Evaluation of Two Integer Ambiguity Resolution Methods for Real Time GPS Positioning

LEANDRO BARONI, HÉLIO KOITI KUGA
Space Mechanics and Control Division
National Institute for Space Research
12227-010, São José dos Campos, SP
BRAZIL
leandrobaroni@gmail.com, hkk@dem.inpe.br

Abstract: - The Global Positioning System (GPS) is a satellite-based navigation system which allows the user to determine position and time with high precision. However, phase measurements has an inherent difficulty, which is the ambiguity determination in number of signal wavelengths. Once ambiguities are resolved to an integer value, positioning can reach sub-meter level in accuracy. Sub-meter positioning accuracy is required in many applications, such as aircraft navigation and landing, attitude and orbit determination of satellites, navigating agricultural vehicles, among others applications. The purpose of this work is to evaluate performance of LSAST and LAMBDA methods for real time integer ambiguity resolution in situations of static and kinematic positioning. Position coordinates of a GPS receiver ("user") are estimated using data of another receiver placed on a landmark with known coordinates ("base"), using phase double difference positioning technique, and an iterated least-squares as float solution estimator. Positioning errors are shown with half meter level for static and less than meter level for kinematic positioning.

Keywords: - Ambiguity resolution, LSAST method, LAMBDA method, Least-squares, Relative positioning.

1 Introduction

The Global Positioning System (GPS) is a satellite-based navigation system which allows the user to determine position and time with high precision. GPS measurements are subject to several error sources. The combined effects of these errors in the propagation signal cause a degradation in precision of positioning. However, using phase measurements, it is possible in certain cases to increase positioning accuracy up to 100 times, if compared with positioning using code pseudorange [1].

However, phase measurements has an inherent difficulty, which is the ambiguity determination in wavelength number of signal. While signal phase changes from epoch to epoch can be measured with high accuracy, cycles integer number along propagation path (integer ambiguity) remains unknown. Once the ambiguities are solved, phase measurements can be used as very precise pseudorange measurements. Therefore, ambiguity resolution is a fundamental issue for sub-meter positioning.

Sub-meter accuracy is required in many applications. For aircraft navigation, high accuracy is required for landing, especially for automatic landings. GPS antennas and receivers can also be mounted on a vehicle or spacecraft so that orbit and attitude information of the vehicle can be derived [2], [3], [4]. Precise kinematic differential GPS will also be useful in navigating agricultural vehicles, playing a role in the distribution of work, navigation of the harvesters, and the guidance of tractors. Vehicle control flow can also be improved [5], as well as time synchronization using GPS signals [6]. Therefore, the objective of this work is to compare two methods of ambiguity resolution, LSAST and LAMBDA, in order to achieve sub-meter accuracies in real time.

The double difference observables are attained through measurements combination from two receivers referring to several GPS satellites simultaneously tracked. The advantage of this method is the elimination of most part of measurements errors, such clock bias, orbital errors and, if the baseline is short, atmosphere errors. This method allows the baseline estimation between receivers [7]. The "user" position is given in relation to the known position of a "base" receiver. Due to proximity between "base" and "user", the positioning principles by double difference presume that environmental effects (mainly troposphere, ionosphere) are the same for short baselines.

In LSAST method (Least Squares Ambiguity Search Technique), ambiguity parameters are divided into two groups: primary ambiguities (typically three double difference ambiguities), and the secondary ambiguities. Only the primary ambiguities are fully searched. For each set of the primary ambiguities, there is a unique set of secondary ambiguities. Therefore, the search dimension is smaller and the computation time is significantly shorter than the full search approach. LAMBDA method (Least-squares AMBiguity Decorrelation Adjustment) is a procedure to estimate ambiguities based on double difference models. This method uses a decorrelating transformation followed by a integer search, reducing computational time because it is not necessary a search through whole space. The estimation is carried out in three steps: float solution, integer solution, and correction of position from resolved integer ambiguities.

In this work, positioning tests using double differenced carrier phase measurements were carried out. LSAST and LAMBDA methods were used in ambiguity resolution process. Two tests were executed: static and kinematic. Static test data were collected by two dual frequency Trimble R8 GPS receivers, and the kinematic test data were collected by two Ashtech Z12 GPS receivers. The positioning errors in these tests are shown to remain with magnitude less than half a meter in static case, and with magnitude less than a meter in kinematic case, using an iterated least-squares estimator for both tests. The results were compared to a known receiver position (in static test) or a reference trajectory (in kinematic test). An off-line adjustment, which leads to smaller errors, is also made for comparison.

2 GPS System

Basically, GPS system consists of a constellation of 27 operational satellites located in orbits around 20000 km altitude, orbital period of approximately 12 hours, and 55° inclination in relation to the equatorial plan, distributed in 6 orbital planes, separated by 60°. These satellites transmit the navigation signals generated on board in two frequencies L_1 (1575.42 MHz) and L_2 (1227.6 MHz), and consisting of two codes with high transmission rate, the C/A code (open) with 1 Mb/s in L_1 and P-code (protected) with 10 Mb/s in L_1 and L_2 frequencies. The GPS modernization program aims to add new civilian signals in L_2 (L2C) and L_5 (1176.45 MHz) frequencies, and implementation is planned to be complete in 2013. These signals provide data on satellite ephemeris (GPS navigation message) and information about atomic GPS time and other informations considered relevant (satellite health, almanac, on board clock derive etc.). The GPS control segment, located in the USA, is responsible for satellite monitoring, maintenance and control in the GPS constellation, and the navigation messages. The user segment is the community of civil and military users, equipped with GPS receivers. The access code signal C/A is open, but the P-code is restricted. Therefore, as the majority of navigation systems, GPS provides distance measurements between unknown user position and system references, i.e., the GPS satellites [8], [9].

3 Double Difference Positioning

The mathematical model for phase pseudorange ϕ_u^i between satellite i and receiver u has the form [9]:

$$\phi_u^i = D_u^i + c \cdot (b_u - B^i) + T_u^i - I_u^i + \lambda N_u^i + \epsilon_u^i \quad (1)$$

where $D_u^i = |\mathbf{R}^i - \mathbf{r}_u|$ is geometric distance, $\mathbf{R}^i$ is satellite position, $\mathbf{r}_u$ is receiver antenna position, b_u is receiver clock bias, B^i is satellite clock bias, T_u^i and I_u^i are tropospheric and ionospheric errors, N_u^i is the ambiguity, ϵ_u^i represents other unmodelled errors, λ is the signal wavelength and c is light speed, 299792458 m/s.

The observable called *single difference* is formed taking the difference of pseudorange measurements between two receivers at a given epoch. Single difference for pseudorange measurement is given by [1]:

$$\begin{aligned} \phi_{ub}^i &= \phi_u^i - \phi_b^i \\ &= (D_u^i - D_b^i) + (b_u - b_b) + (T_u^i - T_b^i) + \\ &\quad + (I_u^i - I_b^i) + \lambda(N_u^i - N_b^i) + (\epsilon_u^i - \epsilon_b^i) \\ &= D_{ub}^i + b_{ub} + T_{ub}^i + I_{ub}^i + \lambda N_{ub}^i + \epsilon_{ub}^i \end{aligned} \quad (2)$$

where $(\cdot)_{ub} = (\cdot)_u - (\cdot)_b$.

