# Swing-By Combined with Impulsive Maneuvers in the Sun-Jupiter System 

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#### Abstract

This paper makes a study of the effects of a Swing-By combined with an impulsive maneuver in the orbit of a spacecraft passing by Jupiter in the Sun-Jupiter system. The Swing-By is a maneuver where the spacecraft approaches a celestial body to gain or lose energy from its gravity. This combined maneuver causes a change in the orbit of the spacecraft. The goal is to calculate the maximum variation of the energy obtained from this combined maneuver. This type of maneuver is important, because it generates a significant fuel economy in space missions. The studies were done with the periapse of the orbit of the spacecraft around Jupiter in different points and with the impulse applied in different directions. The results show that, in general, the application of the impulse in directions that are not tangential to the orbit is more efficient.


Key Words: Swing-By, impulsive maneuver, spacecraft trajectories.

## 1. Introduction

The Swing-By combined with an impulse is a maneuver where the spacecraft passes near a celestial body to gain or lose energy from its gravity field and receives an impulse during this passage. The goal of this type of maneuver is the fuel economy of the spacecraft that performs the maneuver. Recent examples of missions that used this type of maneuver are shown in the internet, like references [1] and [2].

There are several papers in the literature studying this maneuver in more detail or using this method in real missions, like Longuski and Strange [3], who studied graphical methods to plan gravity assisted trajectories, with the goal of lowering the launch mass and the flight time, to make accessible targets of great scientific interests. Araújo, Winter and Prado [4] studied the Swing-By effect in the system Vesta-Magnya. In addition to these works, there are many others available in the literature, like shown in references [5] to [27].

In this paper we study the Swing-By maneuver combined with the application of an
impulse in the periapse of the passage of the spacecraft by Jupiter. In the literature there is a paper of this type applied to the Earth-Moon-spacecraft system [14].

The maneuver is studied in a system composed by three bodies [15], where $\mathrm{M}_{1}$ is the primary body, the one with the largest mass; $\mathrm{M}_{2}$ is the secondary body that orbits $M_{1}$; and $M_{3}$ is a particle with a mass that can be considered negligible that orbits $\mathrm{M}_{1}$ and then makes a Swing-By with $\mathrm{M}_{2}$.

The maneuver is identified by three parameters: Vinf-, the magnitude of the velocity of approach; $\psi$, the approach angle, which defines the position of the periapse; and $\mathrm{r}_{\mathrm{p}}$, the periapse radius, the shortest distance between the spacecraft and the secondary body. To identify the Swing-By with an impulse, in addition to these three parameters, there are two more parameters required to describe the impulse: $\delta \mathrm{V}$, the magnitude of the impulse, in $\mathrm{km} / \mathrm{s}$, and $\alpha$, an angle that defines the direction of the impulse applied.

## 2. Dynamics of the System

The system is composed by three bodies, according to the well known restricted problem of three bodies. Because there is no analytical solution for this system, numerical integration was used.

In the present work, $\mathrm{M}_{1}$ is the Sun, $\mathrm{M}_{2}$ is Jupiter, and $M_{3}$ is the spacecraft that makes a SwingBy with Jupiter and that is supposed to have a negligible mass. The equations of motion are:

$$
\begin{gather*}
\ddot{x}-2 \dot{y}=\frac{\partial \Omega}{\partial x}  \tag{1}\\
\ddot{y}+2 \dot{x}=\frac{\partial \Omega}{\partial y}  \tag{2}\\
\Omega=\frac{1}{2}\left(x^{2}+y^{2}\right)+\frac{1-\mu}{r_{1}}+\frac{\mu}{r_{2}} \tag{3}
\end{gather*}
$$

It is observed that the equations of motion depends on the potential $\Omega$, that depends on $r_{1}$, the distance between $\mathrm{M}_{1}$ and $\mathrm{M}_{3}$; and $\mathrm{r}_{2}$, the distance between $M_{2}$ and $M_{3}$. When $r_{1}$ and $r_{2}$ have values equal or near zero a singularity occurs in the numerical integration. To avoid this singularity we use the method of Lemaître regularization [16]. This method eliminates the singularities by replacing variables.

In this paper we studied the Swing-By combined with the application of an impulse in the periapse of the orbit, that is, when $\theta=0^{\circ}$, as shown in Figure 1. We used $r_{p}=1.1$ Jupiter's radius.


Figure 1 - Geometry of the Swing-By combined with an impulse applied at the periapse.

The algorithm works as follow: 1) Search the point where the impulse will be applied. In this case $\theta$ $=0^{\circ}$, then the impulse will be applied at the periapse, the point P in Figure 1; 2) From the point P , a numerical integration is made in reverse time [17], without applying the impulse, to obtain the data of the first orbit; 3) Then, the impulse is applied at the point P , in the direction defined by the angle $\alpha$ and with a magnitude $\delta \mathrm{V}$, and after that we integrate the equations of motion of the spacecraft forward in time to get the data of the new orbit; 4) Finally, the difference between these energies are calculated. Thus, we can obtain the maximum variation of the energy as a function of the magnitude of the impulse and the angle $\alpha$.

