Recursive Least Squares Algorithms Applied to Satellite Orbit Determination, Using GPS Signals

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Abstract: Using signals of the GPS constellation and least squares algorithms through sequential Givens rotations as a method of estimation, the principal target is to determine the orbit of an artificial satellite. This approach aims to improve the performance of the orbit estimation process and, at the same time, to minimize the computational procedure cost. Perturbations up to high order geopotential and direct solar radiation pressure were taken into account. It was also considered the position of the GPS antenna on the satellite body. An application has been done, using real data from the TOPEX/POSEIDON satellite. In a process of high accuracy orbit determination, frequently a sinusoidal residual behavior is observed during its error analysis. Assuming that we cope with the unmodeled accelerations, which have no direct physical reasons, or that the modeling effort is not worthwhile, these anomalous accelerations will also be analyzed, empirically.

Key-Words: Least Squares Algorithms; Estimation; Orbit Determination; GPS, Satellites; Unmodeled Accelerations; Orbit Perturbations.

1 Introduction

The problem of orbit determination consists essentially of estimating parameters values that completely specify the body trajectory in the space, processing a set of measurements from this body. Such observations can be collected through a tracking network on Earth or through sensors, like the GPS receiver onboard TOPEX/POSEIDON (T/P).

The Global Positioning System (GPS) is a powerful and low cost means to allow computation of orbits for artificial Earth satellites. The T/P satellite is an example of using this system for space positioning.

The orbit determination of artificial satellites is a nonlinear problem in which the disturbing forces are not easily modeled, like geopotential and direct solar radiation pressure. Throughout an onboard GPS receiver is possible to obtain measurements (pseudo-ranges) that can be used to estimate the state of the orbit.

Usually, the iterative improvement of the position parameters of a satellite is carried out using the least squares methods. On a simple way, the least squares estimation algorithms are based on the data equations that describe the linear relation between the residual measurements and the estimation parameters.

2 Recursive Least Squares Using Sequential Givens Rotations

The Givens rotations are used when it is fundamental to cancel specific elements of a matrix. Alternative formulations were developed, based on the QR factorization methods, to solve this deficiency. Using orthogonal transformations, the equation matrix of data can be transformed on a triangular higher form, to which the least squares solution is obtained by a simple substitution. The aim of applying orthogonal transformations in matrices and vectors on the least squares problem is to substitute the matrices inversion by a stronger method, with less numerical errors. The Givens rotations [1] are a method to solve recursive least squares through orthogonal transformations [2].

The Givens rotations are used when is essential to annul specific elements of a matrix. In this procedure, a given matrix becomes triangular by a series of orthogonal matrices. The full transformation generically can be given by:

$$\begin{pmatrix} R \\ 0 \end{pmatrix} = \begin{pmatrix} U_m & U_{m-1} & \dots & U_3 & U_2 \end{pmatrix} H = Q^T H$$

$$\begin{pmatrix} d \\ r \end{pmatrix} = \begin{pmatrix} U_m & U_{m-1} & \dots & U_3 & U_2 \end{pmatrix} y = Q^T y \tag{1}$$

where R is triangular. At each step, the orthogonalization of the H matrix is performed (producing a transformed measurement vector d and r), and the results are stored to the next set of measurements [3, 2].

3 Disturbing Effects Considered

The main disturbing forces that affect the orbit of an artificial satellite are the effects that deviate from a potential and there are also the nongravitational effects.

The disturbing effects are included according to the physical situation presented and to the accuracy that is intended for the orbit determination.

3.1 Perturbations due to Geopotential

The geopotential is a force of gravitational origin that disturbs the orbits of artificial Earth satellites. Earth gravitational field represents one of the main perturbations on the motion of artificial satellites. The potential function is given by [4]:

$$U(r,\phi,\lambda)=\frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_T}{r}\right)^n P_{nm}(\sin\phi)\times \times\left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right) \qquad (2)$$

where μ is Earth gravitational constant; R_T is Earth radius; r is the spacecraft radial distance; ϕ is the geocentric latitude; λ is the longitude on Earth fixed coordinates system; C_{nm} and S_{nm} are the normalized harmonic spherical coefficients; and P_{nm} are the normalized Legendre associated functions, all with n degree and m order.