The satellite clock bias term B_i, which is common for both measurements, is cancelled. The tropospheric and ionospheric terms are differences from corresponding errors at two receivers. The magnitude of these terms depends mainly on separation distance between receivers (baseline). When this distance is small, the tropospheric and ionospheric effects are almost the same and the residuals become negligible, in comparing with errors due to multipath and receiver internal noise. Thus, for a short baseline, the single difference for phase pseudorange is simplified to:

$$\phi_{ub}^i = D_{ub}^i + b_{ub} + \lambda N_{ub}^i + \epsilon_{ub}^i \quad (3)$$

Considering that the baseline is much shorter than distances between receivers and satellites by orders of magnitude, we can define the relation (Fig. 1):

$$D_{ub}^i = D_u^i - D_b^i = \mathbf{1}_b^i \cdot \mathbf{x}_{ub} \quad (4)$$

where $\mathbf{1}_b^i = \begin{bmatrix} -\frac{X^i - x_b}{\rho_b^i} & -\frac{Y^i - y_b}{\rho_b^i} & -\frac{Z^i - z_b}{\rho_b^i} \end{bmatrix}$ is the unity vector pointing from base to satellite i and $\mathbf{x}_{ub}$ represents the baseline between receivers.

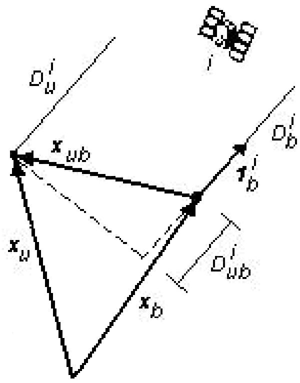


Figure 1: Geometry of an observation [1].

The relative receiver clock bias term b_{ub} is common to all single difference measurements at each epoch. This term may be canceled through *double difference* measurements, which are formed by subtracting two single differences referred to two distinct satellites i and j:

$$\phi_{ub}^{ij} = \phi_{ub}^{i} - \phi_{ub}^{j} \tag{5}$$

which may be rewrited in:

$$\begin{aligned} \phi_{ub}^{ij} &= (D_{ub}^{i} - D_{ub}^{j}) + \lambda(N_{ub}^{i} - N_{ub}^{j}) \\ &\quad + (\epsilon_{\phi,ub}^{i} - \epsilon_{\phi,ub}^{j}) \\ &= D_{ub}^{ij} + \lambda N_{ub}^{ij} + \epsilon_{\phi,ub}^{ij} \end{aligned} \tag{6}$$

The double difference observations are built choosing a master satellite M, generally by the higher elevation criteria and performing subtractions with the other measurements. This procedure assures a linearly independent double difference set [10]. The set is formed by:

$$\phi_{ub}^{Mi} = (\phi_{u}^{M} - \phi_{b}^{M}) - (\phi_{u}^{i} - \phi_{b}^{i}),\ i = 1 \dots m,\ i \neq M \tag{7}$$

where m is the visible satellite number. Thus, we have a set of m - 1 double difference measurements. This equation can be written in function of geometric distances also:

$$\phi_{ub}^{Mi} = (D_{u}^{M} - D_{b}^{M}) - (D_{u}^{i} - D_{b}^{i}) + \lambda N_{ub}^{ij} + \epsilon_{ub}^{Mi} \tag{8}$$

Using the described approximation, (8) becomes linear in relation with baseline $\mathbf{x}_{ub}$:

$$\phi_{ub}^{Mi} = (\mathbf{1}_{b}^{M} - \mathbf{1}_{b}^{i})\mathbf{x}_{ub} + \lambda N_{ub}^{ij} + \epsilon_{ub}^{Mi} \tag{9}$$

This equation constitutes the measurement model to be considered for solving the problem. Therefore, the application of double difference method performs a relative positioning, because permits a baseline estimation.

The covariance matrix for one epoch double difference observation $\mathbf{R}_{DD}$, $m-1 \times m-1$, is given by [11]:

$$\mathbf{R}_{DD} = 2\sigma_0^2 \begin{bmatrix} 2 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 2 \end{bmatrix} \tag{10}$$

where σ_0^2 is the variance of a pseudorange measurement.

4 Iterated least-squares with orthogonal transformations

In non-linear least-squares method, the cost function is euclidean weighted norm by a matrix $\mathbf{W}$, and with *a priori* information $\hat{\mathbf{x}}_0$ and $\mathbf{P}_0$:

$$\begin{aligned} J &= \|\delta\mathbf{y} - \mathbf{H}\delta\mathbf{x}\|_{\mathbf{W}}^2 + \|\delta\hat{\mathbf{x}}_0 - \delta\mathbf{x}\|_{\mathbf{P}_0^{-1}}^2 \\ &= (\delta\mathbf{y} - \mathbf{H}\delta\mathbf{x})^T\mathbf{W}(\delta\mathbf{y} - \delta\mathbf{H}\mathbf{x}) \\ &\quad + (\delta\hat{\mathbf{x}}_0 - \delta\mathbf{x})^T\mathbf{P}_0^{-1}(\delta\hat{\mathbf{x}}_0 - \delta\mathbf{x}) \end{aligned} \tag{11}$$

where $\mathbf{H}$ is observation design matrix and $\mathbf{y}$ is measurements vector. The minimization of cost function gives:

$$\begin{aligned} \delta\hat{\mathbf{x}} &= \hat{\mathbf{P}}(\mathbf{P}_0^{-1}\delta\mathbf{x}_0 + \mathbf{H}^T\mathbf{W}\delta\mathbf{y}) \\ \hat{\mathbf{P}} &= (\mathbf{P}_0^{-1} + \mathbf{H}^T\mathbf{W}\mathbf{H})^{-1} \end{aligned} \tag{12}$$

where $\hat{\mathbf{x}} = \bar{\mathbf{x}} + \delta\hat{\mathbf{x}}$ is the final estimate, and $\hat{\mathbf{P}}$ is the covariance matrix.

The normal equations solution must invert a $n \times n$ matrix, where n is the number of parameters in $\hat{\mathbf{x}}$. These inversions are a potential numerical error source, especially when the matrix is almost singular. However, literature contains several works which intend to increase numerical performance of least-squares [12], [13]. In this work, a matrix triangularization technique is used in matrix $\mathbf{H}$. Equation (11) can be rewritten as:

$$\begin{aligned} J &= \|\mathbf{W}^{1/2}(\delta\mathbf{y} - \mathbf{H}\delta\mathbf{x})\|^2 + \|\mathbf{S}_0^{1/2}(\delta\mathbf{x}_0 - \delta\mathbf{x})\|^2 \\ &= \left\| \begin{bmatrix} \mathbf{S}_0^{1/2}\delta\mathbf{x}_0 \\ \mathbf{W}^{1/2}\delta\mathbf{y} \end{bmatrix} - \begin{bmatrix} \mathbf{S}_0^{1/2} \\ \mathbf{W}^{1/2}\mathbf{H} \end{bmatrix} \delta\mathbf{x} \right\|^2 \end{aligned} \tag{13}$$

where $\mathbf{S}_0 = \mathbf{P}_0^{-1}$.

