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# Attitude determination with magnetometers and accelerometers to use in satellite simulator

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*Abstract: Systems of attitude control of artificial satellites are dependent on a reference attitude, provided by an attitude determination process. This paper presents the implementation and tests of a fully self-contained algorithm for the determination of attitude using magnetometer and accelerometer units, for application on a satellite attitude simulator based on frictionless air bearing tables. However it is known that magnetometers and accelerometers need to be calibrated so as to allow that such measurements are used to their ultimate accuracy. A calibration method is implemented which proves to be an essential part of the procedure to achieve better attitude determination accuracy. For the stepwise real time attitude determination it was used the well-known QUEST algorithm which yields quick response with reduced computer resources. The algorithms are tested and qualified with actual data collected on the streets under controlled situations. For such street runaways the experiment employs an Honeywell (solid-state magneto-resistive technology) magnetometer HMR-2300 and a Crossbow (Xbow-CD-400) IMU navigation block consisting of a triad of accelerometers and a triad of gyros, with MEMS technology. A GPS receiver is used to obtain positional information. The collected measurements are processed through the developed algorithms, and comparisons are made for attitude determination using calibrated and non-calibrated data. The results show that the implemented process of attitude determination reaches the requirements for real-time operation with enough accuracy to a regular standby or emergency mode operation*

***Keywords: Attitude determination, algorithm QUEST, satellite simulator table, magnetometer and accelerometer.***

## INTRODUCTION

Systems of attitude control of artificial satellites are dependent on a reference attitude, provided by an attitude determination process, which involves several satellite components. In situations where high pointing accuracy is not required, as emergency, standby or during satellite orbital maneuvers, one often uses magnetometers and solar sensors, which are simpler and more reliable than more accurate instruments. These instruments are not autonomous, which means that some processing is required to estimate the attitude from their measurements.

This paper presents the implementation and tests of a fully self-contained algorithm for the determination of attitude using magnetometer and accelerometer units, for application on a satellite attitude simulator based on frictionless air bearing tables (Schwartz et al., 2003). Because of the design and physical difficulties to install a solar sensor in this table, one preferred to use accelerometers, which, similarly to the solar sensor, provides one of the directions necessary for the process of attitude determination. It is intended to apply this algorithm in the development and qualification of an on-board 3-axis attitude control system, to support the upcoming Brazilian space missions.

However it is known that magnetometers and accelerometers need to be calibrated so as to allow that such measurements are used to their ultimate accuracy. Therefore a calibration method is implemented (Foster and Elkaim, 2008) so that systematic biases and errors arising from misalignments and scale factors can be removed and the remaining source of errors are, hopefully, minor noises. This proves to be an essential part of the procedure to achieve better attitude determination accuracy. For the stepwise real time attitude determination it was used the well-known QUEST algorithm (Shuster and Oh, 1981) which yields quick response with reduced computer resources.

The algorithms are tested and qualified with actual data collected on the streets under controlled situations (Tagawa et al., 2011). The very final aim is to deploy the ADS (Attitude Determination System) algorithms on a small PC computer board with Linux operational system, operating in real time, in an air bearing table for spacecraft environment simulation. For such street runaways the experiment employs a Honeywell (solid-state magneto-resistive technology) magnetometer HMR-2300 (Honeywell, 1997) and a Crossbow (Xbow-CD-400) IMU navigation block (Crossbow, 2007) consisting of a triad of accelerometers and a triad of gyros, with technology MEMS, both with serial communication interfaces. A GPS receiver (Ashtech, 1999) is used to obtain positional information. The collected measurements are processed through the developed algorithms. Procedure calibration results are shown for both magnetometers and accelerometers, assessing the impacts of estimating biases primarily or biases plus scale factors altogether. Comparisons are made for attitude determination using calibrated and non-calibrated data. It is shown that attitude accuracy may be restrained under lack of calibration with impact on performance. The results show that the implemented process of attitude determination reaches the requirements for real-time operation with accuracy enough to a regular standby or emergency mode operation.

