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ORBITAL EVOLUTION OF PLANET AROUND A BINARY STAR

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We study the secular dynamics of hierarchical (if there is a clearly defined binary and a third body which stays separate from the binary) triple systems composed by a Sun-like central star and a Jupiter-like planet, which are under the gravitational influence of a further perturbing star (brown dwarf). The main goal is to study the orbital evolution of the planet. In special, we investigate the orientation (inclination) and the shape (eccentricity) of its orbit. One key feature explored is the time needed for the first flip in its orientation (prograde to retrograde). The gravitational potential is developed in closed form up to the third order. We have compared the secular evolution of systems with and without the third order term of the disturbing potential. The R_2 (quadrupole) and R_3 (octupole) terms of the disturbing potential are developed without using the elimination of nodes. Numerical simulations were also performed to compare with the analytical model using the N-body simulations with the Mercury code. The results show that the analytical model are in agreement with the numeric simulations.

INTRODUCTION

We study the secular dynamics of hierarchical (if there is a clearly defined binary and a third body which stays separate from the binary[1]) of a triple system composed by a Sun-like central star and a Jupiter-like planet, which are under the gravitational influence of a further perturbing star (brown dwarf). Reference[2] presented a study where the authors took into account the octupole term in the disturbing potential. They showed that the inclination of the inner planet varies from prograde to retrograde for a specific problem, where i is the mutual inclination between the two orbits. The authors showed that, considering only the quadrupole term in the potential, the inclination varies according to the Kozai-Lidov mechanism (see References [2,3,4,5]), i.e., the inclination oscillates with large amplitude when the initial inclination has a value larger than the critical inclination and small amplitudes when the initial inclination has a value smaller than the critical inclination. However, the orbit always remains prograde. When they considered the octupole term in the potential, the inclination grows a lot and can flip from prograde to retrograde trajectories. The authors did not use the elimination of nodes directly in the Hamiltonian which is generally used in the literature. They eliminate the longitude of the ascending node after deriving the equations of motion. For the third order the authors mention that it is possible do that even before deriving the equations of motion, namely the level of the Hamiltonian. Then, for the R_3 term, they use the standard elimination of the nodes. In this work we develop the disturbing potential in closed form up to the third order in a small parameter ($\alpha = a_1/a_2$), where a_1 is the semi-major axis of the planet and a_2 is the semi-major

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axis of the disturbing star, to analyze the effects caused by the terms: quadrupole and octupole on the orbital elements of the planet. The equations of motion are developed in closed form to avoid expansions in power series of the eccentricity and inclination. We compared the models mentioned here in different orders and we also compared with the results obtained by References .[2,3] We investigate what happens in the dynamics when the elimination of nodes is not used, even after deriving the equations of motion for both the terms R_2 and R_3.

EQUATIONS OF MOTION

The triple system under study is characterized by a planet m_1 in an elliptical inner orbit around the center of mass of the system m_0-m_1, orbiting a central star m_0, also moving around the center of mass of the system m_0-m_1 and a further perturbing star (brown dwarf-m_2) moving in an outer elliptical orbit around the center of mass of the system, but with a very distant trajectory that also has large eccentricity. The vector $\mathbf{r_1}$ represents the position of m_1 with respect to the center of mass of the system and the vector $\mathbf{r_2}$ is the position of the body m_2 to the center of mass of the inner orbit. Φ is the angle between $\mathbf{r_1}$ and $\mathbf{r_2}$.

The Hamiltonian of the triple system can be written as follows[6,7,8]

$$F = \frac{Gm_0m_1}{2a_1} + \frac{G(m_0+m_1)m_2}{2a_2} + \frac{G}{a_2} \sum_{j=2}^{\infty} \alpha^j M_j (\tfrac{r_1}{a_1})^j (\tfrac{a_2}{r_2})^{j+1} P_j(\cos\Phi) \tag{1}$$

where G is the gravitational constant, P_j are the Legendre polynomials and

$$M_j = m_0m_1m_2 \frac{m_0^{j-1} - (-m_1)^{j-1}}{(m_0+m_1)^j} \tag{2}$$

We shall deal with the expansion up to the third-order in α.