3.2 Perturbations due to Direct Solar Radiation Pressure

The solar radiation pressure is a force of nongravitational origin that disturbs the translational motion of an artificial satellite. Solar radiation pressure is engendered throughout a continuous flux of photons that stumble at satellite surfaces, which can absorb or reflect such flux. The rate which all incident photons reach the satellite surfaces origins the solar radiation pressure force, what can cause perturbations on the orbital elements.

The components of solar radiation pressure force can be expressed in several systems. Throughout these systems, the orbital elements of the satellite can be connected with sun's position. This procedure was used for the direct solar radiation pressure model adopted for the T/P satellite [5].

3.2.1 Direct Solar Radiation Pressure Model for the TOPEX/POSEIDON Satellite

The force model describes the motion of a satellite's center of mass, but the range measurements are seldom considered at this point. In the case of T/P, they are taken from the location of the center of the antenna. For this reason, it is important the knowledge about satellite attitude motion. Fig.1 shows TOPEX antenna in relation to rest of the spacecraft [6].

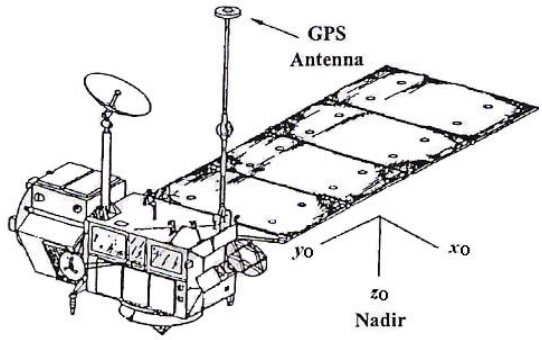

Fig.1 - TOPEX GPS antenna location.

where (x_O, y_O, z_O) is the orbit fixed coordinates system, with origin in the satellite's center of mass.

In estimating the TOPEX state there is a complex, predetermined attitude model being applied. This model was created to maneuver the solar array towards the sun for the most sun-facing surface area while still pointing the altimeter in the nadir direction [7]. In basic terms, this model gives rotation angles about the orbit local coordinates to allow for positioning of the antenna with respect to the center of mass. In truth, the recursive least squares algorithm returns position and velocity coordinates in relation to antenna location. Since the update is only a coordinate translation, it is instead applied to the center of mass.

For completeness, it needs to be stated that the attitude model also gives the orientation of the solar array. This orientation, along with the spacecraft position and the Sun's position are used to compute the direct solar radiation pressure.

The common method for computing the radiation pressure upon orbiting satellites within the orbit determination software had been to ignore rotating, attitude control, geometrically complex shapes and to treat the satellite form as a symmetrically perfect and rotationally invariant sphere (cannonball). The approaches of the cannonball radiation pressure model were not adequate to meet the required error

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budget for modeling the radiation forces acting on T/P over a 10-day period. After considerable analysis of all surface force contributions, resultant models to be used in TOPEX orbit determination were presented [5].

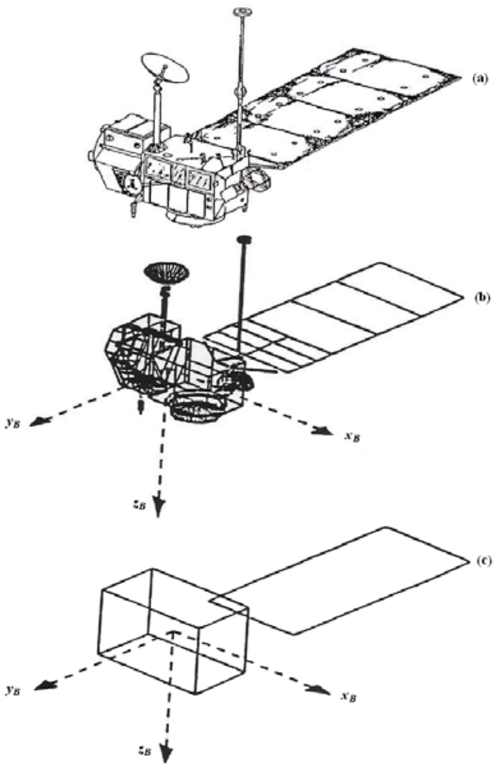

Fig.2 - The TOPEX spacrecraft is shown in (a); the corresponding micromodel in (b); and the corresponding macromodel in (c).

