

## DIAGNOSIS OF INCIPIENT TIME DEPENDING FAULTS BASED ON INVERSE PROBLEM FORMULATION AND STOCHASTIC ALGORITHMS

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### Abstract

This work is focused on the formulation of fault diagnosis (FDI) by an inverse problem methodology. It has been shown that this approach allows diagnosing with an adequate balance between robustness and sensitivity. The main contribution of this paper is related to the diagnosis of incipient faults, which are time depending. The FDI problem is formulated as an optimization problem, which is solved with the stochastic algorithm Differential Evolution, and its variation Differential Evolution with Particle Collision. The proposed approach is tested using simulated data of the Two Tanks system, which is recognized as a benchmark for control and diagnosis. The results indicate the suitability of the proposed approach.

**Keywords:** computational cost, differential evolution, fault diagnosis, inverse problem, sensitivity, robustness.

### 1. Introduction

The automatic early detection, isolation, and identification of faults is named Fault diagnosis, FDI [1]. This is an important task for improving reliability and safety in the industry [1,2,3].

The FDI methods should guarantee the fast detection of the faults, while rejecting false alarms attributable to different causes such as uncertainties in the measurements; external disturbances or spurious signal. It leads to the necessity of sensitive and robust FDI methods [1-3].

An adequate balance of these properties is the key for practical applications of FDI methods [1,2]. Furthermore, it is still considered as a main limitation of the current FDI methods [2-4].

Within the methods for Fault Diagnosis, we find the analytical model based methods [5]. The analytical model of the system can incorporate the dynamics of the faults that can eventually affect the system [6,7]. Such dynamics can be modeled by means of a fault vector. The determination of the fault vector, when the system outputs and inputs are measured, is an inverse problem [8].

It has been previously shown that the formulation of the fault diagnosis by an inverse problem methodology, allows to obtain an appropriate balance between robustness and sensitivity [9,10]. In these papers, the FDI inverse problem is established as an optimization problem, which is solved with stochastic algorithms. In all these works, the faults keep a constant magnitude in time.

This work is also focused on the formulation of the fault diagnosis by an inverse problem methodology. The main contribution of this paper is the diagnosis of incipient faults, which are time dependent. The application of results from the diagnosis area, namely results related with structural detectability and structural separability of faults [11], allows obtaining information concerning the inverse problem under study [12].

The optimization problem is solved with the stochastic algorithms Differential Evolution, DE [13] and its modified version Differential Evolution with Particle Collision, DEwPC [9]. The proposal is tested using simulated data of the Two Tanks system, which is recognized as a benchmark for control and diagnosis [14]. The test cases results show the suitability of the proposal.

The remaining content of this paper is organized as follows. In Section 2, the FDI formulated as an inverse problem is presented. Differential Evolution and Differential Evolution with Particle Collision are briefly explained in Section 3. Afterward, Section 4 details the case of study and its simulations. The other sections present the Experimental Methodology and Results, following the same order. In Section 7, some concluding comments and remarks are presented.

## 2. Fault Diagnosis as an Inverse Problem

FDI based on model parameters requires online parameters estimation methods. These parameters can be determined with parameters estimation methods by measuring the input vector  $u(t)$  and the output vector  $y(t)$ , if the basic model structure is known [1].

The models used for describing the systems vary depending on the dynamics of the process, and the objective to be reached with the simulation. The typically used model is the linear time invariant (LTI) which has two representations: the transfer function or transfer matrix, and the state space representation.

The state representation is also valid for nonlinear models. Thus in this paper, this representation is considered in order to decrease the uncertainties due the linearization. These nonlinear models can be described as:

$$\begin{aligned} \dot{x}(t) &= g(x(t), u(t), \theta) \\ y(t) &= h(x(t), u(t)) \\ x(t_0) &= x_0 \end{aligned} \quad (1)$$

Where  $x(t) \in \mathbb{R}^n$  is the state variables vector;  $x_0$  is the initial state;  $\theta \in \mathbb{R}^l$  is the parameter vector of the model and  $t \in [t_0, t_f]$ . The input  $u(t) \in \mathbb{R}^p$  and the output  $y(t) \in \mathbb{R}^m$  are measured with sensors.