In References[3,9] the authors show that, in the three-body problem, when the three planets (or stars) have mass, is incorrect to use the elimination of the nodes[10] for the triple system. According to the authors, the $h_1 - h_2 = \pi$ term (often used in the literature) can not be replaced at the Hamiltonian level. It is possible to use it only after deriving the equations of motion. With this, they show that the mutual inclination can flip from prograde to retrograde trajectories when it is taken into account only the quadrupole term. Here we present a different approach, the nodes will not be eliminated before or after deriving the equations of motion. To verify this fact we developed the disturbing potential taking into account the expression for $\cos\Phi$ written in the form[11]

$$\begin{aligned}
\cos(\Phi) = {} & 1/4\,(-1+c_2)(-1+c_1)\cos(f_1+g_1-h_1-f_2-g_2+h_2)+ \\
& 1/4\,(1+c_1)(1+c_2)\cos(f_1+g_1+h_1-f_2-g_2-h_2)- \\
& 1/4\,(1+c_2)(-1+c_1)\cos(f_1+g_1-h_1+f_2+g_2+h_2)- \\
& 1/4\,(-1+c_2)(1+c_1)\cos(f_1+g_1+h_1+f_2+g_2-h_2)+ \\
& 1/2\,s_1\,s_2\,(\cos(f_1+g_1-f_2-g_2)-\cos(f_1+g_1+f_2+g_2))
\end{aligned} \tag{3}$$

where we will use the shortcut $s_1 = \sin i_1$, $c_1 = \cos i_1$, $s_2 = \sin i_2$, and $c_2 = \cos i_2$. Here i_1, g_1, h_1 and f_1 are the inclination, argument of the periastron, longitude of the ascending node and true anomaly of the inner orbit, respectively, and i_2, g_2, h_2 and f_2 are the inclination, argument of the periastron, longitude of the ascending node and true anomaly of the outer orbit, respectively. Those equations are written in a inertial reference system that has the equator of the main body in the $x-y$ plane.

In Reference[12] the disturbing potential was developed up to the fourth order, where was used the equation $g_2 = \pi - h_1$ given by Reference[13] and the Kozai classical expression[4] for the $\cos(\Phi)$ term.

For the model considered in the present paper, it is necessary to calculate the terms R_2 and R_3 of the disturbing function due to P_2 and P_3 terms, respectively. We get,

$$R_2 = \frac{G}{a_2}\alpha^2 M_2 (\frac{r_1}{a_1})^2 (\frac{a_2}{r_2})^3 P_2(\cos\Phi) \tag{4}$$

$$R_3 = \frac{G}{a_2}\alpha^3 M_3 (\frac{r_1}{a_1})^3 (\frac{a_2}{r_2})^4 P_3(\cos\Phi) \tag{5}$$

The disturbing potential given by Eq. (1) can be written as

$$F = R_0 + R_2 + R_3 \tag{6}$$

where $R_0 = \frac{G m_0 m_1}{2a_1} + \frac{G(m_0+m_1)m_2}{2a_2}$.

To eliminate the short-period terms of the potential given by Eq. (8), the double-averaged method is applied with respect to the eccentric anomaly and of the true anomaly. We make the average over the eccentric anomaly of the planet and the true anomaly of the perturbing body. This is done by using known equations from the celestial mechanics which are:

$$\sin(f) = (\sqrt{1-e^2}\sin(E))/(1-e\cos(E)); \tag{7}$$

$$\cos(f) = (\cos(E) - e)/(1 - e\cos(E)) \tag{8}$$

$$r/a = 1 - e\cos(E) \tag{9}$$

$$a/r = (1 + e\cos(f))/(1 - e^2) \tag{10}$$

$$dl = (1 - e\cos(E))dE \tag{11}$$

We also used the area integral, in the form[14]

$$dl = \frac{1}{\sqrt{1-e^2}}\frac{r^2}{a^2}df \tag{12}$$

Using the expressions given by Eqs. (1)-(5), and taking into account the known relationships from the celestial mechanics mentioned above, we integrate the equations with respect to the true and eccentric anomalies to eliminate short-period terms. Thus, we obtain the disturbing potential expanded up to the third order in a small parameter. The long-period disturbing potential (R_2) can be written as

$$
\begin{aligned}
R_{2C} = & -\frac{45}{2}\frac{\beta_3 L_1{}^4}{L_2{}^6(1-e_2{}^2)^{3/2}}\times \\
& (1/6\, e_1{}^2(c_2-1)(c_2+1)(c_1+1)^2\cos(2\,g_1-2\,h_2+2\,h_1)+ \\
& 1/6\, e_1{}^2(c_2-1)(c_2+1)(c_1-1)^2\cos(2\,g_1+2\,h_2-2\,h_1)+ \\
& 2/3\, c_2\, s_1\, s_2\, e_1{}^2(c_1-1)\cos(2\,g_1-h_1+h_2)+ \\
& 2/3\, c_2\, s_1\, s_2\, e_1{}^2(c_1+1)\cos(2\,g_1+h_1-h_2)- \\
& 1/5\,(c_2+1)(c_1-1)(c_1+1)(c_2-1)(2/3+e_1{}^2)\times \\
& \cos(-2\,h_2+2\,h_1)-4/5\, c_1\, s_2\, s_1\,(2/3+e_1{}^2)c_2\times \\
& \cos(h_1-h_2)+(-1/3+c_2{}^2)((c_1{}^2-1)e_1{}^2\cos(2\,g_1)- \\
& 3/5\,(c_1{}^2-1/3)(2/3+e_1{}^2)))
\end{aligned}
\tag{13}
$$