This concept is based on approximating the satellite shape with a combination of flat plates, as seen in Fig.2. For TOPEX, a box-wing shape was chosen, with the plates aligned along the satellite body-fixed coordinate system (x_B, y_B, z_B).

3.2.2 Radiant Energy of the Sun

The major source of radiant energy which T/P will encounter is the sun. The sun emits a nearly constant amount of photons per unit of time, varying less than 0.2%, that acts on the surfaces of artificial satellites [7]. Also, for TOPEX/POSEIDON, it is the largest nongravitational force acting on the satellite. This is the reason for considering only this parcel of the radiation forces herein.

The model of force acting on each plate is [5]:

$$\vec{F} = \frac{GA\cos\theta}{c}\left[2\left(\frac{\delta}{3} + \rho\cos\theta\right)\hat{n} + (1-\rho)\hat{s}\right] \quad (3)$$

where G is solar radiant flux; A is the surface area of each plate; δ is difusive reflectivity; ρ is specular reflectivity; $\hat{n}$ is surface normal vector; $\hat{s}$ is source incidence vector; θ is the angle between surface normal and solar incidence; and c is the speed of light.

There are 8 plates in the model developed for TOPEX, according to the box-wing shape chosen. So, it is necessary to compute independently the direct solar radiation force acting on each surface. All plate interaction effects, such as shadowing, reflection, and conduction are ignored. Mathematically:

$$\vec{F}_k = \frac{GA_k\cos\theta_k}{c}\begin{bmatrix} 2\left(\frac{\delta_k}{3} + \rho\cos\theta_k\right)\hat{n}_k + \\ +(1-\rho_k)\hat{s} \end{bmatrix} \quad (4)$$

$$\Rightarrow \qquad \vec{F} = \sum_{k=1}^{8}\vec{F}_k$$

where subscript k varies from 1 to 8, representing each plate, and $\vec{F}$ is the total direct solar radiation force acting on the satellite.

4 Unmodeled Accelerations

Sometimes after launch, ground based orbit determination solutions do not provide the level of accuracy expected. After verifying all known dynamic models, there may be a residual signature in the orbit as result of unmodeled accelerations. This leads to attempt to estimate anomalous accelerations during the orbit fit, if sufficient data exist. If successful, the acceleration estimates can improve the fit residuals, and also results in better orbital position estimates [8].

Unmodeled accelerations may have many reasons: truncation of geopotential field; limitations of modeling solar pressure, Earth albedo, Earth infrared radiation, drag; and others.

The use of periodic accelerations, with a period near once per revolution of the satellite orbit, has been used within precision orbit determination programs to improve the accuracy of the derived ephemeris.

4.1 Modeling Anomalous Accelerations

When defining an anomalistic or periodic acceleration, one must consider three aspects: the subarc interval, the type of function, and the coordinate frame.

The subarc interval is the time of duration or number of revolutions for a given acceleration to be active. As it name implies, it is usually a subset of the total arc. A reason to break an arc into a subarc is to allow for better overall fits.

The type of function is the most basic: a constant function with a constant force in a specific direction. And the periodic functions (sine or cosine) have amplitude, frequency, and phase associated with them. The periodic functions are written as [8]:

$$acc = A\sin(\omega t + \phi_A) \text{ or } acc = B\cos(\omega t + \phi_B) \quad (5)$$

where *A* and *B* are amplitudes; ω is the frequency; *t* is the time elapsed since the start of the periodic function reference point or subarc interval; and, ϕ_A and ϕ_B are the phase offsets. Either of these accelerations can be rewritten as:

$$acc = A'\sin(\omega t) + B'\cos(\omega t) \quad (6)$$

where for a sine acceleration the phase are $A' = +A\cos\phi_A$; $B' = +A\sin\phi_A$, and, for a cosine, they are $A' = -B\cos\phi_B$; $B' = +B\sin\phi_B$. When estimated, the amplitudes A' and B' will adjust themselves to produce an effective phase offset.

