Static Load Distribution in Ball Bearings

Mário Ricci*

Abstract

A numerical procedure for computing the internal loading distribution in statically loaded, single-row, angular-contact ball bearings when subjected to a known combined radial and thrust load is presented. The combined radial and thrust load must be applied in order to avoid tilting between inner and outer rings. The numerical procedure requires the iterative solution of $Z + 2$ simultaneous nonlinear equations – where Z is the number of the balls – to yield an exact solution for axial and radial deflections, and contact angles. Numerical results for a 218 angular-contact ball bearing have been compared with those from the literature and show significant differences in the magnitudes of the ball loads, contact angles, and the extent of the loading zone.

Introduction

Ball and roller bearings, generically called *rolling bearings*, are commonly used machine elements. They are employed to permit rotary motions of, or about, shafts in simple commercial devices and also used in complex engineering mechanisms.

This work is devoted to the study of the internal loading distribution in statically loaded single-row angular-contact ball bearings. Several researchers have studied the subject [1] [2] [3] [4]. The methods developed by them to calculate distribution of load among the balls and rollers of rolling bearings can be used in most bearing applications because rotational speeds are usually slow to moderate. Under these speed conditions, the effects of rolling element centrifugal forces and gyroscopic moments are negligible. At high speeds of rotation, these body forces become significant, tending to alter contact angles and clearance. Thus, they can affect the static load distribution to a great extent.

Harris [5] described methods for internal loading distribution in statically loaded bearings addressing pure radial; pure thrust (centric and eccentric loads), and combined radial and thrust load. These methods use radial and thrust integrals introduced in [2] and those initially due to [3] for ball bearings under combined radial, thrust, and moment load.

There are many works describing the parameters variation models under static loads but few demonstrate such variations in practice, even under simple static loadings. The author believes that the lack of practical examples is mainly due to the inherent difficulties of the numerical procedures that, in general, deal with the resolution of various non-linear algebraic equations that must to be solved simultaneously.

In an attempt to cover this gap, studies are being developed in parallel [6] [7]. In this work, a numerical procedure is described for an internal load distribution computation in statically loaded, single-row, angular-contact ball bearings when subjected to a known external combined radial and thrust load. The novelty of the method is in the choice of the set of the nonlinear equations, which must be solved simultaneously. The author did not find in the literature the solution of this problem using the same set of equations.

* National Institute for Space Research, São José dos Campos, São Paulo State, Brazil

Proceedings of the 40th Aerospace Mechanisms Symposium, NASA Kennedy Space Center, May 12-14, 2010

NASA/CP-2010-216272

Static Load Distribution under Combined Radial and Thrust Load in Ball Bearings

It is possible to consider how the bearing load is distributed among the rolling elements having defined in other works analytical expressions for geometry of bearings and for contact stress and deformations for a given ball or roller-raceway contact (point or line loading) in terms of load. In this section, a specific load distribution consisting of a combined radial and thrust load must be applied to the inner ring of a statically loaded ball bearing so that no tilt is allowed between inner and outer rings.

Let a ball bearing with a number of balls, Z, symmetrically distributed about a pitch circle according to Figure 1, to be subjected to a combined radial and thrust load, so that a *relative axial displacement*, δ_a, and a *relative radial displacement*, δ_r, between the inner and outer ring raceways may be expected. Let $\psi = 0$ to be the angular position of the maximum loaded ball.

Figure 2 shows the initial and final curvature centers positions at angular position ψ, before and after loading, considering the centers of curvature of the raceway grooves fixed with respect to the corresponding raceway. If δ_a and δ_r are known, the contact angle at angular position ψ, after the combined load has been applied, is given by

$$\beta(\psi) = \cos^{-1}\left(\frac{A\cos\beta_f + \delta_r\cos\psi}{A + \delta_n}\right), \tag{1}$$

where A is the distance between raceway groove curvature centers for the unloaded bearing, β_f is the free-contact angle, and δ_n is the total normal deflection at the contacts.