## CHARACTERISTICS OF THE SENSORS

In order to carry out the experiment, three types of sensors have been utilized. A 3-axis magnetometer, an IMU (Inertial Measurement Unit) with 3-axis gyros and accelerometers, and a GPS receiver for positioning data. The used sensors are described in the sequel.

### IMU – Inertial Measurement Unit

The IMU-MEMS used in this work is a Crossbow IMU model CD400-200 (Crossbow, 2007), which measures linear accelerations around 3 orthogonal axes, and rotations rate also in 3 orthogonal axes, aligned properly. Figure 1 shows the main characteristics of the IMU. Being a COTS (Commercial Off-The-Shelf) equipment, it is a low cost IMU with limited range of applications (not very accurate), notwithstanding, just enough for the application intended herein. The data is output through a RS-232 serial interface in several sampling rates (from 1 to 120Hz). In this experiment the accelerometer data was sampled at 20Hz.


| Characteristics of Gyros | Value | Characteristics of Accelerometers | Value |
|---|---|---|---|
| Range Roll, Pitch, Yaw (°/s) | ± 200 | Range X, Y, Z (g) | ± 4 |
| Bias Roll, Pitch, Yaw (°/s) | < ± 1.0 | Bias X, Y, Z (mg) | < ± 12 |
| Scale Factor Accuracy (%) | < 1 | Factor Accuracy (%) | < 1 |
| Non-Linearity (% FS) | < 0.3 | Non-Linearity (% FS) | < 1 |
| Resolution (°/sec) | < 0.05 | Resolution (mg) | < 0.6 |
| Bandwidth (Hz) | > 25 | Bandwidth (Hz) | > 75 |
| Random Walk ($^o/hr^{1/2}$) | < 4.5 | Random Walk ($m/s/hr^{1/2}$) | < 1 |

**Figure 1 – Characteristics of the CrossBow IMU-MEMS**

### Three-axis Magnetometer

The magnetometer used herein is a Honeywell model HMR-2300 (Honeywell, 1997), which measures the local magnetic field along the 3-directions. The HMR-2300 possesses 3 magneto-resistive sensors mounted tri-orthogonally as in Figure 2. The data is obtained through a RS-232 serial port at configurable sampling rates, and one used 50Hz in the experiment. The full scale covers the whole Earth surface magnetic fields and the mean performance has an accuracy of 0.1% of 1G (Gauss), translating to 1mG (milli-G) at 25°C.


| Features | Range | Values | Units |
|---|---|---|---|
| Full scale | -2 to +2 | | Gauss |
| Resolution | -2 to +2 | <70 micro | Gauss |
| Accuracy | ±1 at 25°C | 0.5% | Gauss |
| Accuracy | ±2 at 25°C | 2% | Gauss |
| Accuracy | RSS | 0.1% | Gauss |
| Sampling | 10-154Hz | Selectable | |
| Output | 3-axis | BCD ASCII or Binary | |
| Interface | Serial 9600-19200 | RS-232 or RS-485 | bauds |
| Power supply | +6 to +15 | | Volts |

**Figure 2 – Honeywell HMR 2300 3-axis magnetometer**

## GPS Receiver

The GPS receiver used is the Ashtech model Z12 (Ashtech, 1999), which provides 12 channels for GPS satellites tracking, and provides measurements in two-frequencies (L1 and L2), see Figure 3. It makes full use of the GPS constellation and yields navigation accuracy with aeronautical quality, although such accuracy is not needed herein. The data were stored internally in its mass memory and retrieved further through a RS-232 serial port. Actually, it may provide real time PVT (Position, Velocity, Time) navigation solution at 1-2Hz at most, besides the GPS time (accurate to 10micro-seconds) and DOP (Dilution Of Precision) quality flag which indicates the geometrical strength of the solution. The PVTs were obtained in this work at 1Hz rate, allowing to compute the theoretical geomagnetic field at that position, which is an important information to the calibration and to the attitude determination system.


