divided into three parts:

- First Part: A situation of single fault is considered, and only the actuator fault  $f_1$  can affect the system. The output of the system is corrupted with 5% level noise in order to analyze robustness. The faulty situation is the Case 1 shown in Tab. 2.
- Second Part: A situation of multiple faults is considered, the faults  $f_1$  and  $f_3$  are the only affecting the system. The output of the system is corrupted with 5% noise level in order to analyze robustness. The faulty situation is the Case 2 shown in Tab. 2.

- Third Part: A situation of multiple faults is considered, with the faults  $f_1$ ,  $f_2$  and  $f_3$  affecting the system. The output of the system is corrupted with 5% noise level in order to analyze robustness. The faulty situation is the Case 3 shown in Tab. 2.

All the parts considered the three sets of values for each algorithm, as shown in Tab. 1, in order to analyze their influence in the diagnosis properties.

Table 2: Faulty situations for the numerical experiments

	$f_1$	$f_2$	$f_3$
Case 1	0.5	-	-
Case 2	0.5	-	0.02
Case 3	0.5	0.01	0.02

Each experiment was repeated 30 times with the intention of making statistically valid the description of the results by means of computation of the arithmetic average of the parameters estimates. The abbreviations that were used in the tables and figures are:  $\hat{f}_i$  and  $\sigma_{\hat{f}_i}$  for the mean and variance, respectively, of the estimations for the fault  $f_i$ ,  $\bar{I}ter$  for the arithmetic average of the number of iterations that were achieved, and  $\bar{t}$  for the arithmetic average of the computing time, in seconds. The computational effort of the algorithm is analyzed based on the number of iterations.

In Tabs. 3- 5 are shown the results of the estimations for each case considered in Tab. 2.

The results in Tab. 3 show that when only a detectable fault is considered to be affecting the system, the diagnosis is correct. The best results are for ACO-1 and ACO-d1, indicating that the major exploration of the search space provides better results in the diagnosis.

Table 3: Results of the diagnosis for the Case 1 of the Tab. 2

	$\hat{f}_1$	$\sigma_{\hat{f}_1}$	$\hat{f}_2$	$\hat{f}_3$	$\hat{t}(sec)$	$\bar{I}ter$
ACO-1	0.4903	1.2e-006	-	-	35.007	32
ACO-2	0.4755	2.1e-005	-	-	50.620	48
ACO-3	0.4606	1.5e-005	-	-	45.4218	41
ACO-d1	0.4980	1.0e-006	-	-	34.6556	30
ACO-d2	0.4795	4.2e-005	-	-	39.0320	38
ACO-d3	0.4723	3.1e-005	-	-	48.5302	47

The results in Tab. 4 show that when two separable faults are affecting the system, the diagnosis is also correct. The best results are, again, for ACO-1 and ACO-d1. The number of iterations was higher than those for the previous case.