Also, from Figure 2,

$$\delta_a = (A + \delta_n)\sin\beta - A\sin\beta_f\ , \tag{2}$$

and we can arrive in the expression for the extent of the *loading zone*, that is given by

$$\psi_l = \cos^{-1}\left\{\frac{A}{\delta_r}\left[\cos\left(\sin^{-1}\left(\frac{\delta_a + A\sin\beta_f}{A}\right)\right) - \cos\beta_f\right]\right\}. \tag{3}$$

From (1), the total normal approach between two raceways at angular position ψ, after the combined load has been applied, can be written as

$$\delta_n(\psi) = A\left(\frac{\cos\beta_f}{\cos\beta} - 1\right) + \frac{\delta_r\cos\psi}{\cos\beta}. \tag{4}$$

From Figure 2 and (4) it can be determined that s, the distance between the centers of the curvature of the inner and outer ring raceway grooves at any rolling element position ψ, is given by

$$s(\psi) = A + \delta_n = A\frac{\cos\beta_f}{\cos\beta} + \frac{\delta_r\cos\psi}{\cos\beta}. \tag{5}$$

From (2) and (5) yields, for $\psi = \psi_j$,

$$\delta_a - \delta_r\tan\beta_j\cos\psi_j - A\frac{\sin(\beta_j - \beta_f)}{\cos\beta_j} = 0\ , \quad j = 1,\ldots,Z. \tag{6}$$

From the load-deflection relationship for ball bearings and (4) yields, for $\psi = \psi_j$,

$$Q_j = K_{nj}\left[A\left(\frac{\cos\beta_f}{\cos\beta_j} - 1\right) + \frac{\delta_r \cos\psi_j}{\cos\beta_j}\right]^{3/2}, \quad j = 1,\ldots,Z. \tag{7}$$

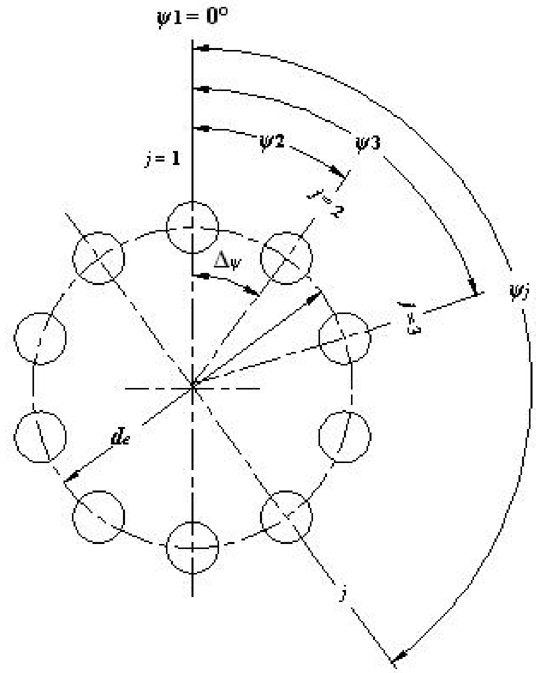

Figure 1. Ball angular positions in the radial plane that is perpendicular to the bearing's axis of rotation, $\Delta\psi = 2\pi/Z$, $\psi_j = 2\pi/Z(j-1)$