The faults affecting the system may eventually change the parameters values in the vector  $\theta$ . The main disadvantage of this approach is that the model's parameters should have physical meaning, i.e., they should match with the parameters of the system. Furthermore, the fault isolation may become extremely difficult because model parameters do not uniquely match with those of the system [1,4].

Instead, it can be considered a model that directly includes the dynamics of the faults, by means of the fault vector  $f$  [6, 7]:

$$\begin{aligned} \dot{x}(t) &= g(x(t), u(t), f) \\ y(t) &= h(x(t), u(t), f) \\ x(t_0) &= x_0 \end{aligned} \quad (2)$$

The fault vector  $f = [f_a \ f_p \ f_s]^t \in \mathbb{R}^{p+q+m}$ , where  $f_a \in \mathbb{R}^p$ ;  $f_p \in \mathbb{R}^q$  and  $f_s \in \mathbb{R}^m$  are the faults affecting the actuator, process and sensors, respectively [6, 7]. These three parts establish the level of abstraction for the diagnosis.

In such cases the diagnosis can be directly obtained from the estimation  $\hat{f}$  of the fault vector. This inverse problem of parameter estimation can be formulated as an optimization problem:

$$\begin{aligned} \min \quad & F(\hat{f}) = \sum_{t=1}^S [\mathbf{y}(f, t) - \hat{\mathbf{y}}(\hat{f}, u(t), t)]^2 \\ \text{s.t.} \quad & f_{min} \leq \hat{f} \leq f_{max} \end{aligned} \quad (3)$$

where  $S$  is the number of sampling instants,  $\hat{\mathbf{y}}(\hat{f}, u(t), t)$  is the estimated vector output in each instant of time and it is obtained from the model (2) and using the measurements of the output  $u(t)$ ;  $\mathbf{y}(f, t)$  is the output vector, which is measured by the sensors at the same time instant  $t$ .

In recent works, this approach has been applied to FDI [9, 10]. In these previous works, the faults were assumed constant throughout the process. The main contribution of this paper, and its difference comparing with these works, is that faults considered are time dependent. This intends to include the faulty situations that affect systems in practical situation.

For this purpose it has been assumed that the dynamics of a fault  $f_i$  can be described by means of the well known ramp function:

$$f_i(t) = m_i t \quad (4)$$

Being the optimization problem:

$$\begin{aligned} \min \quad & F(m) = \sum_{t=1}^S [\mathbf{y}(m, t) - \hat{\mathbf{y}}(\hat{m}, t)]^2 \\ \text{s.t.} \quad & m_{\min} \leq \hat{m} \leq m_{\max} \end{aligned} \quad (5)$$

Where  $m_{\min}, m_{\max}$  are functions of  $f_{\min}, f_{\max}$ , respectively. This problem is similar to other parameters estimation inverse problems. Furthermore, considering the reported applications of stochastic algorithms to this kind of problem [15], we are also interested about solving problem in Eq. (5) by means of stochastic algorithms.

## 2.1 Structural Analysis

For obtaining some prior information about the uniqueness, or not, of the set of fault vectors that can justify the observed behavior of the system, some results related with sensor placement for faults detectability and separability are applied [11].

These results are based on the structural representation of the model, and on the Dulmage-Mendelsohn decomposition. In [12] it is shown how it can be understood as an alternative sensitivity analysis for parameter estimation inverse problems, when the model of the systems is represented by ordinary differential equations.

## 3. Differential Evolution and Differential Evolution with Particle Collision

Differential Evolution, DE, was proposed in 1995 for optimization problems [13]. Some of the most important advantages of DE are: simple structure, simple computational implementation, speed and robustness [13].

DE is based on three operators: Mutation, Crossover and Selection [13]. These operators are based on vector operations, which is the main difference comparing with Genetic Algorithms.