## 3. Results

We study the case Sun-Jupiter-spacecraft in the situation where $\theta=0^{\circ},-180^{\circ}<\alpha<180^{\circ}$, varying in steps of 0.1 for different values of $\psi$. Since the goal is to analyze the maximum variation of energy, we concentrate the study in the region $180^{\circ}<\psi<360^{\circ}$, because this is a region where energy is gained due to the Swing-By. For $\psi=270^{\circ}$ we have the maximum gain of energy. It is possible to see this observation from Equation 5 [18]:

$$
\begin{equation*}
E_{o}=E_{i}-2 V_{2} V_{\text {inf }-} \sin \delta \sin \psi \tag{5}
\end{equation*}
$$

This equation calculates the energy of the spacecraft after the close approach with $\mathrm{M}_{2}$. In equation $5, \mathrm{E}_{\mathrm{i}}$ is the energy before the close approach; $\mathrm{V}_{2}$ is the linear velocity of $\mathrm{M}_{2}$ relative to the center of mass of $M_{1}$ and $M_{2} ; V_{\text {inf- }}$ is the magnitude of the velocity of approach; $\delta$ is half of the curvature angle of the first orbit; and $\psi$ is the angle of approach. The studies were done for $\psi=180^{\circ}, \psi=225^{\circ}, \psi=270^{\circ}$ and $\psi=315^{\circ}$, that covers the interval where there are energy gains.

In the cases where $\psi=180^{\circ}$ and $\psi=225^{\circ}$, the maximum variation of energy occurs for $\alpha$ negative. This fact occurs because, when $\alpha$ is negative, the spacecraft goes to a trajectory that passes closer to Jupiter, so increasing the effects of the Swing-By. Then the energy lost with the impulse applied out of the periapse is compensated by this gain. Figures 2 to 7 are made for $\psi=180^{\circ}, \psi=225^{\circ}$ and $\psi=270^{\circ}$ considering different values for the magnitude of impulse.


Figure 2 - Graphics of $\alpha$ vs. $\Delta$ E, with $\psi=180^{\circ}$ and $\delta \mathrm{V}=1.0 \mathrm{~km} / \mathrm{s}$.


Figure 3 - Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=180^{\circ}$ and $\delta \mathrm{V}=1.5 \mathrm{~km} / \mathrm{s}$.


Figure 4 - Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=225^{\circ}$ and $\delta \mathrm{V}$ $=1.0 \mathrm{~km} / \mathrm{s}$.

$\theta=0^{\circ}, \psi=225^{\circ}$ e $\delta V=1.5 \mathrm{~km} / \mathrm{s}$
Figure 5-Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=225^{\circ}$ and $\delta \mathrm{V}$ $=1.5 \mathrm{~km} / \mathrm{s}$.


Figure 6-Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=270^{\circ}$ and $\delta \mathrm{V}$ $=1.5 \mathrm{~km} / \mathrm{s}$.


Figure 7 - Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=270^{\circ}$ and $\delta \mathrm{V}$ $=2.0 \mathrm{~km} / \mathrm{s}$.

For the cases where $\psi=270^{\circ}$, it is possible to notice that the maximum variation of energy occurs for negative values of $\alpha$. This fact happens because for values of $\alpha$ negative, there is a decrease in $r_{p}$, so the effect of the Swing-By is increased. Figures 8 to 11 are made for $\psi=315^{\circ}$, for different values of the magnitude of impulse.


Figure 8-Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=315^{\circ}$ and $\delta \mathrm{V}$ $=0.5 \mathrm{~km} / \mathrm{s}$.


Figure 9 - Graphics of $\alpha$ Vs $\Delta$ E, with $\psi=315^{\circ}$ and $\delta \mathrm{V}=1.0 \mathrm{~km} / \mathrm{s}$.


Figure 10-Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=315^{\circ}$ and $\delta \mathrm{V}=1.5 \mathrm{~km} / \mathrm{s}$.


Figure 11 - Graphics of $\alpha$ vs. $\Delta \mathrm{E}$, with $\psi=315^{\circ}$ and

$$
\delta \mathrm{V}=2.0 \mathrm{~km} / \mathrm{s} .
$$

In these figures and in Table 1, it is possible to see that the maximum variation of energy grows up as the magnitude of the impulse applied increases, what is expected, because more energy is transferred to the spacecraft.