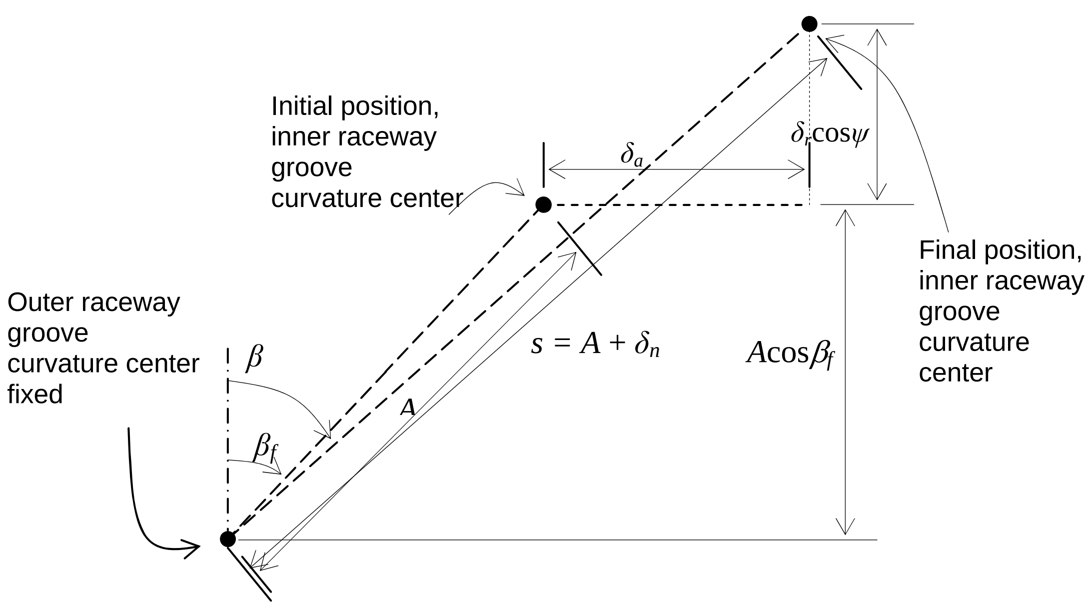

Figure 2. Initial and final curvature centers positions at angular position ψ, with and without applied load

If a thrust load, F_a, and a radial load, F_r, are applied then, for static equilibrium to exist

$$F_a = \sum_{j=1}^{Z} Q_j \sin\beta_j \ , \tag{8}$$

$$F_r = \sum_{j=1}^{Z} Q_j \cos\beta_j \cos\psi_j \, . \tag{9}$$

Additionally, each of the normal ball load components produces a moment about of the inner ring center of mass in the plane that passes through the bearing rotation axis and contains the external radial load (moments about the other two perpendicular planes are self-equilibrating). For static equilibrium, the thrust load, F_a, and/or the radial load, F_r, must exert a moment, M, about the inner ring center of mass that must be equal to the sum of the moments of each rolling element load, that is,

$$M = -\sum_{j=1}^{Z} Q_j \sin\beta_j \left[\left(R_i + \delta_r \cos\psi_j \right) \cos\psi_j - \delta_r \right], \tag{10}$$

where

$$R_i = d_e / 2 + \left(f_i - 0.5 \right) D \cos\beta_f$$

expresses the locus of the centers of the inner ring raceway groove curvature radii.

Substitution of (7) into (8) yields

$$F_a - \sum_{j=1}^{Z} K_{nj} \sin\beta_j \left(A \left(\frac{\cos\beta_f}{\cos\beta_j} - 1 \right) + \frac{\delta_r \cos\psi_j}{\cos\beta_j} \right)^{3/2} = 0 \tag{11}$$

Similarly,

$$F_r - \sum_{j=1}^{Z} K_{nj} \cos\psi_j \cos\beta_j \left(A \left(\frac{\cos\beta_f}{\cos\beta_j} - 1 \right) + \frac{\delta_r \cos\psi_j}{\cos\beta_j} \right)^{3/2} = 0 \tag{12}$$

Equations (6), (11) and (12) are $Z + 2$ simultaneous nonlinear equations with unknowns δ_a, δ_r, and β_j, $j = 1,\ldots,Z$. Since K_{nj} are functions of final contact angle, β_j, the equations must be solved iteratively to yield an exact solution for δ_a, δ_r and β_j.

Numerical results

A numerical method (the Newton-Rhapson method) was chosen to solve the simultaneous nonlinear equations (6), (11) and (12). To show an application of the theory developed in this work, a numerical example is presented. I have chosen the 218 angular-contact ball bearing that was also used by [5]. Thus, the results generated here can be compared to a certain degree with the Harris results.

Figures 3 - 5 show some parameters as functions of the applied thrust load under a radial load of 17,800 N. We can observe a substantial difference between results found in this work and those found by Harris, for a thrust load of 17,800 N.

Figure 3 shows the normal ball loads, Q. Harris found the following ball load magnitudes: 6571; 5765; 3670; and 1200 N, for the balls located at angular positions: $|\psi| = 0$; 22.5^{o}; 45^{o}; and 67.5^{o}, respectively, and found zero ball load magnitudes for the balls located at angular positions $|\psi| \geq 90^{o}$ (p. 262). This work found the following ball loads magnitudes: 5997; 5395; 3807; 1820; and 239 N, for the balls located at angular positions: $|\psi| = 0$; 22.5^{o}; 45^{o}; 67.5^{o}; and 90^{o}, respectively, and found zero ball load magnitudes for the balls located at angular positions $|\psi| > 90^{o}$.