The algorithm generates at each iteration *Iter* a new population of *Z* feasible solutions  $X_{Iter}^1, X_{Iter}^2 \dots X_{Iter}^Z$  with the application of the three operators on the current population. This mechanism can be summarized with the notation:

$$DE/X_{Iter}^\delta / \gamma / \lambda \quad (6)$$

where  $\gamma$  indicates the number of pair of solutions of the current solution to be used for the perturbation of the current solution  $X_{Iter}^\delta$ ;  $\lambda$  represents the distribution function to be used during the crossover. In this work was applied the scheme  $DE/X^{best} / 2/bin$ , being *bin* a notation for the binomial distribution function and the Mutation is described by:

$$\hat{X}_{Iter}^z = X^{best} + C_{scal} (X_{Iter-1}^{\alpha 1} - X_{Iter-1}^{\alpha 3} + X_{Iter-1}^{\alpha 2} - X_{Iter-1}^{\alpha 4}) \quad (7)$$

where  $X^{best}, X_{Iter-1}^{\alpha 1}, X_{Iter-1}^{\alpha 3}, X_{Iter-1}^{\alpha 2}, X_{Iter-1}^{\alpha 4} \in \mathbb{R}^n$  are solutions of the current population and  $C_{scal}$  is a parameter of the algorithm, called *Scaling factor*. In the other hand, the *Crossover* and *Selection* operator can be described as:

- *Crossover*:

$$\hat{x}_{(Iter)n}^z = \begin{cases} \hat{x}_{(Iter)n}^z & \text{if } q_{rand} \leq C_{cross} \\ \hat{x}_{(Iter-1)n}^\delta & \text{otherwise} \end{cases} \quad (8)$$

where  $\hat{x}_{(Iter)n}^z$  are the components of the vector  $\hat{X}_{Iter}^z$ ;  $0 \leq C_{cross} \leq 1$  is another parameter of the algorithm: *crossover factor*; and  $q_{rand}$  is a random number which is generated by means of the distribution represented by  $\lambda$ .

- *Selection*: The vector  $X_{Iter}^z$ , to be part of the new population, is selected following the rule:

$$X_{Iter}^z = \begin{cases} \hat{X}_{Iter}^z & \text{if } F(\hat{X}_{Iter}^z) \leq F(X_{Iter-1}^\delta) \\ X_{Iter-1}^\delta & \text{otherwise} \end{cases} \quad (9)$$

A general description of the algorithm for DE is given in Fig. 1.

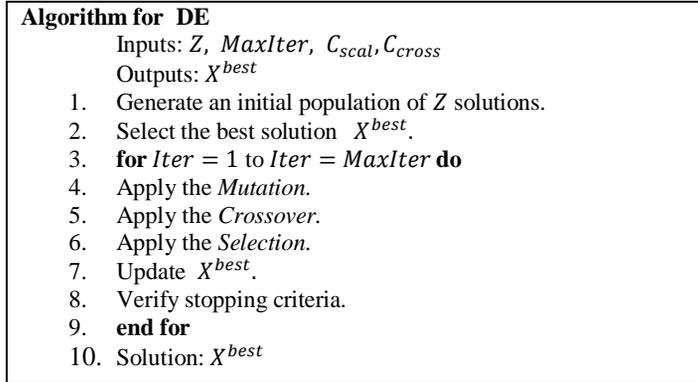


Fig. 1. Algorithm for DE

The more successful variants of DE are focused on variations of the *Mutation* operator and in the self adaptation of the parameters  $C_{cross}$  and  $C_{scal}$ .

### 3.1 Differential Evolution with Particle Collision

The new algorithm Differential Evolution with Particle Collision, DEwPC [9], has the objective to improve the performance of DE based on the incorporation of some ideas of the Particle Collision Algorithm, PCA [16], in order to improve its capacity of escaping from local optimum.

DEwPC keeps the same structure of the operators Mutation and Crossover in DE, while introduces a modification in the Selection operator [9]. This modification adds a new parameter  $MaxIter_c$ .

The new Selection operator takes the ideas of Absorption and the Scattering from PCA. The adaptation of this operator to the DEwPC has been called *Selection with Absorption- Scattering with probability* and can be established as:

*Selection with Absorption- Scattering with probability*

- If  $F(\hat{X}_{Iter}^z) \leq F(X_{Iter-1}^\delta)$  then the operator Absorption is applied to  $\hat{X}_{Iter}^z$
- If  $F(\hat{X}_{Iter}^z) > F(X_{Iter-1}^\delta)$  then the operator Scattering with probability is applied to  $\hat{X}_{Iter}^z$ .