where

$$L_1 = \frac{m_0\, m_1\,\sqrt{G(m_0+m_1)a_1}}{m_0+m_1} \tag{14}$$

$$L_2 = \frac{m_2\,(m_0+m_1)\sqrt{G(m_0+m_1+m_2)a_2}}{m_0+m_1+m_2} \tag{15}$$

$$\beta_3 = \frac{1}{16}\,\frac{G^2(m_0+m_1)^7 m_2{}^7}{(m_0+m_1+m_2)^3 m_0{}^3 m_1{}^3} \tag{16}$$

$$\beta_4 = \frac{1}{4}\,\frac{G^2(m_0+m_1)^9 m_2{}^9(m_0-m_1)}{(m_0+m_1+m_2)^4 m_0{}^5 m_1{}^5} \tag{17}$$

Note that the potential (three degrees of freedom) depends on the longitude of the ascending node of the inner (h_1) and the outer (h_2) orbits. Note also that the argument of periastron of the outer orbit does not appear in the second order of the potential, it is usually removed during the process of double-averaged. With this we get $de_2/dt = 0$.

The potential due to the R_3 term will not be presented here because it is too long, it depends on the g_1, g_2, h_1 and h_2 terms (four degrees of freedom). Our disturbing potential is put in the form

$$R = R_{2C} + R_{3C} \tag{18}$$

APPROACH WITH RESPECT TO THE ELIMINATION OF NODES

Here we replaced Eq. (18) in the Lagrange planetary equations[15] and numerically integrated the set of nonlinear differential equations using the software Maple to analyze the orbital behavior of the planet for some particular cases. To investigate the effects of R_2 and R_3 we use the following initial conditions. The star has mass $1M_{\odot}$, the planet has mass $1M_J$ and the outer brown dwarf has mass $40M_J$. The inner orbit has $a_1 = 6$ AU, $e_1 = 0.001$ and the initial value for the relative inclination is $i = 65°$. These initial conditions were obtained from.[2]

Figure 1 shows a comparison between the disturbing potential models taking into account only the R_2 term. The red curve represents the potential when the elimination of the nodes is used in the Hamiltonian.[4] The blue curve show the results when the nodes have not been eliminated. For the chosen values of the longitude of the ascending nodes h_1 and h_2, the inclination oscillates in a prograde orbit. We verified that, for this dynamical system, the inclination inversion phenomenon does not occur considering only the R_2 term. When the development is done without the elimination of nodes, the system thus defined is more accurate because it avoids some simplifications. Note that, taking into account the potential given by Reference [4] the inclination remains in a prograde orbit with constant amplitude (Figure 1 red line) and, when considering the potential given by Eq. (18), the inclination also remains in a prograde orbit but with variable amplitude (Figure 1 blue line).