The selection of the start of the subarc can be important, especially for non-circular orbits. Conventionally, equator crossings, argument of perigee, mean anomaly or orbit angle have been used as reference point.

5 Results

Here, the tests and analysis from the algorithm developed to compute direct solar radiation pressure are presented. The algorithm was implemented through FORTRAN language.

To analyze and to validate the purposed method, real data from the T/P satellite were used. Position and velocity to be estimated were compared with TOPEX precise orbit ephemeris (POE), from JPL/NASA.

In a first step, it was analyzed the effect of including the considered perturbations in orbit propagation, before orbit determination through least squares estimation. Fig. 3 and Fig. 4 show the behavior of the error, in meters, in RNT (radial, normal, and transverse) system, along a 24 hours period, which is a meaningful interval of time in case of orbit propagation.

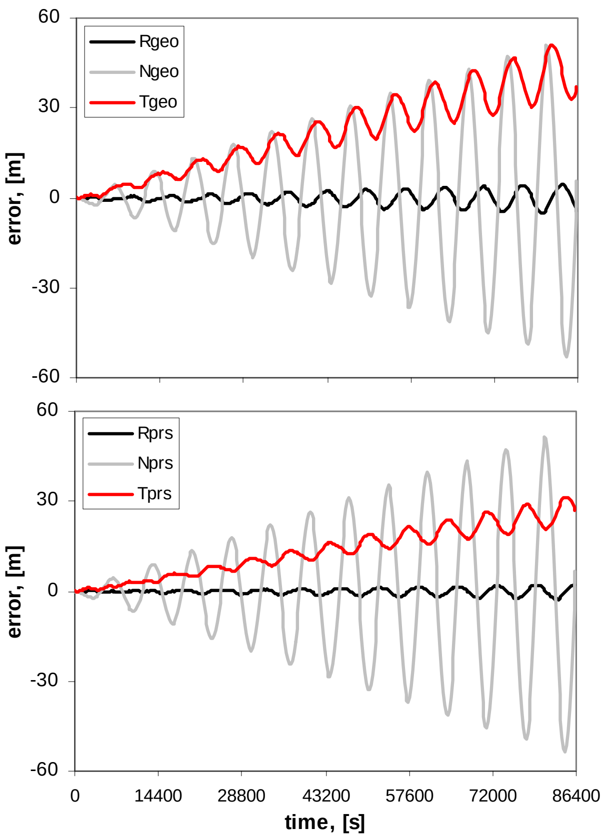

Fig.3 - Orbit propagation per 24 hours period for 11/18/93.