Table 4: Results of the diagnosis for the Case 2 of the Tab. 2

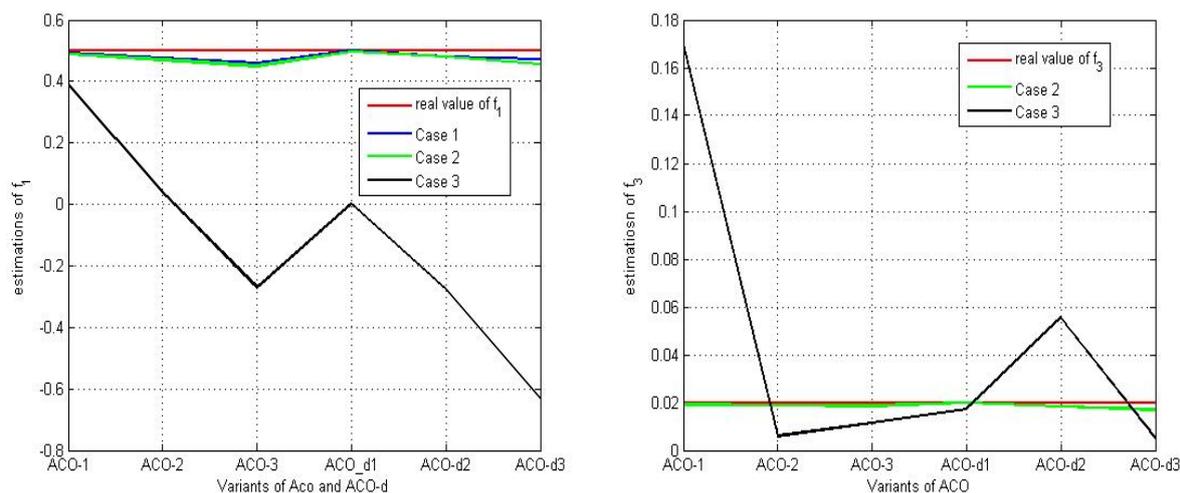
	$\hat{f}_1$	$\sigma_{\hat{f}_1}$	$\hat{f}_2$	$\hat{f}_3$	$\sigma_{\hat{f}_3}$	$\hat{t}(sec)$	$\bar{I}ter$
ACO-1	0.4891	9.3e-006	-	0.0190	1.4e-008	46.154	43
ACO-2	0.4701	7.4e-005	-	0.0188	1.0e-007	59.962	57
ACO-3	0.4499	7.6e-005	-	0.0185	3.1e-007	60.005	58
ACO-d1	0.4958	8.1e-006	-	0.0198	1.2e-008	45.094	42
ACO-d2	0.4797	7.4e-005	-	0.0182	2.8e-007	57.082	55
ACO-d3	0.4555	8.0e-005	-	0.0170	9.0e-008	61.9925	60

The results in Tab. 5 show that diagnosis is not correct when the three faults are affecting the system at the same time. These results are in agreement with the analysis of separability for the study case shown in section 4. The estimations permit to detect the faults affecting the system, but do not allow the other requirements for a diagnosis of the system. In these cases the diagnosis via faults estimation is not feasible.

The results presented in the Tabs. 3- 5 are summarize in the Fig. 5. In the Fig. 5 is shown that for the Cases 1 and 2 the diagnosis is completed, while for the Case 3 the diagnosis is not possible.

Table 5: Results of the diagnosis for the Case 3 of the Tab. 2

	$\hat{f}_1$	$\sigma_{\hat{f}_1}$	$\hat{f}_2$	$\sigma_{\hat{f}_2}$	$\hat{f}_3$	$\sigma_{\hat{f}_3}$	$\hat{t}(sec)$	$\bar{I}ter$
ACO-1	0.3892	1.1e-004	0.07	1.8e-007	0.17	2.0e-007	138.0004	98
ACO-2	0.0381	9.2e-004	0.015	9.5e-007	0.0060	1.1e-006	122.0973	85
ACO-3	-0.2707	9.2e-004	0.0001	1.8e-007	0.0115	1.3e-006	100.50	77
ACO-d1	0.0013	3.8e-004	0.0001	9.0e-007	0.0173	1.1e-007	136.865	97
ACO-d2	-0.276	9.7e-003	0.0009	9.3e-007	0.0555	3.1e-006	127.72	89
ACO-d3	-0.631	1.2e-003	0.0063	1.4e-006	0.0051	3.4e-006	109.634	70

Figure 5: Comparison between the estimations of  $f_1$  and  $f_3$  for the faulty situations described in Tab. 2

## 7 CONCLUSIONS

This study indicates that the formulation and solution of an FDI based on fault estimation is feasible. The results on structural detectability and separability give a prior information for determining the limitations of the diagnosis via fault estimation ones the inverse problem is formulated.

The study of the influence of the parameters of the algorithms indicated that the best set of parameter values for ACO and ACO-d correspondes to the version that makes a major exploration of the search space,  $q = 0.15$ . Following that experience is evident to obtain a better diagnosis with ACO-1 than with ACO-d1. This fact is because of ACO-d conception is based on a pheromone dispersion that helps in the exploration of the search space.

The results confirm the results of the section 4: the faults in the IPS can be detected via direct fault estimation in the IPS but the diagnosis is only feasible for the situation with one or two faults at the same time. For the case with the three faults affecting the system at the same time the diagnosis is not reliable due to the lack of separability between the faults.

## 8 ACKNOWLEDGEMENTS

The authors acknowlegde the support provided by FAPERJ, Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro, CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico, and CAPES, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior.

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