As $\mathbf{H}$ is $m \times n$, with $m > n$, be $\mathbf{T}$ an orthogonal matrix $m \times m$ which triangularizes $\mathbf{H}$:

$$\begin{aligned} \mathbf{T}\mathbf{H} &= \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{0} \end{bmatrix} \begin{matrix} \leftarrow \quad n \times n \\ \leftarrow (m-n) \times n \end{matrix} \\ \mathbf{T}\mathbf{y} &= \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \begin{matrix} \leftarrow \quad n \times 1 \\ \leftarrow (m-n) \times 1 \end{matrix} \end{aligned} \tag{14}$$

where $\mathbf{H}_1$ is triangular superior (result from triangularization), and m is the number of measurements.

As multiplication by orthogonal matrix does not change the norm, (13) can be given by:

$$\begin{aligned} J &= \left\| \mathbf{T} \begin{bmatrix} \mathbf{S}_0^{1/2}\delta\mathbf{x}_0 \\ \mathbf{W}^{1/2}\delta\mathbf{y} \end{bmatrix} - \mathbf{T} \begin{bmatrix} \mathbf{S}_0^{1/2} \\ \mathbf{W}^{1/2}\mathbf{H} \end{bmatrix} \delta\mathbf{x} \right\|^2 \\ &= \left\| \begin{bmatrix} \delta\mathbf{y}_1 \\ \delta\mathbf{y}_2 \end{bmatrix} - \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{0} \end{bmatrix} \delta\mathbf{x} \right\|^2 \end{aligned} \tag{15}$$

Therefore, cost function becomes:

$$J = \|\delta\mathbf{y}_1 - \mathbf{H}_1\mathbf{x}\|^2 + \|\delta\mathbf{y}_2\|^2 \tag{16}$$

and whose minimum is $J = \|\delta\mathbf{y}_2\|^2$. The solution obtained by described method is the least-squares solution. In (16), the solution $\delta\hat{\mathbf{x}} = \mathbf{H}_1^{-1}\delta\mathbf{y}_1$ does not need explicit inverse of $\mathbf{H}_1$, because this matrix is triangular superior. The solution $\delta\hat{\mathbf{x}}$ is obtained by backwards substitution. In this work, the Householder transformation is used. This technique triangularizes matrix by succession of orthogonal transformations, which are numerically efficient.

This method is iterative, once current estimative $\hat{\mathbf{x}}$ can be used as a new reference:

$$\hat{\mathbf{x}} = \bar{\mathbf{x}} + \delta\hat{\mathbf{x}}, \quad \hat{\mathbf{x}} \rightarrow \hat{\mathbf{x}}_0$$

5 LSAST Method

LSAST method (Least-Squares Ambiguity Solution Technique) was proposed in [14]. This method involves a modified sequential least-squares technique, in which ambiguity parameters are divided into two groups: primary ambiguities (typically three double difference ambiguities), and the secondary ambiguities. Only the primary ambiguities are fully searched. For each set of the primary ambiguities, there is a unique set of secondary ambiguities. Therefore, the search dimension is smaller and the computation time is significantly shorter than the full search approach.

The choice of primary group measurements is based on GDOP value. Satellites with low GDOP will lead to a search with less potential solutions. However, GDOP cannot be very low, in order to avoid the position uncertainty include more than one solution for secondary group measurements. The procedure is to choose primary group satellites which have a reasonable GDOP. Several methods are available to calculate GDOP values, e.g., as in [15].

5.1 Potential Solutions

Equations for primary group solution with three double difference are:

$$\begin{bmatrix} \phi^1 + N^1 \\ \phi^2 + N^2 \\ \phi^3 + N^3 \end{bmatrix} = \begin{bmatrix} C_i^1 & C_j^1 & C_k^1 \\ C_i^2 & C_j^2 & C_k^2 \\ C_i^3 & C_j^3 & C_k^3 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} \tag{17}$$

where ϕ is phase double difference, N is ambiguity, C represents the direction cosines to the satellites, δx, δy and δz are estimated baseline, subscripts i, j and k designate x, y and z directions and superscripts designate the satellites.

In (17), we have only three ambiguities, and the corresponding solution from any specific choice of three ambiguity values results in only one solution. Rewriting (17) in matrix form gives:

$$\mathbf{y}_p = \mathbf{H}_p\mathbf{x}_p \tag{18}$$

where $\mathbf{y}$ is the measurement vector, $\mathbf{H}$ is direction cosines matrix, $\mathbf{x}$ is the solution vector and subscript p designates primary group.

The solution for $\mathbf{x}_p$ is:

$$\mathbf{x}_p = \mathbf{H}_p^{-1}\mathbf{y}_p \tag{19}$$

For all potential solutions corresponding to all different choices of $\mathbf{y}$ due to different combination of ambiguity N, the value of $\mathbf{H}_p^{-1}$ does not change. This allows the potential solutions to arise from a combination of three basis vectors:

$$\begin{aligned} \mathbf{y}_1^T &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \\ \mathbf{y}_2^T &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\ \mathbf{y}_3^T &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{20}$$

Using vectors from (20) in (19), the solutions are:

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{H}_p^{-1}\mathbf{y}_1 \\ \mathbf{x}_2 &= \mathbf{H}_p^{-1}\mathbf{y}_2 \\ \mathbf{x}_3 &= \mathbf{H}_p^{-1}\mathbf{y}_3 \end{aligned} \tag{21}$$

Thus, the general measurement vector is:

$$\mathbf{y}_p^T = \begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix} \tag{22}$$

With solutions from (21), the solution for the measurement given by (22) is:

$$\mathbf{x}_p = \alpha\mathbf{x}_1 + \beta\mathbf{x}_2 + \gamma\mathbf{x}_3 \tag{23}$$

Making α, β e γ values vary in loops, it is possible to generate a set of potential solutions covering an extended volume of space.

5.2 Eliminating incorrect potential solutions

In order to eliminate unnecessary storing information, the secondary group could be used to test the potential solutions as they are formed in the loop. Those which do not agree with the additional measurements could be eliminated.

Firstly, the innovation vectors for the secondary group are calculated:

$$\mathbf{u}_s = \mathbf{y}_s - \mathbf{H}_s\mathbf{x}_p \tag{24}$$

where $\mathbf{u}$ is the innovation vector and subscript s refers to secondary group. The innovations corresponding to primary group are zero.

$$\Delta\mathbf{x} = (\mathbf{H}_c^T\mathbf{H}_c)^{-1}\mathbf{H}_s^T\mathbf{y}_s = \mathbf{B}\mathbf{u}_s \quad (25)$$

where subscript c refers to complete set of double differences.

The residuals are necessary in order to quantify the quality of solutions. The residual vector $\mathbf{v}$ is given by:

$$\mathbf{v} = \mathbf{u}_c - \mathbf{H}_c^T\Delta\mathbf{x} \quad (26)$$

The vector $\mathbf{u}_c$ is the innovations for the secondary group $\mathbf{u}_s$, plus three zeros in elements corresponding to primary group innovations.