This means that Harris calculation has overestimated (or underestimated) the normal ball loads for balls located at angular positions satisfying |ψ| < 45º (|ψ| ≥ 45º); representing an error of 9.56% in the determination of maximum normal ball load and errors of 6.86%, –3.59%, –34.06% and –100% in the determination of normal ball load for the eight balls immediately about the most heavily loaded ball, respectively.

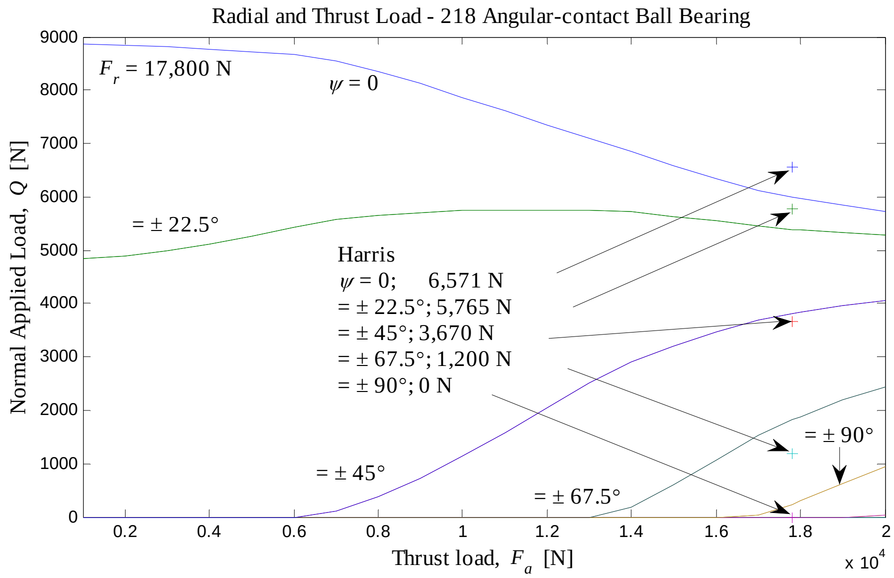

Figure 3. Normal ball load, *Q*, for 17,800 N radial load, as a function of the thrust load, F_a.

Figure 4 shows the contact angle, β. While Harris assumed a contact angle magnitude of 40° for all balls (p. 260), contact angles ranging from 38.2° to 42.7° were found in this work, while ψ were varied from ψ = 0º to ±180°, respectively. This represents errors between 4.71% and –6.28% in the contact angles determination, meaning that Harris assumption has overestimated (underestimated) the contact angles for balls located at angular positions satisfying |ψ| < 90º (|ψ| ≥ 90º).

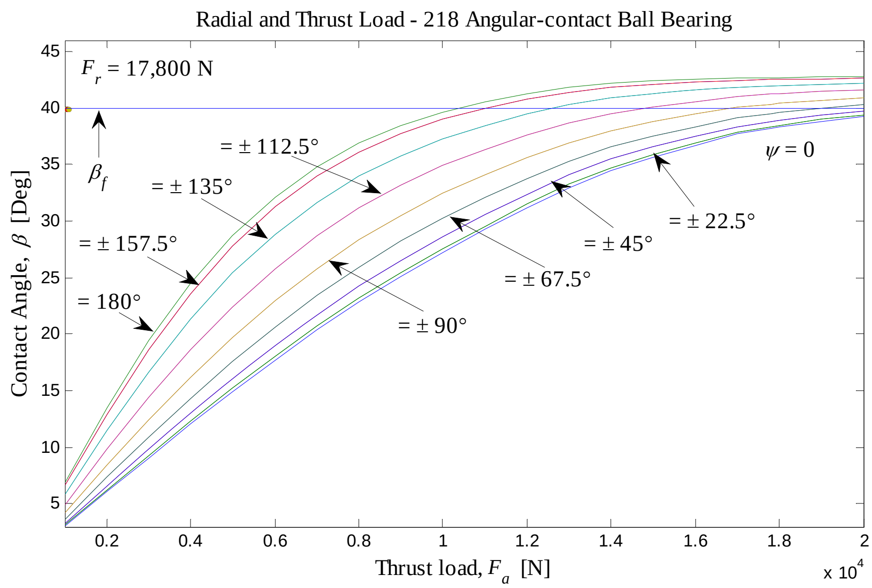

Figure 4. Contact angle, β, for 17,800 N radial load, as a function of the thrust load, F_a.