**Figure 3 – Ashtech Z12 dual frequency GPS receiver**

## CALIBRATION METHOD

The calibration method implemented does not require any special laboratory framework, as it is independent of the attitude knowledge. Clearly stating, one does not need to know the attitude if sufficient data is provided so that observability of the biases are guaranteed. The procedure only needs to know the magnitude of the phenomen being measured, such as geomagnetic field, or gravity. Let $B$ be the magnitude of the $\mathbf{B} = (B_x \quad B_y \quad B_z)$ field:

$$B^2 = B_x^2 + B_y^2 + B_z^2 \tag{1}$$

If $\mathbf{b} = (b_x \quad b_y \quad b_z)$ is the bias vector, $\mathbf{K} = (K_x \quad K_y \quad K_z)$ the scale factors, and $(\rho \quad \phi \quad \lambda)$ the misalignment (small) angles, then the corrected measured values are (Foster and Elkaim, 2008):

$$\hat{B}_x = K_x B_x + b_x \tag{2}$$

$$\hat{B}_y = K_y \left( B_y \cos\rho + B_x \sin\rho \right) + b_y \tag{3}$$

$$\hat{B}_z = K_z \left( B_z \cos\phi \cos\lambda + B_x \sin\phi \cos\lambda + B_y \sin\lambda \right) + b_z \tag{4}$$

Using the magnitude squared and the definitions one arrives at a equation like:

$$A\hat{B}_x^2 + B\hat{B}_x\hat{B}_y + C\hat{B}_x\hat{B}_z + D\hat{B}_y^2 + E\hat{B}_y\hat{B}_z + F\hat{B}_z^2 + G\hat{B}_x + H\hat{B}_y + I\hat{B}_z + J = 0 \tag{5}$$

where *A, B, C, D, E, F, G, H, I, J* are functions of $\mathbf{b}, \mathbf{K}, (\rho \quad \phi \quad \lambda)$. Now ordering adequately the eq. (1) with the unknowns *A, B, C, D, E, F, G, H, I, J*, one obtains a single equation relating them with the measured values $(B_x \quad B_y \quad B_z)$, nice and clearly explained in Foster and Elkaim (2008). Then by accumulating more measurements where the magnitude of the measured vector is known, one obtains enough equations to statistically solve the problem, by a standard least squares method.

The procedure was coded with slight improvements like introducing the weights related to the standard deviations of the measurements, and considering the magnitude changes dynamically due to motion of the measurement sensor. A care to be taken is to make sure that the system is observable enough so that meaningful results are obtained, as is pointed for instance in Alonso and Shuster (2003).

## QUEST ATTITUDE DETERMINATION METHOD

The attitude determination problem consists of finding the attitude matrix **A** which relates two reference frames as:

$$\mathbf{v}_b = \mathbf{A}\mathbf{v} \tag{6}$$

where for example $\mathbf{v}_b$ is vector of sensor measurements in the body frame and $\mathbf{v}$ is the vector in a reference frame (e.g. inertial frame). Normally one defines a suitable quadratic loss function $J$ to be minimized:

$$J = \frac{1}{2}\sum w \left\| \mathbf{v}_b - \mathbf{A}\mathbf{v} \right\|^2 \tag{7}$$

which is commonly referred to as the Wahba‘s problem. The $w$ are adequate weigths to be defined later. The minimization of $J$ corresponds to the application of the q-method, where the attitude matrix is written in terms of quaternions $\overline{\mathbf{q}} \equiv \left( \mathbf{q}^T \;\; q_4 \right)$ so that:

$$\mathbf{A} = \left( q_4^2 - \mathbf{q}^T \mathbf{q} \right)\mathbf{I} + 2\,\mathbf{q}\mathbf{q}^T - 2q_4 \mathbf{q}\times \tag{8}$$