The estimated variance is used for measuring the quality of potential solutions:

$$q = \frac{\mathbf{v}^T\mathbf{v}}{m-3} \quad (27)$$

where m is the total number of double differences. Only solutions with value of q greater than a selected threshold are retained as potential solutions.

The greater the number of double differences the higher the probability that only one solution will remain as solution which agrees with all measurement data. In addition, even when several solutions repeat as potential solutions, only the true solution will remain as the satellite geometry changes.

6 LAMBDA Method

LAMBDA method (*Least-squares AMBiguity Decorrelation Adjustment*) is a procedure for integer ambiguity estimation in carrier phase measurements. After applying a decorrelating transformation, a sequential conditional adjustment is made upon the ambiguities. As a result, integer least-squares estimates for the ambiguities are obtained. This method was introduced in [16] and [17]. [18] and [19] show computational implementation aspects and ambiguity search space reducing.

These double difference observation equations are appropriate models for short baselines. The linearized equations are given by:

$$\mathbf{y} = \mathbf{A}\mathbf{x}_a + \mathbf{B}\mathbf{x}_b + \epsilon \quad (28)$$

where $\mathbf{y}$ is observed minus computed double differences, $\mathbf{x}_a$ is integer ambiguity double difference vector, $\mathbf{x}_b$ is baseline increments vector, $\mathbf{A}$ and $\mathbf{B}$ are design matrix for ambiguity and baseline and ϵ is an unmodeled errors vector.

The LAMBDA method takes as starting point (28), using least-squares method as estimator for obtaining $\mathbf{x}_a$ and $\mathbf{x}_b$. The minimization criterium for solving (28) is:

$$\min_{\mathbf{x}_b,\mathbf{x}_a} \|\mathbf{y} - \mathbf{A}\mathbf{x}_a - \mathbf{B}\mathbf{x}_b\|^2_{\mathbf{Q}_\mathbf{y}^{-1}}, \text{with } \begin{cases} \mathbf{x}_b \in \mathbb{R}^p \\ \mathbf{x}_a \in \mathbb{Z}^m \end{cases} \quad (29)$$

where $\|\cdot\|^2_{\mathbf{Q}_y^{-1}} = (\cdot)^T\mathbf{Q}_y^{-1}(\cdot)$ and $\mathbf{Q}_y^{-1}$ is covariance matrix of double difference observables. The ambiguity number m is equal to the number of satellites minus one, multiplied by the used frequencies values, and the number of baseline components $\mathbf{x}_b$ is three, in case of a static receiver, or a multiple of three, in case of a moving one.

One can notice that (29) is an integer least-squares problem, because of the restriction $\mathbf{x}_a \in \mathbb{Z}^m$. This problem can be solved in three steps. The first one, or float solution, consists of solving (29) with $\mathbf{x}_a \in \mathbb{R}^m$ by means of a common least-squares method. As result, we have $\hat{\mathbf{x}}_a$ e $\hat{\mathbf{x}}_b$ as real values. The second step, or integer solution, consists of solving the minimization problem:

$$\min_{\mathbf{x}_a} \|\hat{\mathbf{x}}_a - \mathbf{x}_a\|^2_{\mathbf{Q}_{\hat{a}}^{-1}}, \quad \text{with } \mathbf{x}_a \in \mathbb{Z}^m \quad (30)$$

followed by the third step, which is a correction of baseline $\hat{\mathbf{x}}_b$ by difference between $\hat{\mathbf{x}}_a$ and the result of minimization (30).

In fact, the second problem consists of minimizing (30), resulting in an integer estimative of ambiguity vector $\mathbf{x}_a$. In this step is utilized the LAMBDA method [17]. The two main features of this method are: (i) ambiguity decorrelation, carried through a reparametrization (Z-transform); (ii) the actual integer ambiguity estimation.

With Z-transform, ambiguities and their covariance matrix are transformed according to:

$$\mathbf{x}_z = \mathbf{Z}^T\mathbf{x}_a \quad \text{and} \quad \mathbf{Q}_{\hat{z}} = \mathbf{Z}^T\mathbf{Q}_{\hat{a}}\mathbf{Z} \quad (31)$$

Minimization itself is made upon transformed ambiguities. The minimization (30) consists of a search over grid points inside the m-dimensional ambiguity hyper-ellipsoid, defined by:

$$(\hat{\mathbf{x}}_z - \mathbf{x}_z)^T\mathbf{Q}_{\hat{z}}(\hat{\mathbf{x}}_z - \mathbf{x}_z) \leq \chi^2 \quad (32)$$

The volume of the ellipsoid and the number of candidates can be controlled by setting the value for χ^2.

Prior to the integer estimation, the ambiguities are decorrelated by application of the Z-transform:

Minimization is then carried through transformed ambiguities. The output consists of $\check{\mathbf{x}}_z$ together with their respective norms. Using the Z-transform, they can be transformed back to the original ambiguities:

$$\check{\mathbf{x}}_a = \mathbf{Z}^{-T}\check{\mathbf{x}}_z \quad (33)$$

Estimative of $\check{\mathbf{x}}_b$ and its covariance matrix $\mathbf{Q}_{\check{b}}$ are obtained from:

$$\begin{aligned} \check{\mathbf{x}}_b &= \hat{\mathbf{x}}_b - \mathbf{Q}_{\hat{b}\hat{a}}\mathbf{Q}_{\hat{a}}^{-1}(\hat{\mathbf{x}}_a - \check{\mathbf{x}}_a) \\ \mathbf{Q}_{\check{b}} &= \mathbf{Q}_{\hat{b}} - \mathbf{Q}_{\hat{b}\hat{a}}\mathbf{Q}_{\hat{a}}^{-1}\mathbf{Q}_{\hat{a}\hat{b}} \end{aligned} \tag{34}$$

The least-squares estimates $\check{\mathbf{x}}_b$ and $\check{\mathbf{x}}_a$ are the solution to the constrained minimization (29).

6.1 Z-transform

We can decompose covariance matrix $\mathbf{Q}_{\hat{a}}$ in:

$$\mathbf{Q}_{\hat{a}} = \mathbf{L}^{-T}\mathbf{D}^{-1}\mathbf{L}^{-1} \tag{35}$$

Note that this corresponds to the inverse of the $\mathbf{LDL}^T$ decomposition of $\mathbf{Q}_{\hat{a}}^{-1}$ which is easily derived from a Cholesky factorization. The principle of the decorrelation is to find a matrix $\mathbf{Z}$ which is an integer approximation of matrix $\mathbf{L}$. If we would be able to find an integer matrix $\mathbf{Z}$ that fulfills the requirements in [20] and that exactly equals $\mathbf{L}$, then with (31):

$$\mathbf{Q}_{\hat{z}} = \mathbf{Z}^T\mathbf{Q}_{\hat{a}}\mathbf{Z} = \mathbf{Z}^T\mathbf{L}^{-T}\mathbf{D}^{-1}\mathbf{L}^{-1}\mathbf{Z} = \mathbf{D}^{-1} \tag{36}$$