The *Absorption- Scattering with probability* operator, as well as the algorithm for DEwPC are represented in Fig. 2.

The operator Small Search indicates a small stochastic perturbation around a solution. The Search indicates a stochastic perturbation around a solution [16].

<p><b>Algorithm for Absorption</b> Input: <math>\hat{X}_{Iter}</math> Output: <math>X_{Iter}</math></p> <ol style="list-style-type: none"> <li>1. <math>X_{Iter} = \hat{X}_{Iter}</math></li> <li>2. Small Search (<math>X_{Iter}</math>)</li> </ol>	<p><b>Algorithm for DEwPC</b> Inputs: <math>Z, MaxIter, C_{scal}, C_{cross}, MaxIter_c</math> Output: <math>X^{best}</math></p> <ol style="list-style-type: none"> <li>1. Generate an initial population of <math>Z</math> solutions.</li> <li>2. Select the best solution <math>X^{best}</math>.</li> <li>3. <b>for</b> <math>Iter = 1</math> to <math>Iter = MaxIter</math> <b>do</b></li> <li>4. Apply the <i>Mutation</i>.</li> <li>5. Apply the <i>Crossover</i>.</li> <li>6. <b>for</b> <math>j=1</math> to <math>j=Z</math></li> <li>7. <b>if</b> <math>rand &lt; 0.7</math> <b>do</b></li> <li>8. Apply the operator <i>Absorption-Scattering with probability</i> to <math>\hat{X}_{Iter}^{(j)}</math></li> <li>9. <b>else</b></li> <li>10. Apply <i>Selection</i> to <math>\hat{X}_{Iter}^{(j)}</math></li> <li>11. <b>end if</b></li> <li>12. <b>end for</b></li> <li>13. Update <math>X^{best}</math></li> <li>14. Verify stopping criteria.</li> <li>15. <b>end for</b></li> <li>16. Solution: <math>X^{best}</math></li> </ol>
<p><b>Algorithm for Scattering with probability</b> Inputs: <math>\hat{X}_{Iter}, F(X^{best})</math> Output: <math>X_{Iter}</math></p> <ol style="list-style-type: none"> <li>1. Compute <math>F(\hat{X}_{Iter})</math></li> <li>2. Compute <math>p_{r(Iter)} = 1 - \frac{F(X^{best})}{F(\hat{X}_{Iter})}</math></li> <li>3. Generate a random number <math>q</math></li> <li>4. <b>if</b> <math>q &lt; p_{r(Iter)}</math> <b>then</b></li> <li>5. <math>X_{Iter} = \hat{X}_{Iter}</math></li> <li>6. Search (<math>X_{Iter}</math>)</li> <li>7. <b>else</b></li> <li>8. <math>X_{Iter} = X_{Iter-1}</math></li> <li>9. <b>end</b></li> </ol>	

Fig. 2. Algorithms for the Absorption and Scattering with probability operators and DEwPC

#### 4. Two Tanks system

The Two Tanks system is a simplified version from the Three Tanks system [14]. Both are benchmarks for control and diagnosis. The system is formed by two tanks of liquid that can be filled with two similar and independent pumps acting on tanks 1 and 2, respectively. The tanks have the same cross section  $S_1 = S_2 = 2.54 \text{ m}^2$ . The pumps deliver the flow rate  $q_1$  in tank 1 and  $q_2$  in tank 2. The tanks are interconnected to each other through lower pipes, see Fig. 3. All the pipes have the same cross section  $S_p = 0.1 \text{ m}^2$ . The liquid level  $L_1$  and  $L_2$  at tank 1 and tank 2, respectively, are the outputs of the system. The control variables  $q_1$  and  $q_2$  are chosen to control the levels of tank 1 and tank 2 ( $\tilde{L}_1 = 4.0 \text{ m}$  and  $\tilde{L}_2 = 3.0 \text{ m}$ ).