where $\times$ is the vector product. That way, minimizing $J$ means maximizing the gain function $g$:

$$g(\mathbf{A}) = \sum w\, \mathbf{v}_b^T \mathbf{A}\mathbf{v} = g(\overline{\mathbf{q}}) = \overline{\mathbf{q}}^T \mathbf{K} \overline{\mathbf{q}}\ , \tag{9}$$

where **K** is a 4x4 matrix defined by:

$$\mathbf{K} \equiv \begin{bmatrix} \mathbf{S} - \sigma \mathbf{I} & \mathbf{Z} \\ \mathbf{Z}^T & \sigma \end{bmatrix}, \quad \mathbf{B} = \sum w \left( \mathbf{v}_b \mathbf{v}^T \right), \quad \mathbf{S} = \mathbf{B} + \mathbf{B}^T\ , \quad \mathbf{Z} = \left[ B_{23} - B_{32} \quad B_{31} - B_{13} \quad B_{12} - B_{21} \right]^T, \quad \sigma = \mathrm{tr}[\mathbf{B}] \tag{10}$$

In other words, this is now a problem of solving for eigenvectors and eigenvalues. Details of this algebra may be found in Shuster and Oh (1981), who after proving that que optimal gain is:

$$g \approx \sum w \tag{11}$$

arrives at the QUEST solution:

$$\overline{\mathbf{q}} = \frac{1}{\sqrt{1 + \mathbf{p}^T \mathbf{p}}} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} \tag{12}$$

where **p** is the vector of Rodrigues parameters as computed in Shuster and Oh (1981):

$$\mathbf{p} = \left[ (g + \sigma)\mathbf{I} - \mathbf{S} \right]^{-1} \mathbf{Z} \tag{13}$$

It means that given at least two $\mathbf{v}_b$ measurement vectors (linearly independent of course), it is possible to quickly find the quaternion solution $\overline{\mathbf{q}}$ from where the attitude matrix may be completely recovered and so the corresponding Euler angles (roll, pitch, yaw).

## TEST CONDITIONS

The experimental assembly is shown in the right picture of Figure 4, where one sees the laboratory trailer (non-magnetic fiber structure) with the GPS unity, IMU, power supply, and a notebook for data recording. The magnetometer unity is hidden because it is fixed on top of the roof outside the trailer. The axes of the IMU and magnetometer are aligned horizontally and with the longitudinal axis of the trailer motion. The trailer was towed by a car along a path of 542m depicted in the left of Figure 4, from (45.86138141W, 23.20844975S, 609.222m) to (45.85760508W, 23.21187649S, 613.866m), recording all the data from the IMU, GPS, and magnetometer units. The path was made in a very slow movement so as not to excite longitudinal accelerations on the accelerometer. Because the calibration is, in principle, an off-line procedure, the different batches of different sensors were all correlated in time (GMT), so as to guarantee a synchronization when all the data are assembled and merged. Unreliable GPS navigation solution with PDOP (Position Dilution Of Precision) ≥ 5 were disregarded and not used to tag the position.


**Figure 4 – Trajectory of the experiment and assembly**

## RESULTS

The Foster and Elkaim (2008) procedure was used to calibrate the biases and scale factors of the magnetometers and the accelerometers. The misalignments for both were also estimated but they barely played an important role in the experiment. Table 1 shows the bias and scale factor values for the HMR-2300 magnetometer, in the 3 axes, along with the standard deviations of the calibration algorithm. The mean value of the geomagnetic field as per the IGRF (International Geomagnetic Reference Field) model was around 230mG along the path of the car. One can see that the bias values are quite pronounced (42mG magnitude) which is almost 20% of the total geomagnetic field. The scale factors are nearly unity meaning that the performance of the AD (Analog Digital) circuit inside is quite good.