The transformed ambiguities $\hat{\mathbf{x}}_z$ are fully decorrelated and the integer minimization reduces to a simple rounding of the real valued estimates. In practice a complete decorrelation will not be possible due to the integer requirement. The result of the decorrelation process is the square $m \times m$ transformation matrix $\mathbf{Z}$. The estimate $\hat{\mathbf{x}}_z$ follows from $\hat{\mathbf{x}}_z = \mathbf{Z}^T\hat{\mathbf{x}}_a$. The factors of the covariance matrix are updated such as:

$$\mathbf{Q}_{\hat{z}} = \tilde{\mathbf{L}}^{-T}\tilde{\mathbf{D}}^{-1}\tilde{\mathbf{L}}^{-1} \tag{37}$$

The problem (30) has now been transformed into the following minimization:

$$\min_{\mathbf{x}_z} \; \|\hat{\mathbf{x}}_z - \mathbf{x}_z\|^2_{\mathbf{Q}_{\hat{z}}^{-1}}, \quad \text{com } \mathbf{x}_z \in \mathbb{Z}^m \tag{38}$$

7 Results

The results for both cases (static and kinematic) were obtained from the same process of estimation, using an iterated least-squares with *a priori* information (section 4) processing code and carrier phase measurements.

Used measurements are code and carrier phase double differences observables and are related to the parameters to be estimated $\mathbf{x}$ as follows:

$$\begin{bmatrix} [\rho]_{m\times 1} \\ [\phi]_{m\times 1} \end{bmatrix} = \begin{bmatrix} [\mathbf{1}_b^M - \mathbf{1}_b^i]_{m\times 3} & \mathbf{0}_{m\times 3} & \mathbf{0}_{m\times m} \\ [\mathbf{1}_b^M - \mathbf{1}_b^i]_{m\times 3} & \mathbf{0}_{m\times 3} & \lambda\mathbf{I}_{m\times m} \end{bmatrix} \cdot \mathbf{x} \tag{39}$$

where $[\rho]_{m\times 1}$ and $[\varphi]_{m\times 1}$ represent vectors with m double difference measurements from code and carrier phase respectively, $\mathbf{1}_b^i$ is the line of sight vector between "base" receiver and satellite i and λ is L_1 wavelength. The vector $\mathbf{x}$ consists of baseline components between the receivers δx, δy and δz and the ambiguities $N_1, \ldots, N_m$ corresponding to each carrier phase double difference measurement. In this method, measurement standard deviation for code pseudorange was set to 1.0 m, and for carrier phase was set to 0.005 m.

Both ambiguity resolution methods use least-squares as float solution estimator. The search for ambiguity integer values is based on float ones, which define a search space. Once ambiguities are resolved, one can obtain the phase pseudorange, which position solution is calculated.

The LSAST method divides the satellites in primary and secondary groups. Primary group defines the search space of the three primary ambiguities in 20 cycles, in this case, around the corresponding float ambiguity, after rounded to the nearest integer. The decision about the choice is made using ambiguities from the secondary group, which must meet the criteria defined in section 5.

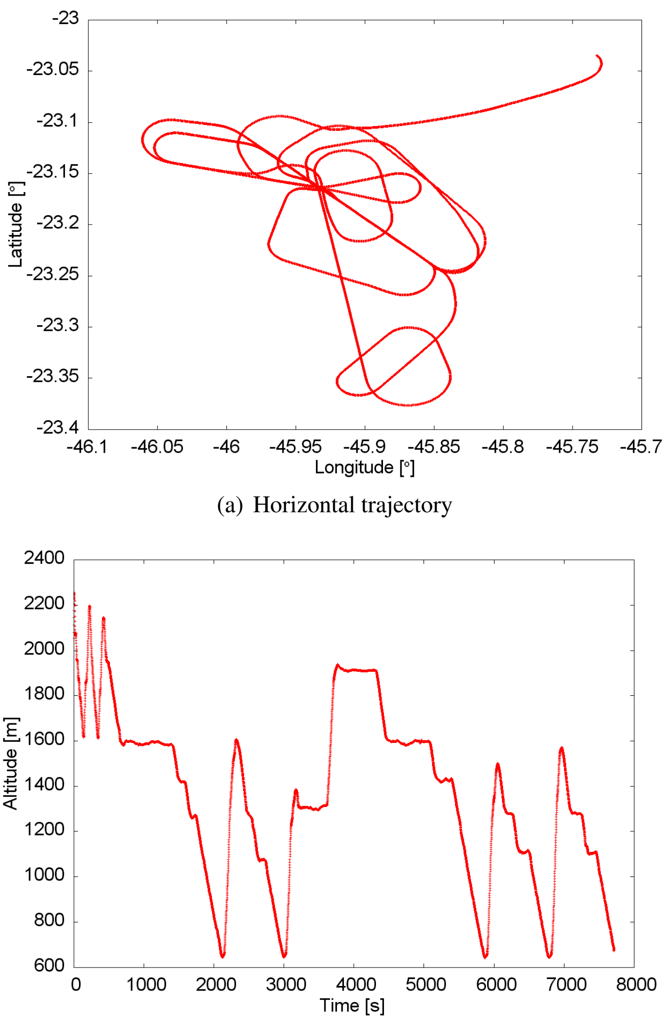
The LAMBDA method makes the integer ambiguity estimation through a Z-transform, in which ambiguities are decorrelated before the integer values search process. Then, minimization problem is approached as a discrete search inside an ellipsoidal region. This method has as result a least-squares estimative for ambiguities based on float ambiguities and their covariance matrix.

The first data set was collected by static receivers, which remain stationary at precisely known positions, to verify the quality of the proposed algorithm. The data were collected by two Trimble R8 receivers, and 1 Hz of sampling rate. Base receiver was placed on a reference landmark with coordinates N 51° 04' 45.94126", W 114° 07' 58.29947" and 1116.617 m, in ECEF coordinates of WGS-84 system, and user receiver was placed in another landmark, 2.944 m from base. The solution was attained through a iterated least-squares method with *a priori* information (section 4), processing code and carrier phase measurements. The standard deviation for code measurement was set to 1.0 m, and phase, 0.005 m.

Baseline components were calculated epoch by epoch, applying both LSAST and LAMBDA methodologies, using float ambiguity values from measurement processing in each epoch by the least-squares method. Graphics in Fig. 3(a)-3(c) show user position error component behavior related to the base when LSAST was applied. The error statistics for each baseline component were -0.009±0.400 m on south, -0.063±0.562 m on east, and 0.080±0.709 m on vertical directions. Graphics in Fig. 3(d)-3(f) show user position errors when using LAMBDA method, and the error statistics were 0.228±0.555 m on south, -0.015±0.387 m on east, and 0.294±1.154 m on vertical directions.

These methods build search space in different manner, so LSAST processing time is considerably longer than LAMBDA one. For LSAST method, processing time in each epoch was 0.078 s, whereas LAMBDA method was 0.002 s, for resolving 5 ambiguities.

The second data set was collected by an aircraft during a flight test. These data were collected by a receiver installed on an aircraft and a fixed receiver as base. The base position coordinate are given by S 23° 13' 42.9859", W 45° 51' 23.4615" and 686.227 m, in ECEF coordinates of WGS-84 system, and sample rate was 2 Hz. For analysis purposes, the results were compared to a trajectory obtained post processing the data, which were considered precise enough for this purpose. Graphics in Fig. 2 show the aircraft trajectory during the test.