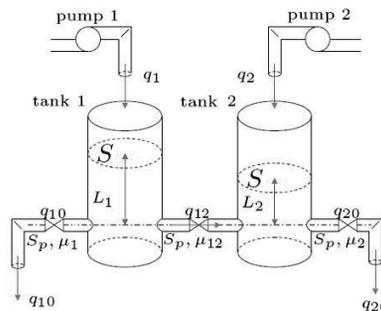


Fig.3. Two tanks system

The system can be affected by two faults  $f_{p(1)}(t)$  and  $f_{p(2)}(t)$ , which model a leak in Tank 1 and in Tank 2, respectively. Both faults are under the following restrictions:

$$f_{p(1)}, f_{p(2)} \in \mathbb{R}: 0 \leq f_{p(1)}, f_{p(2)} \leq 1 \text{ m}^3/\text{s} \quad (10)$$

The model of the system can be derived from the application of fundamental laws, along with Torricelli's law:

$$\begin{cases} \dot{L}_1 = \frac{q_1}{s_1} - \frac{c_1}{s_1} \sqrt{L_1} - \frac{c_3}{s_1} \sqrt{|L_1 - L_2|} \operatorname{sig}(L_1 - L_2) - \frac{f_{p(1)}}{s_1} \\ \dot{L}_2 = \frac{q_2}{s_2} - \frac{c_2}{s_2} \sqrt{L_2} + \frac{c_3}{s_2} \sqrt{|L_1 - L_2|} \operatorname{sig}(L_1 - L_2) - \frac{f_{p(2)}}{s_2} \\ y_1 = L_1 \\ y_2 = L_2 \end{cases} \quad (11)$$

The model is nonlinear;  $C_i = \mu_i S_p \sqrt{2g}$ , with  $i = 1, 2$ , are set as  $C_1 = C_2 = C_3 = 0.3028 \text{ m}^{\frac{3}{2}}/\text{s}$ . It has been designed a PID controller with parameters  $K_p = [12 \ 14]$ ,  $K_i = [1.15 \ 0.3]$  and  $K_d = [1.0 \ 1.5]$ .

We considered that the faults are incipient and change on time following a ramp function:

$$f_{p(1)} = m_1 t, \quad f_{p(2)} = m_2 t, \quad \text{with } 0 \leq m_1 \leq \frac{1}{t_f}, \quad 0 \leq m_2 \leq \frac{1}{t_f} \quad (12)$$

The inverse problem of estimating the fault vector is formulated as an optimization problem:

$$\begin{aligned} \min \|F(\hat{m}_1, \hat{m}_2)\|_{\infty} &= \left\| \sum_{t=1}^S \left( L(t, m_1, m_2) - \hat{L}(t, q_1, q_1 \hat{m}_1, \hat{m}_2) \right)^2 \right\|_{\infty} \\ \text{s.t.} \quad & 0 \leq \hat{m}_1 \leq \frac{1}{t_f} \\ & 0 \leq \hat{m}_2 \leq \frac{1}{t_f} \end{aligned} \quad (13)$$

where  $L^T = (L_1 \ L_2) \in \mathbb{R}^2$  and  $\|g\|_{\infty} = \max_i g_i$ . The optimization problem is solved with the application of stochastic algorithms. In this case, we applied DE and DEwPC.

#### 4.1 Structural Analysis

The structural representation of the model and its Dulmage-Mendelsohn decomposition are represented in matrix in Eq. (14). It has been used the notation  $e_i$ ,  $i = 1, 2, 3, 4$  for the equations 1, 2, 3 and 4, respectively from model (11). The equations  $e_1$  and  $e_2$  are directly affected for the faults  $f_{p(1)}$  and  $f_{p(2)}$ , respectively. The equations  $e_1$  and  $e_2$  are within the over-determined part of the model. Results related with structural separability and detectability [11,12], indicate that in this case the two faults can be estimated at the same time with the available measurements of the system.

$$\begin{array}{cc} eq / \text{var} & L_1 \quad L_2 \\ e_1 & 1 \quad 1 \\ e_2 & 1 \quad 1 \\ e_3 & 1 \quad 0 \\ e_4 & 0 \quad 1 \end{array} \quad (14)$$

### 5. Experimental Methodology

With the aim of analyzing the feasibility of the inverse problem methodology for diagnosing time dependent incipient faults, three criteria have been considered: quality of the estimations, robustness and computational cost.