When $\psi=315^{\circ}$ the spacecraft is in the region where the spacecraft gains energy from the Swing-By, but is travelling to go near the frontier where the gains becomes losses $\left(\psi=360^{\circ}\right)$. So, the values for $\alpha$ is around zero, with the intention of minimizing the change of the geometry of the system to avoid that the spacecraft reaches a region where there is a loss in the energy. In Figs. 8, 9 and 10 the value of $\alpha$ for the maximum variation of energy is $0.9^{\circ}$ and in Fig. 11 the maximum variation of energy occurs for $\alpha=0.8^{\circ}$.

Table 1 - Maximum variation of energy and their data, for $r_{p}=1.1$ Jupiter's radius, with $\theta$ equal to $0^{\circ}$.

| $\psi$ | $\boldsymbol{\delta V}(\mathrm{km} / \mathrm{s})$ | $\Delta \mathbf{E}_{\text {máx }}$ | $\boldsymbol{\alpha}$ |
| :---: | :---: | :---: | :---: |
| $180^{\circ}$ | 1.5 | 19.9740 | $-20.8^{\circ}$ |
| $180^{\circ}$ | 2.0 | 28.8649 | -19.7 ${ }^{\circ}$ |
| $180^{\circ}$ | 2.5 | 38.7032 | $-18.8^{\circ}$ |
| $180^{\circ}$ | 3.0 | 49.4267 | $-17.9^{\circ}$ |
| $180^{\circ}$ | 3.5 | 60.9842 | $-17.1^{\circ}$ |
| $180^{\circ}$ | 4.0 | 73.3319 | $-16.4^{\circ}$ |
| $225^{\circ}$ | 1.5 | 258.9481 | -7.9 ${ }^{\circ}$ |
| $225^{\circ}$ | 2.0 | 283.0748 | $-9.0^{\circ}$ |
| $225^{\circ}$ | 2.5 | 306.5115 | -9.9 ${ }^{\circ}$ |
| $225^{\circ}$ | 3.0 | 329.5409 | $-10.7^{\circ}$ |
| $225^{\circ}$ | 3.5 | 352.3514 | $-11.4^{\circ}$ |
| $225^{\circ}$ | 4.0 | 375.0731 | $-12.0^{\circ}$ |
| $270^{\circ}$ | 1.5 | 398.5889 | $-2.4{ }^{\circ}$ |
| $270^{\circ}$ | 2.0 | 441.2306 | $-2.9^{\circ}$ |
| $270^{\circ}$ | 2.5 | 481.9640 | $-3.5^{\circ}$ |
| $270^{\circ}$ | 3.0 | 521.2718 | $-3.9{ }^{\circ}$ |
| $270^{\circ}$ | 3.5 | 559.4926 | -4.4 ${ }^{\circ}$ |
| $270^{\circ}$ | 4.0 | 596.8742 | $-4.9^{\circ}$ |
| $315^{\circ}$ | 1.5 | 356.6317 | $0.9{ }^{\circ}$ |
| $315^{\circ}$ | 2.0 | 410.5652 | $0.8^{\circ}$ |
| $315^{\circ}$ | 2.5 | 462.6676 | $0.7^{\circ}$ |
| $315^{\circ}$ | 3.0 | 513.3236 | $0.6{ }^{\circ}$ |
| $315^{\circ}$ | 3.5 | 562.8091 | $0.4{ }^{\circ}$ |
| $315^{\circ}$ | 4.0 | 611.3335 | $0.3{ }^{\circ}$ |

In every figure, the regions where there are no curves are regions where captures of the spacecraft by

Jupiter occurred. This happens because, for $\alpha$ around $-180^{\circ}$ and $180^{\circ}$, it means that the impulse was applied in the opposite direction of the motion of the spacecraft. Then the spacecraft has its velocity decreased and tends to go into a lower orbit, which can cause the capture of the spacecraft by $\mathrm{M}_{2}$.

Table 1 shows the maximum variation of energy for different values of the magnitude of the impulse and the angle of approach $\psi=180^{\circ}, \psi=$ $225^{\circ}, \psi=270^{\circ}$ and $\psi=315^{\circ}$. In Table $1 \Delta \mathrm{E}_{\text {máx }}$ is the maximum variation of energy. With this study we verified the importance of the parameter $\alpha$ in this type of maneuver. It defines the curvature of the second orbit. If the goal is to decrease $r_{p}$ and so to increase the effect of the Swing-By, $\alpha$ determines how the spacecraft will approach $\mathrm{M}_{2}$ to obtain the maximum energy without colliding with the body.

## 4. Conclusions

From the studies and results obtained in the present paper, it is concluded that the best direction to apply an impulse is in directions that are not tangential to the orbit, so $\alpha \neq 0^{\circ}$. A maneuver with these characteristics tends to make better use of the Swing-By combined with the impulse applied, therefore larger magnitudes for the variation in energy is obtained. The best values for each circumstance are shown, as a function of the parameters that defines the trajectory.

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