(a) Horizontal trajectory

(b) Vertical trajectory

Figure 2: Aircraft trajectory during the flight.

Graphics in Fig. 4(a)-4(c) show positioning error in each direction (south, east, and vertical), compared to the reference trajectory, using LSAST method. The errors for each component were -0.399±0.735 m on south, -0.472±0.720 m on east, and -0.446±1.928 m on vertical directions. With LAMBDA method, these errors were -0.650±0.646 m on south, 0.124±0.321 m on east, and -1.553±1.174 m on vertical directions (Fig. 4(d)-4(f)).

Compared to static, the kinematic test showed larger errors due to the aircraft motion and maneuvers, and distance to base (up to 25 km). These aspects may lead to ambiguities values within ±4 cycles (∼0.8 m) from correct ones. It is expected, with further improvements, these values remains within ±1-2 cycles.

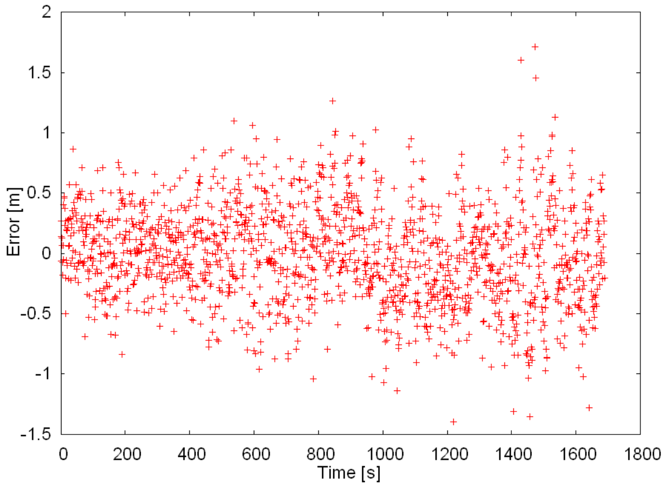
In order to evaluate the maximum accuracy which this method can reach in real time when all features are implemented, an off-line adjustment of ambiguities based on LAMBDA results was carried out. Graphics in Fig. 5 show error with this adjustment for each direction: -0.244±0.339 m for south, -0.015±0.183 m for east, and -0.655±0.899 m for vertical directions. In kinematic case, processing time for LSAST method was 0.094 s per epoch, and for LAMBDA method was 0.003 s per epoch, for resolving 6 ambiguities.

8 Conclusions and Future Works

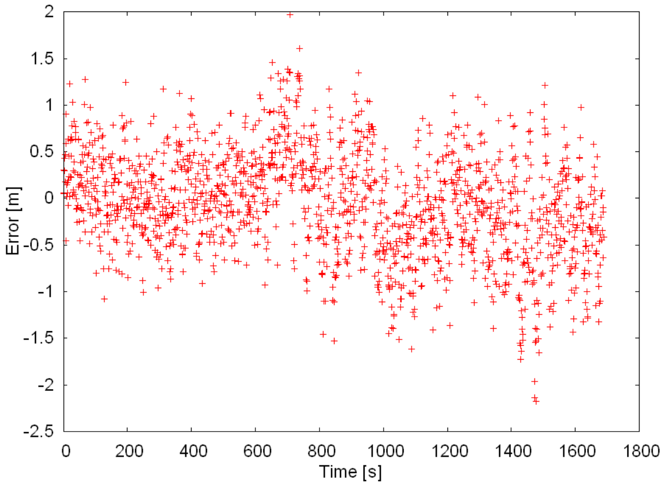
The positioning techniques were carried out through phase measurements processing, using the LSAST and LAMBDA approach.

In static case, base receiver was placed on a landmark, and user receiver was on another landmark at 2.944 m from base, both with known positions. Measurements were processed at each epoch, through an iterated least-squares algorithm. Baseline error were, in each direction, -0.009±0.400 m on south, -0.063±0.562 m on east, and 0.080±0.709 m on vertical directions for LSAST method and 0.228±0.555 m on south, -0.015±0.387 m on east, and 0.294±1.154 m on vertical directions for LAMBDA method.

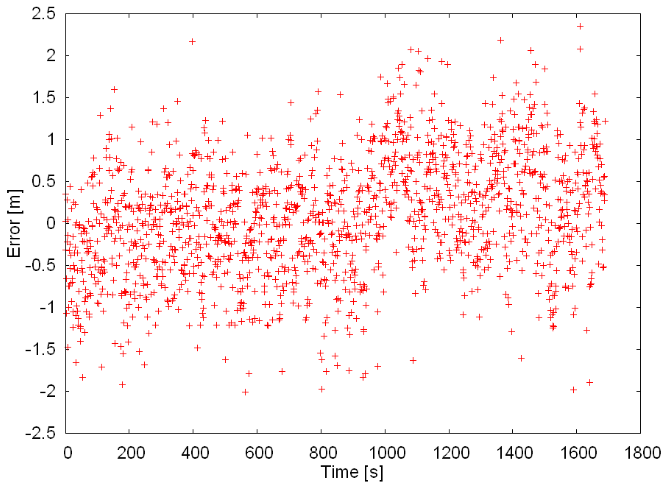
In kinematic test, user receiver was mounted on an aircraft during a test flight. The baseline error values were -0.399±0.735 m on south, -0.472±0.720 m on east, and -0.446±1.928 m on vertical directions with LSAST and -0.650±0.646 m on south, 0.124±0.321 m on east, and -1.553±1.174 m on vertical directions with LAMBDA method. These values were obtained through a iterated least-squares algorithm, in each epoch. An off-line ambiguity adjusted result showed the level of accuracy which can be at-