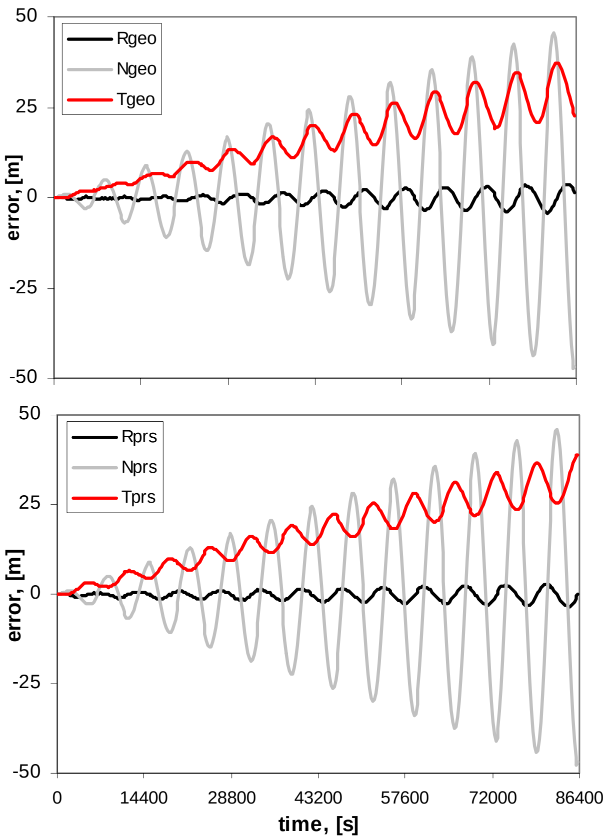

Fig.4 - Orbit propagation per 24 hours period for 11/19/93.

The force model includes perturbations due to high order geopotential (50 × 50), with harmonic coefficients from JGM-2 model, and due to direct solar radiation pressure.

The obtained data were evaluated through error in position as the parameter, given by:

$$\Delta \vec{r} \equiv \begin{bmatrix} x - \hat{x} \\ y - \hat{y} \\ z - \hat{z} \end{bmatrix} \quad (7)$$

which were after translated to radial, normal, and transverse components of orbit fixed system [9]. In Eq. (7), x_i and $\hat{x}_i$ are the position reference and the position estimated components, respectively, in the orbit fixed reference frame.

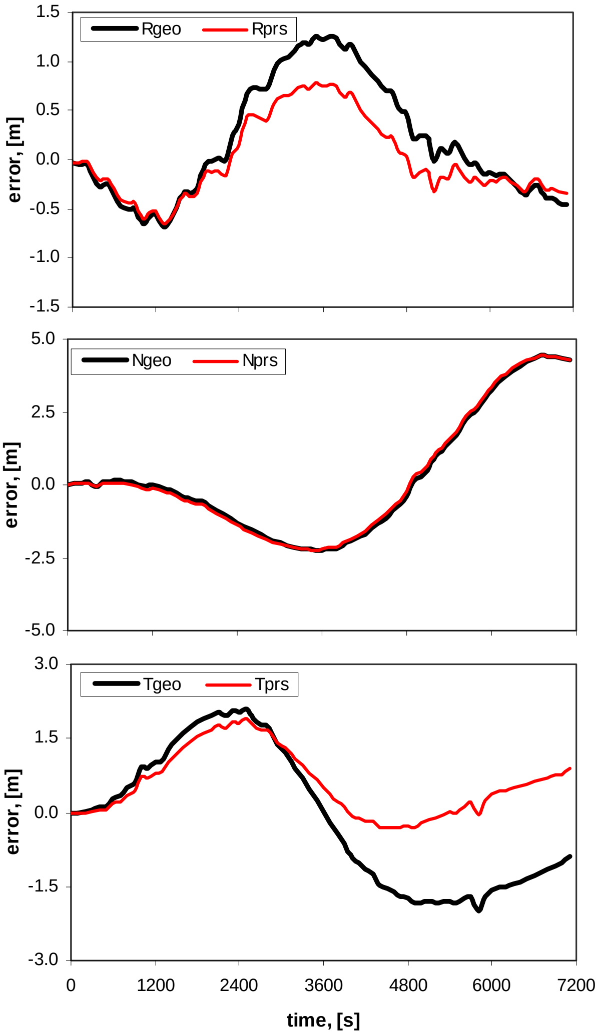

Fig. 5 - Error in position for 2 hours, comparing effects of geopotential and solar radiation pressure.

Fig. 5 and Fig. 6 show the behavior of the error in position along time, considering only geopotential, and geopotential and direct solar radiation pressure effects, in two different curves. The data used are from 18/11/1993.

The subscript “geo” means perturbations due to geopotential only; and “prs”, perturbations due to geopotential and direct solar radiation pressure.

Next, Table 1 shows the maximum and minimum values of the obtained errors for each of the perturbations considered.