**Table 1 – Calibration results of the 3-axis magnetometer: biases and scale factors**

| Variable | Values (mG) | Standard deviation (mG) |
|---|---|---|
| Bias N | -25.93094 | 0.00041 |
| Bias E | 2.61724 | 0.00059 |
| Bias D | -33.46204 | 0.00044 |
| Scale Factor N | 1.00047 | 0.00000 |
| Scale Factor E | 0.99929 | 0.00000 |
| Scale Factor D | 1.00000 | 0.00000 |

Table 2 shows the bias and scale factors values for the Crossbow model CD-400-200 accelerometers in the 3 axes, along with the standard deviations obtained from the calibration algorithm. The mean value of the gravity acceleration in total is around 1g, because of the very slow movement of the car. It is noticed that the bias values are rather small in agreement with the 12mg of the IMU data sheet. The bias level (36mg) contributes with only 4% of the total magnitude. The scale factors for this sensor were also almost unity.

**Table 2 – Calibration results of the 3-axis accelerometers: biases and scale factors**

| Variable | Values (g) | Standard deviation (g) |
|---|---|---|
| Bias N | 0.03572 | 0.00080 |
| Bias E | -0.02001 | 0.00088 |
| Bias D | -0.00328 | 0.00008 |
| Scale Factor N | 1.00000 | 0.00000 |
| Scale Factor E | 1.00000 | 0.00000 |
| Scale Factor D | 1.00000 | 0.00000 |

In the attitude determination algorithm, the quaternions were computed by the algorithm QUEST (Shuster and Oh, 1981) based on the magnetometer and accelerometer measurements. The weights of the sensors were chosen somewhat ad-hoc based on the accuracy, that is, 5mG in 300mG scale to magnetometer and 0.01g in 1g scale to accelerometer. Therefore the magnetometer measurements contributes with 62.5% and the accelerometer with 37.5% weights in the QUEST algorithm. Figure 5 shows the attitude determination results for calibrated (biases taken into account) and uncalibrated (raw biased data) altogether.