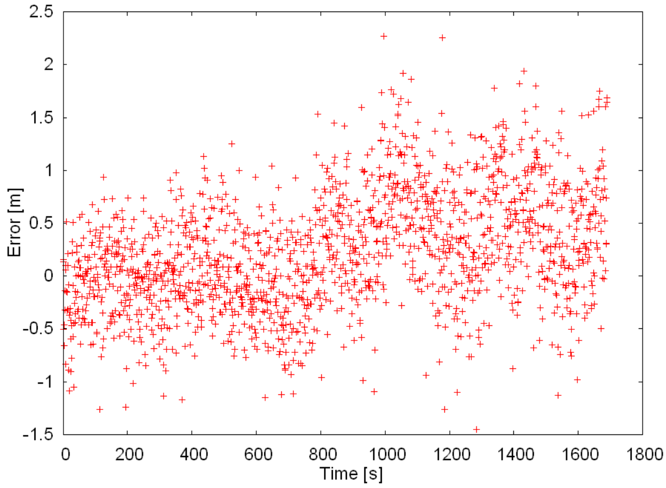
(a) South



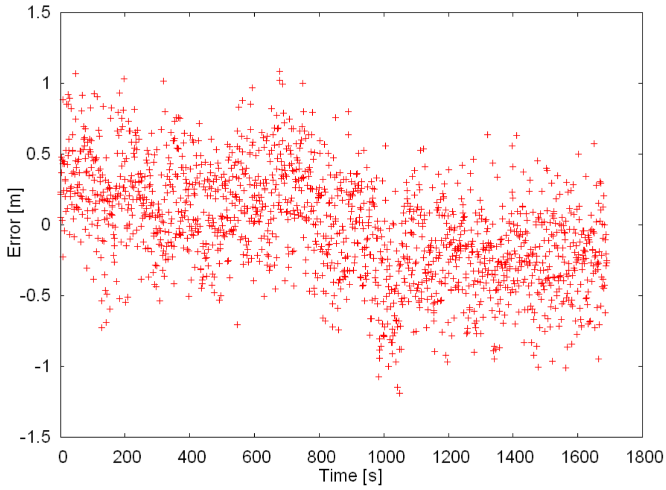
(b) East



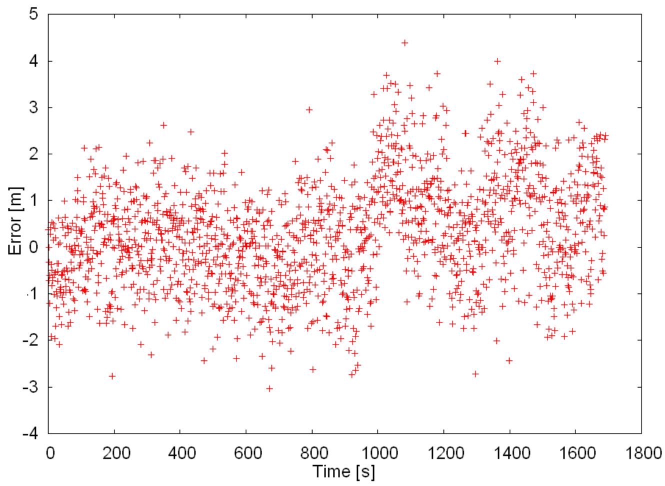
(c) Vertical



(d) South



(e) East



(f) Vertical

Figure 3: Error components using LSAST (Figs. a-c) and LAMBDA (Figs. d-f) methods for static test..

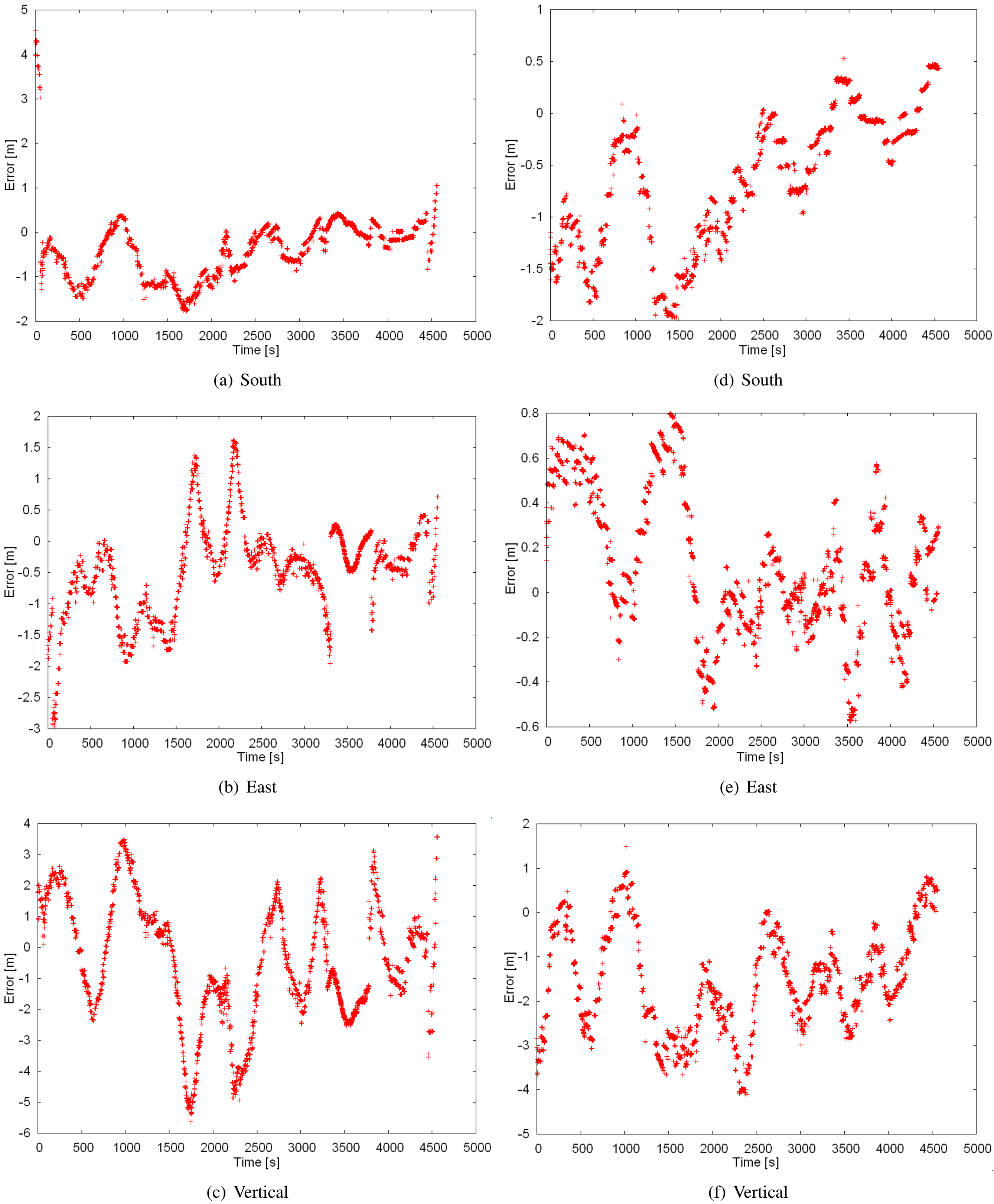


Figure 4: Error components using LSAST (Figs. a-c) and LAMBDA (Figs. d-f) methods for kinematic test.