The dynamics of the faults are described by Eqs. (11-12). In Table 1 are represented the different cases to be diagnosed during the experiments. Cases 1, 3, 5, 7 and Cases 2, 4, 6, 8 represent the same fault situation, respectively, but with dissimilar level noise in the measurements of  $L_1$  and  $L_2$  (up to 2%, 5%, 8% and 10%) in order to evaluate robustness. The faulty situations are intended to simulate incipient faults, which means that at the end of the sample time,  $t_f = 100 \text{ s}$ , the value of the fault is small, and its effect on the outputs of the test system may be masked by the effect of noise.

Table 1. Cases considered in the numerical experiments

Case	1	2	3	4	5	6	7	8
$m_1$	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
$m_2$	0	0.004	0	0.004	0	0.004	0	0.004
Noise %	2	2	5	5	8	8	10	10

The computational cost is compared taking as criterion the number of objective function evaluations. This aspect is important because it determines the diagnosis time, which is a requirement for on line processes. For this analysis the algorithms are executed under two stopping criteria: maximum number of iterations and error of the estimations. For each case were executed 25 independent runs of each algorithm. The indicators Success Rate, SR, and Success Performance, SP, were computed. The first indicate the percent of successful runs while  $SP = \frac{\overline{Eval}(EE)}{SR}$  ( $\overline{Eval}(EE)$  is the average of the number of objective function evaluations for the successful runs). A successful run is the one that finished because the error on the estimations of both parameters are  $e \leq 2\%R$ , being R the real value of the parameter.

**Implementation of DE:** It is based on the algorithm in Fig. 1. The parameters values are:  $Z = 20$ ,  $C_{cross} = 0.9$  and  $C_{scal} = 0.5$ . The stopping criteria are  $MaxIter = 100$  or  $F(\hat{m}_1, \hat{m}_2) = M(noise)$ .

**Implementation of DEwPC:** It is based on the algorithm in Fig. 2. The parameters values are:  $Z = 10$ ,  $C_{cross} = 0.9$ ,  $C_{scal} = 0.5$ ,  $MaxIter_c = 5$ . The stopping criteria are  $MaxIter = 100$  or  $F(\hat{m}_1, \hat{m}_2) = M(noise)$ .

## 6. Results

In Fig.4 are shown the average of the relative error for the estimations obtained by each algorithm, at Cases 1, 3, 5 and 7. The results indicate that with the increase of the noise level, both algorithms leads to false alarms due to over estimation of the fault parameters. In all tested cases, the algorithms reached  $MaxIter$ . Therefore, we decided to study the values of the objective function for different levels of noise when the parameter estimations are correct.

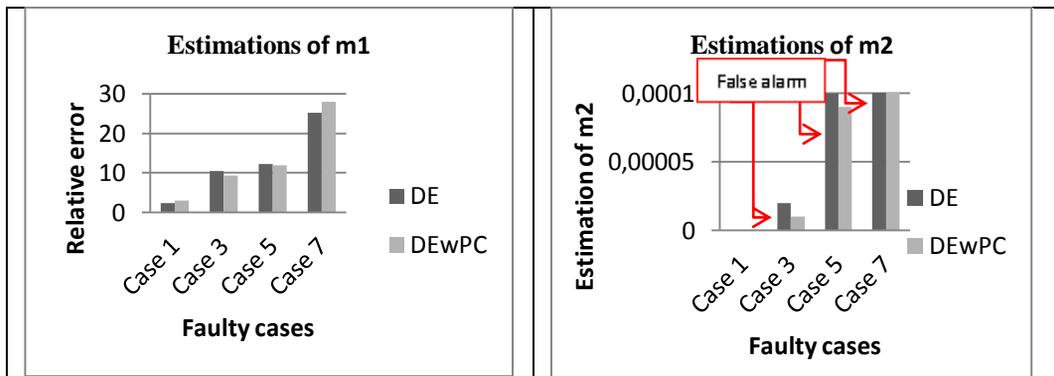


Fig. 4. Results of the estimations for the Cases 1,3,5 and 7

In Fig. 5 are shown the values of the objective function at different levels of noise. It was considered that no faults are affecting the system. This allows to change the stopping criterion  $F(\hat{m}_1, \hat{m}_2)$  and makes it dependent on the noise affecting the system  $F(\hat{m}_1, \hat{m}_2) = M(noise)$ . This implies the necessity to have some prior information concerning the noise level affecting the measurements.