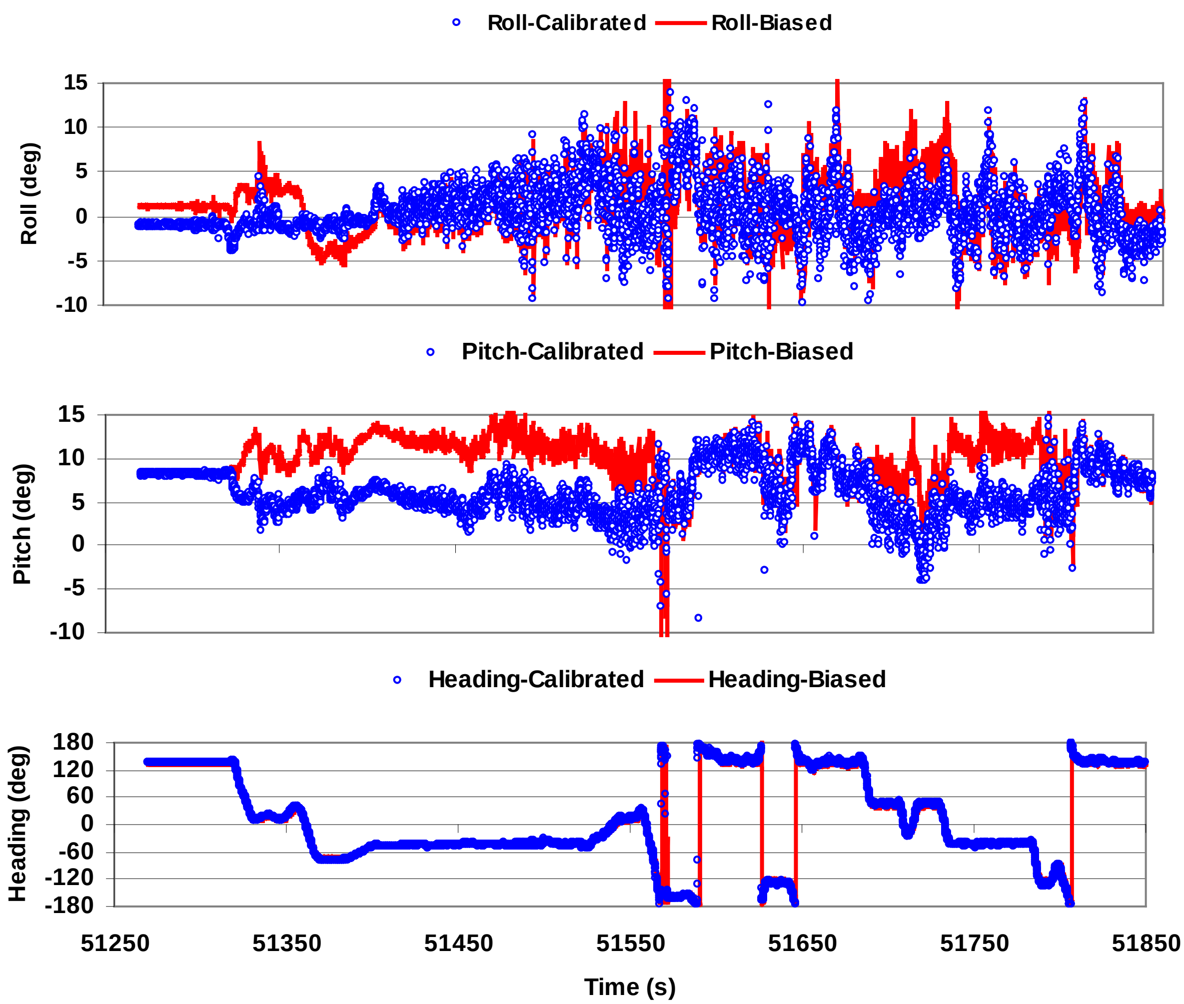


**Figure 5 – Attitude angles computed from calibrated and uncalibrated sensors**

The roll and pitch angles are small meaning that the vehicle stayed close to horizontal (leveled). Because pitch is almost always positive, the mobile is moving slightly in uphill ascent direction. The roll angle reflects the car rolling around the longitudinal motion axis. The heading (azimuth) angle differences are not perceptible is the scale of Figure 5. It is remarkable the difference in the pitch angles computed by both sources of data (raw and calibrated ones). This fact was somehow expected because of the higher bias level of magnetometer in roll (N) and yaw (D) directions (angle on xz plane), according to Table 1.

Figure 6 shows the difference between attitude angles computed using calibrated and raw measurements of the magnetometers and accelerometers. Most of the time the angles are confined to ±10°, with some ripples in some periods (see e. g. between 51550 and 51650s), mainly due to high frequency noise of the sensors. It depicts a clear idea of which attitude errors one could obtain without proper calibration of the sensors. The minimum and maximum errors were approximately –5° and 3°, -7° and 1°, and -7° and 5°, for roll, pitch, and heading angles respectively. On the other hand, the noises of the sensors should be adequately dealt with by some real time pre-processing such as smoothing, low-pass filter or some similar technique to discard bad data. If angular rates are available, e.g. from gyros, then a simple quaternion propagator would solve the issue, by comparison with estimated quaternion (by QUEST algorithm), and rejection when abnormal difference due to data outliers are present.