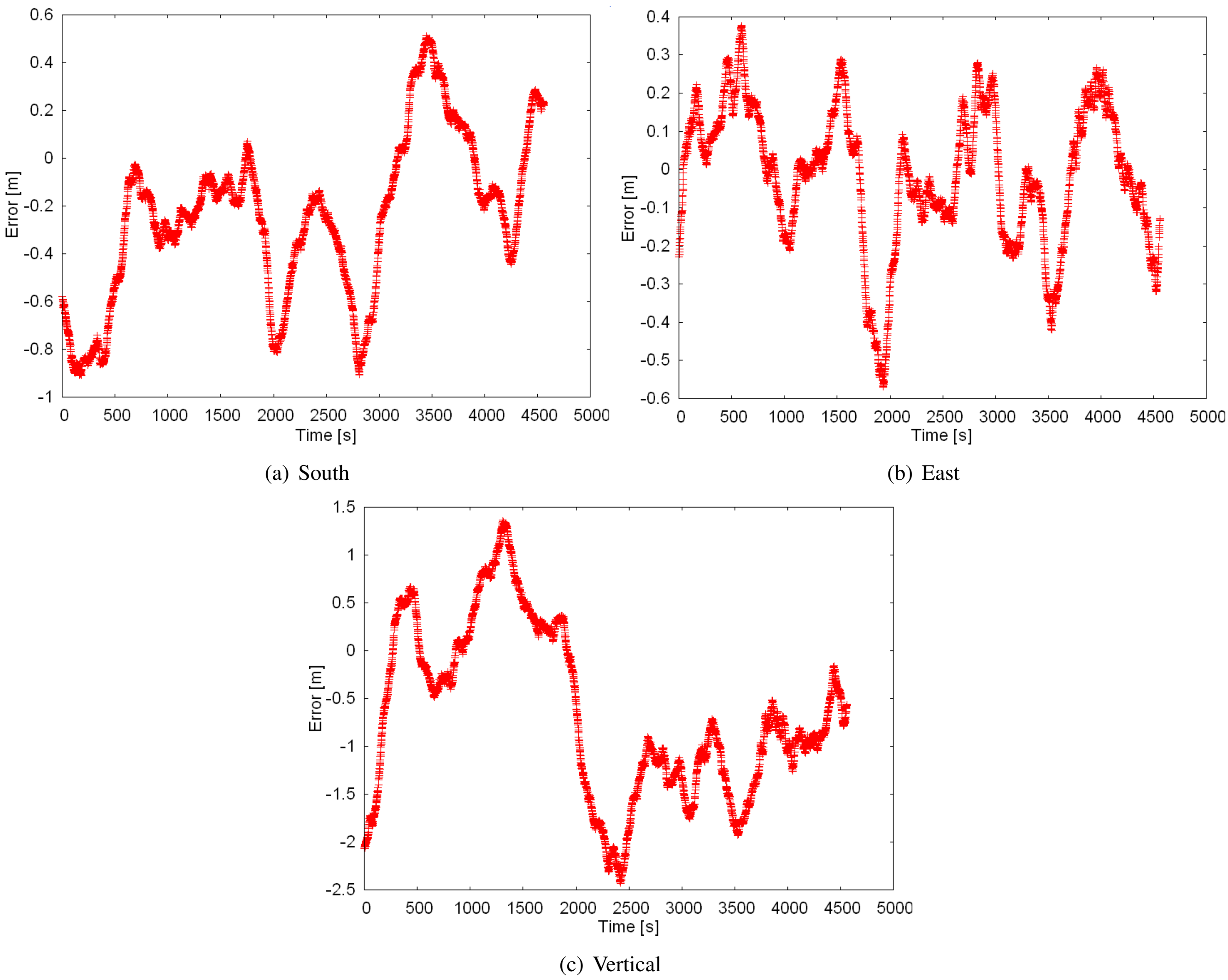


Figure 5: Error components for offline adjusted data.

tained, with the improvements to be implemented in real time. In both tests, LAMBDA is about 100 times faster than LSAST in computer processing.

This work is part of a investigation to develop a differential GPS, using carrier phase measurements in real time. Further developments consider: (i) include better dynamics in the estimation process; (ii) a better filter tunning for carrier phase measurements; (iii) add other measurements combination, such as L_2 frequency and widelane combination; (iv) validate ambiguities after resolving; (v) cycle slips detection and correction.

References:

[1] P. Misra and P. Enge, *Global positioning system: Signals, measurements and performance*, Lincoln MA: Ganga-Jamuna Press, 2001, 408 p.

[2] P. C. P. M. Pardal, H. K. Kuga and R. V. Moraes, "Recursive least-squares algorithms applied to satellite orbit determination, using GPS signals", *Recent Advances in Signal Processing, Robotics and Automation*, Cambridge, UK: WSEAS Press, ISBN 978-960-474-054-3, 2009, pp. 167-172.

[3] V. M. Gomes, A. F. B. A. Prado and H. K. Kuga, "Orbital maneuvers Using Low Thrust", *Recent Advances in Signal Processing, Robotics and Automation*, Cambridge, UK: WSEAS Press, ISBN 978-960-474-054-3, 2009, pp. 120-125.

[4] G.-S. Huang, "Application of the vehicle navigation via GPS carrier phase", *6th WSEAS Int. Conf. on Robotics, Control and Manufacturing Technology*, Apr. 16-18, Hangzhou, China, 2006, pp. 218-223.

[5] G.-S. Huang, "Control the vehicle flow via GPS monitor center", *WSEAS Transactions on Signal Processing*, Issue 8, vol. 2, 2006, pp. 174-181.

[6] M. Sadeghi and M. Gholami, "Time Synchroniz-

ing Signal by GPS Satellites", *WSEAS Transactions on Communications*, Issue 5, Vol. 7, 2008.

[7] L. Baroni and H. K. Kuga, "Analysis of navigational algorithms for a real time differential GPS system", *18th International Congress of Mechanical Engineering*, São Paulo: ABCM, 2005.

[8] J. F. G. Monico, *Posicionamento pelo GNSS: Descrição, fundamentos e aplicações*, 2 Ed., São Paulo: Editora UNESP, 2008, 476 p.

[9] B. W. Parkinson and J. J. Spilker Jr., *Global positioning system: Theory and applications*, Washington: AIAA, v. 1-2, 1996.

[10] A. Saalfeld, *Generating basis sets of double differences*, *Journal of Geodesy*, 73, 1999, pp. 291-297.

[11] G. Strang and K. Borre, *Linear algebra, geodesy, and GPS*, Cambridge Press, Wellesley, 1997, 614 p.

[12] C. L. Lawson and R. J. Hanson, *Solving least-squares problems*, New Jersey: Prentice Hall, 1974. 340 p.

[13] G. J. Bierman, *Factorization methods for discrete sequential estimation*, New York: Academic Press, 1977, 241 p.

[14] R. R. Hatch, "Instantaneous ambiguity resolution", *KIS '90*, Banff, Canada:Springer-Verlag, 1990, pp. 299-308.

[15] H.-S. Wang, "A Behavioral Approach to GNSS Positioning and DOP Determination", *WSEAS Transactions on Systems*, Issue 1, Vol. 7, 2008

[16] P. J. G. Teunissen, "Least-squares estimation of the integer GPS ambiguities", *IAG General Meeting*, Beijing, China: IAG, 1993.

[17] P. J. G. Teunissen, "A new method for fast carrier phase ambiguity estimation", *IEEE PLANS 1994*, April 11-15, Las Vegas, 1994, pp. 562-573.

[18] P. J. de Jonge, C. C. J. M. Tiberius, *The LAMBDA method for integer ambiguity estimation: implementation aspects*, Delft University of Technology, 1996, 59 p. (LGR-Series, n. 12).

[19] P. J. de Jonge, C. C. J. M. Tiberius and P. J. G. Teunissen, "Computational aspects of the LAMBDA method for GPS ambiguity resolution", *9th ION-GPS*, Kansas City:ION, 1996, pp. 935-944.

[20] P. J. G. Teunissen, "The invertible GPS ambiguity transformations", *Manuscripta Geodaetica*, v. 20, n. 6, 1995, pp. 489-497.