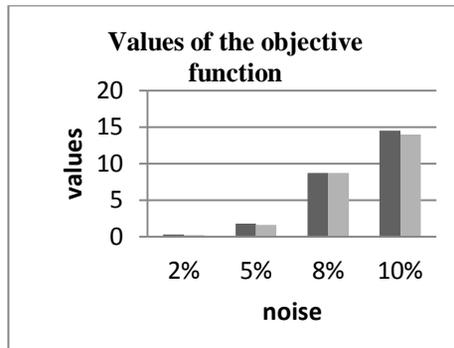


Fig. 5. Values of the objective function at different noise levels and correct parameters estimation

With the adjustment of the stopping criterion  $F(\hat{m}_1, \hat{m}_2)$ , the experiments were repeated. The results are shown in Figs. 6 and 7. In this case, the false alarms have decreased; it means that the robustness has been increased. For a noise level up to 8%, the relative error in the average of estimations is kept under 10%. For a noise level up to 10%, the relative error is kept under 13% for DE and near to 10% for DEwPC. These results indicate that for higher noise environments, the DEwPC estimations are more accurate, which implies that the diagnosis is more robust.

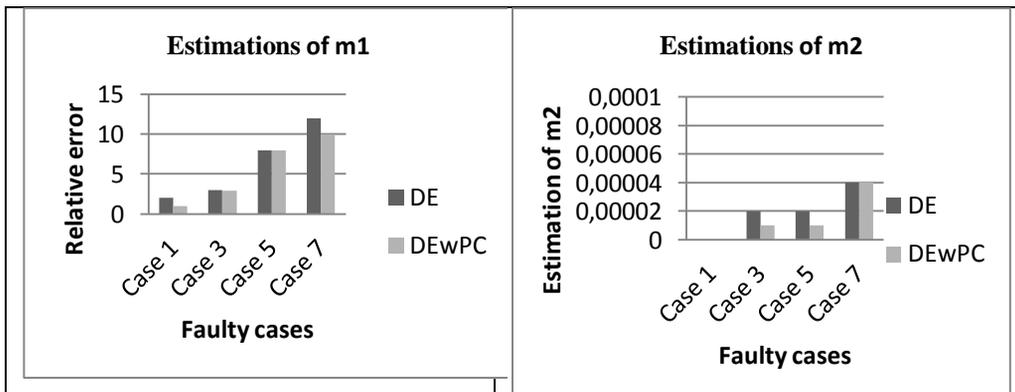


Fig. 6. Results of the estimations for the Cases 1,3,5 and 7

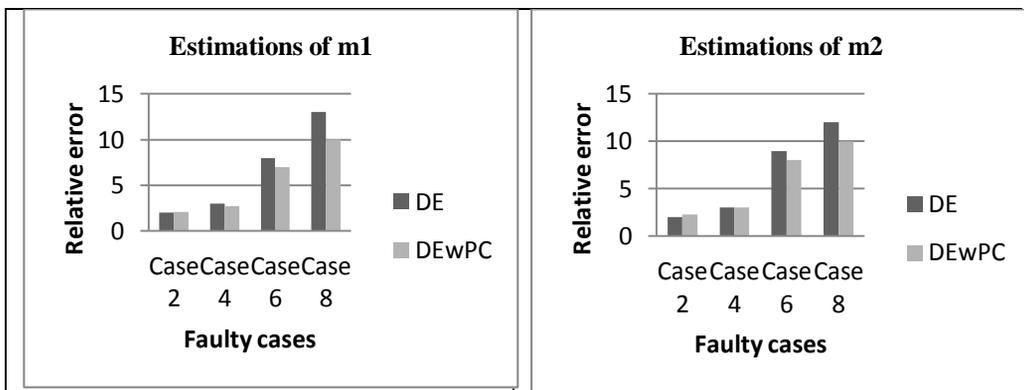


Fig. 7. Results of the estimations for the Cases 2, 4, 6 and 8

Concerning the computational cost, Table 2 shows the indicators SR and SP. In all tested cases, the best SP is observed for DEwPC. It is also shown that SR is decreasing with the increment of the noise level. For the case of noise level up to 10%, SR is around 64% for DEwPC but less than 60% for DE. This