Figure 5 shows the loading zone, ψ_l. While Harris found a loading zone of 84.84º (p. 262), this work found a loading zone of 97.74º. This represents an error of –13.2% in the loading angle, meaning that the Harris calculation has underestimated the effect of the loading.

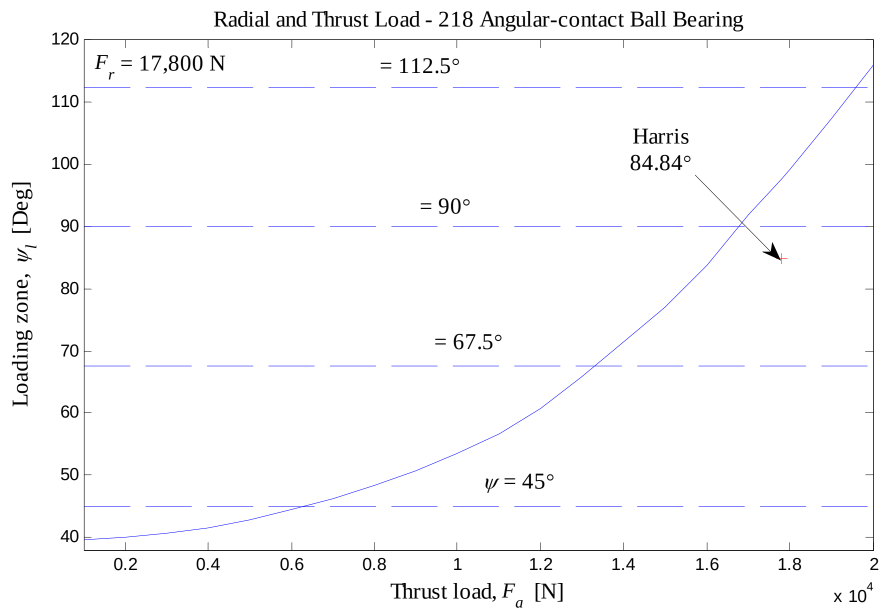

Figure 5. Contact zone, ψ_l, for 17,800 N radial load, as a function of the thrust load, F_a.

Conclusion

A numerical procedure for computing the internal loading distribution in statically loaded, single-row, angular-contact ball bearings when subjected to a known combined radial and thrust load has been presented. The combined radial and thrust load must be applied in order to avoid tilting between inner and outer rings. The procedure requires the iterative solution of $Z + 2$ simultaneous nonlinear equations with unknowns δ_a, δ_r, and β_j, $j = 1,\ldots,Z$. Numerical results for a 218 angular-contact ball bearing have been compared with those from the literature and show significant differences in the magnitudes of the ball loads, contact angles, and extent of the loading zone.

References

1. Stribeck, R. “Ball Bearings for Various Loads.” *Trans. ASME*, 29 (1907), 420-463.
2. Sjoväll, H. “The Load Distribution within Ball and Roller Bearings under Given External Radial and Axial Load.” *Teknisk Tidskrift, Mekanik*, häfte 9 (1933), 97-102. (http://runeberg.org/tektid/1933m/0099.html)
3. Jones, A. *Analysis of Stresses and Deflections*. Bristol: New Departure Engineering Data, ©1946.
4. Rumbarger, J. “Thrust Bearings with Eccentric Loads.” *Machine Design* (15 February 1962).
5. Harris, T. *Rolling Bearing Analysis*. 4th Ed., New York: John Wiley & Sons, Inc., ©2001.
6. Ricci, M. C., “Ball bearings subjected to a variable eccentric thrust load.” *DINCON’09 Proceedings of the 8th Brazilian Conference on Dynamics, Control and Applications*, (May 2009). ISBN: 978-85-86883-45-3.
7. Ricci, M. C. “Internal loading distribution in statically loaded ball bearings subjected to a combined radial, thrust, and moment load.” *Proceedings of the 60th International Astronautical Congress*, (October 2009). ISSN 1995-6258.