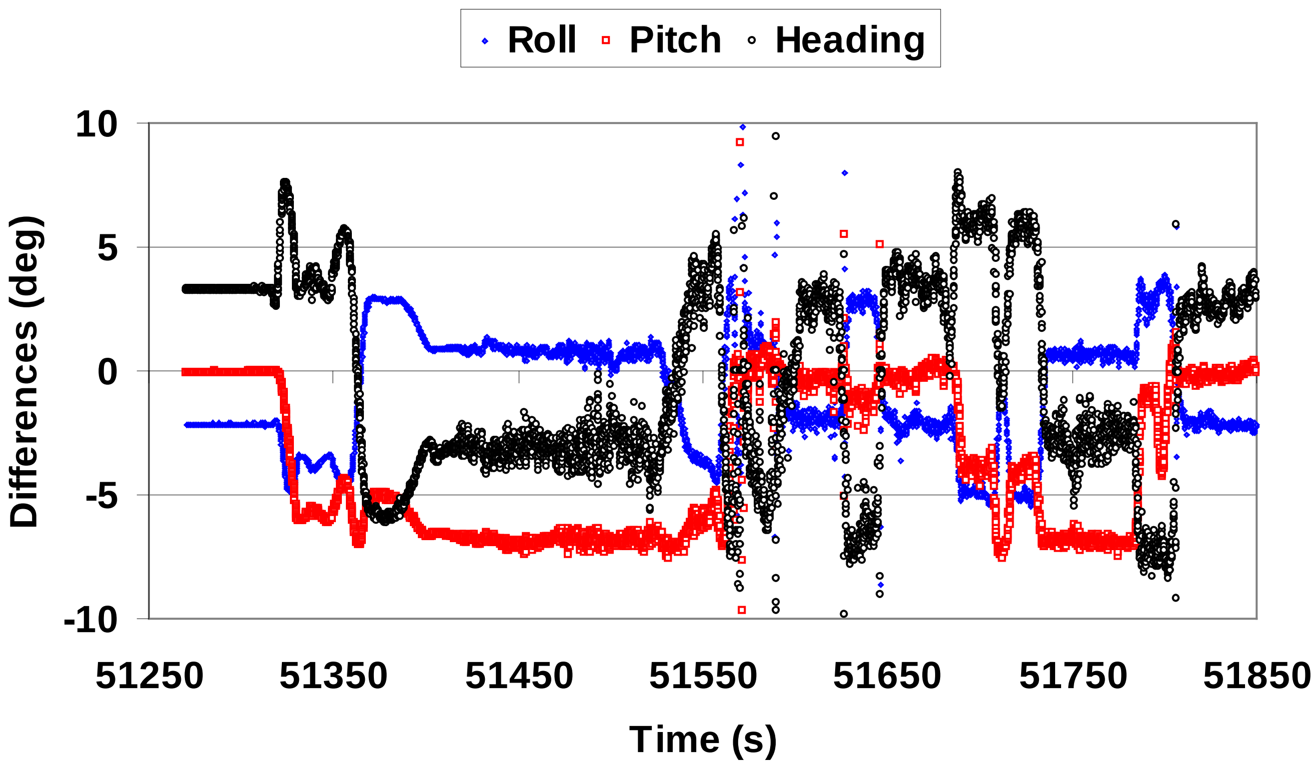


**Figure 6 – Differences between calibrated and uncalibrated attitude angles**

## CONCLUSIONS

This work presented an experimental calibration and attitude determination scheme for a moving mobile (trailer towed by a car). The calibration procedure is based on the Foster and Elkaim (2008) algorithm which does not need the attitude knowledge so that it can be applied without special approaches or environment. Magnetometer was the most biased sensor as expected and the major source of attitude angles error (62.5%). Accelerometer biases contributed only with 37.5% of the weight in the attitude determination algorithm. There was some noise present in the sensors measurements and in the future this must be addressed properly in order not to impair the performance of attitude determination. The presence of gyros, commonly present in IMU-MEMS, may be beneficial to avoid the data outliers to be input to the ADS (Attitude Determination System). For the test results presented herein the errors in the attitude angles were limited roughly to less than 10°. Thus the ADS without calibration presented an error magnitude which can not be endured and definitely makes the calibration (primarily bias removal) a mandatory step in the implementation of ADS in the air bearing table to act as a spacecraft simulator environment. Future work points to the need of porting such scheme to the real air bearing table and test it exhaustively so that its ADS performs smoothly and reliably.

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