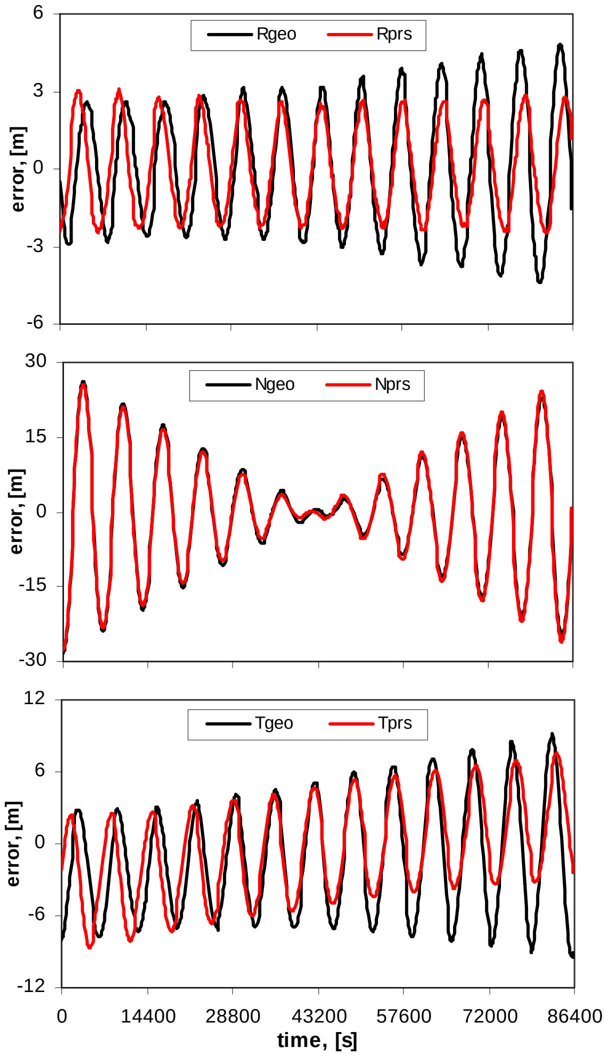

Fig. 6 - Error in position for 24 hours, comparing effects of geopotential and solar radiation pressure.

Table 1 - Maximum and minimum values for errors.

Error (m)		2 hours			24 hours		
	value	R	N	T	R	N	T
geo	*Max*	1.48	4.44	2.19	5.70	26.26	9.27
	Min	-0.75	-2.28	-2.09	-4.67	-28.63	-12.43
prs	*Max*	0.84	4.49	1.89	4.90	25.60	7.70
	Min	-0.71	-2.21	-0.16	-2.69	-27.92	-8.70

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As Table 1 shows, for 2 hours period, solar radiation pressure decreases up to 43% the radial component value and up to 16% the transverse. And for 24 hours, it reduces up to 42% the radial component value and up to 30% the transverse. Direct solar radiation pressure does not act meanwhile on the normal component.

6 Conclusions

The principal aim here was to determine the orbit of an artificial satellite. Using signals of the GPS constellation and least squares algorithms and sequential Givens rotations as the method of estimation, the analysis period covered a short period (near once T/P period) and a long period (one day) of orbit determination. Real time requirements were not present; meantime, they were appropriate to keep low computational cost.

Geopotential and direct solar radiation pressure were taken into consideration and the analysis occurred without selective availability on the GPS signals measurements.

The results were compared with real data from TOPEX POE/JPL. For short period orbit determination, the magnitude of error in position varied from 4.6 m to 4.2 m, and for long period, the magnitude varied from 29.3 m to 27.8 m, according to the model's complexness increase. As the numbers show, the model that includes direct solar radiation pressure decreases at most around 5% the precision in position.

It happens because of the appearance of remaining errors due to both perturbations. They have periodic nature, with a frequency near the orbital period, due to unmodeled residual accelerations, which appear by different reasons. In case of geopotential it may be caused by truncation of the harmonics of the geopotential field; whereas in the solar radiation pressure the possible causes are mismodeled attitude, self-shadowing, and differences between physical and simplified derived models.

Throughout the results, it was found that least squares method through sequential Givens rotations and positioning using GPS showed trustfulness and accuracy enough for artificial satellites orbit determination.

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