indicates that for such level noise, the diagnosis of the incipient faults that change on time is not very reliable. Due to the design of the stopping criterion for these experiments, the SP provides a hint for the number of objective function evaluations needed for reaching an error less than 2% in the estimation of each parameter. In all cases, the SP from DEwPC is the lowest, which means that the computational cost of DEwPC is lower than DE.

Table 2. Results of the comparison for the computational cost

		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
<b>DE</b>	<b>SR</b>	92	100	80	92	72	72	56	54
	<b>SP</b>	1913	1829	2375	2045	2756	2626	3571	3703
	<b>SP/SPbest</b>	1.46	1.51	1.78	1.52	1.47	1.68	1.41	1,40
<b>DEwPC</b>	<b>SR</b>	92	100	92	96	80	80	64	64
	<b>SP</b>	1304	1211	1336	1344	1875	1561	2534	2656
	<b>SP/SPbest</b>	1	1	1	1	1	1	1	1

In Fig. 8. (a-b) is shown the evolution of the average of the objective function that DE and DEwPC achieves for Case 4, respectively. DEwPC reaches a smaller value of the objective function. DEwPC achieves the better estimations in a lower number of iterations than DE.

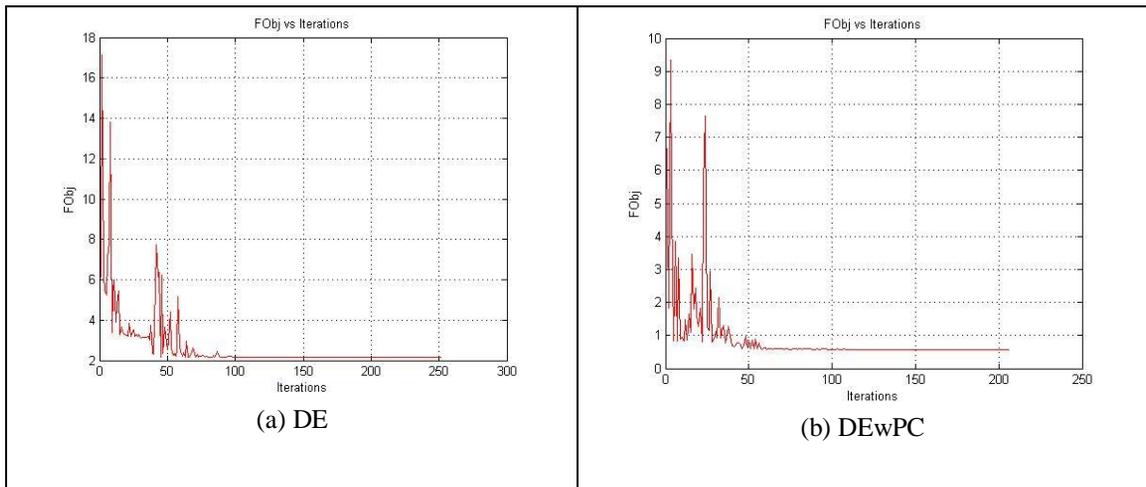


Fig. 8. Evolution of the average of the objective function, Case 4

## 7. Conclusions

This paper presents the formulation of FDI with an inverse problem methodology. The principal contribution of the paper is to consider incipient faults that change with time, which is a more realistic description of practical situations.

In that sense, the experiments confirmed the suitability of the inverse problem methodology, in particular its formulation as an optimization problem, for developing robust and sensitive methods. Moreover, the results have also shown that the diagnosis of incipient faults that change on time is also possible.

The comparison of the results for the benchmark Two tanks has also shown that DEwPC allows reducing the computational cost required by DE.

## Acknowledgements

The authors are grateful for support provided by the Brazilians Agencies FAPERJ, Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro; CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico; CAPES, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior and MES/CUBA, Ministerio de Educación Superior de Cuba.

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