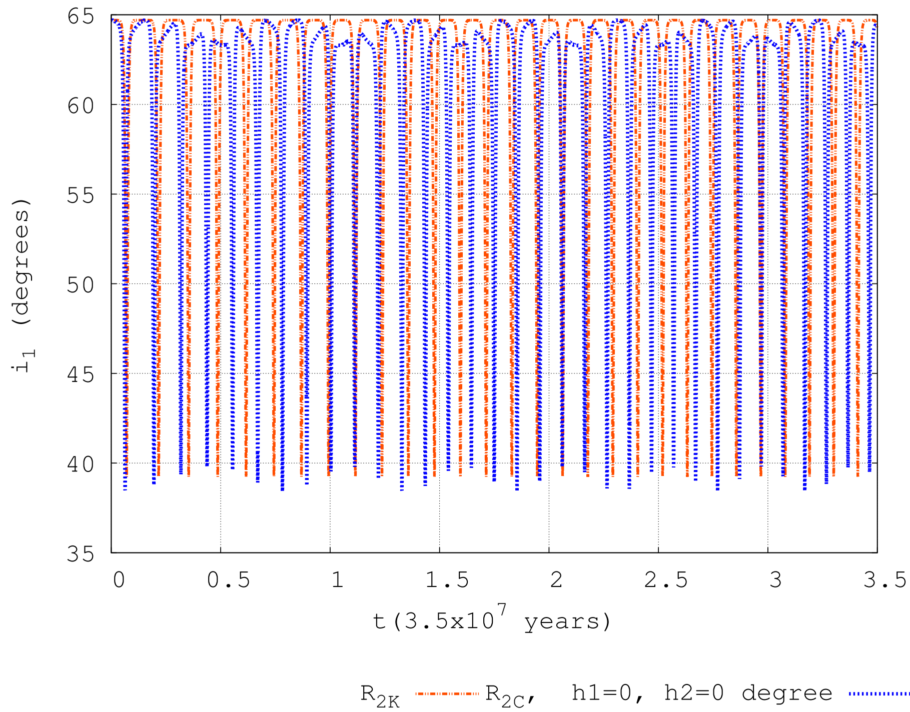

Figure 1. Temporal evolution of the inclination. Initial conditions: $a_1 = 6$ **AU,** $a_2 = 100$ **AU,** $e_1 = 0.001$**,** $e_2 = 0.6$**,** $i_1 = 64.7°$**,** $i_2 = 0.3°$**,** $g_1 = 45°$**,** $g_2 = 0°$**.**

Where R_2 term (red line) is the long-period disturbing potential (quadrupole) given by Kozai.[4] Figures 2(b) and 2(a) show the effect of the eccentricity with respect to the inclination. Figure 2(b) was plotted using the potential given by Eq. (13) and Figure 2(a) was plotted using the long-period disturbing potential (quadrupole) given by Kozai.[4] Note that the figures have the same characteristics, but with a slight difference. Figure 2(b) shows a thicker line than Figure 2(a). This fact happens because the amplitude of the inclination that is varying in Figure 1 (blue line) is different from the red line (Figure 1), that is constant. This variation of the inclination amplitude is due to the nodes.

Figure 3 shows the behavior of the inclination with respect to time. Note that the curves represented in Figure 3 shows different behaviors due to the initial conditions considered for the nodes. The phenomenon needs more time (1.5×10^7) to make the first inversion ($h_1 = 0°$ and $h_2 = 90°$), when other initial conditions are considered for the nodes. On the other hand, when it is taken into account the initial conditions $h_1 = 43°$ and $h_2 = 0°$ the first inversion occurs at $\sim 6 \times 10^6$.

For the numerical simulations (Figures 4 and 5), we adapted the Mercury integrator package[16] for

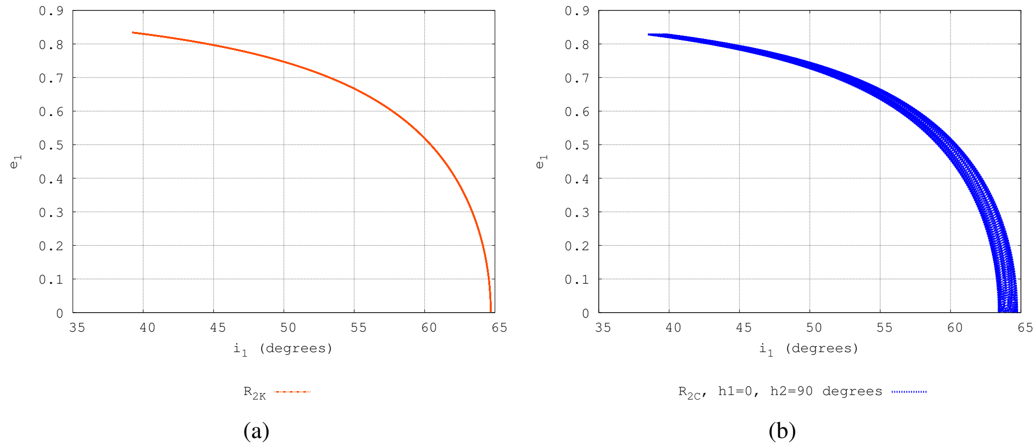

Figure 2. Temporal evolution of the inclination. Initial conditions: $a_1 = 6$ AU, $a_2 = 100$ AU, $e_1 = 0.001$, $e_2 = 0.6$, $i_1 = 64.7°$, $i_2 = 0.3°$, $g_1 = 45°$, $g_2 = 0°$. (a) R_2 given by Kozai. (b) R_{2C}.

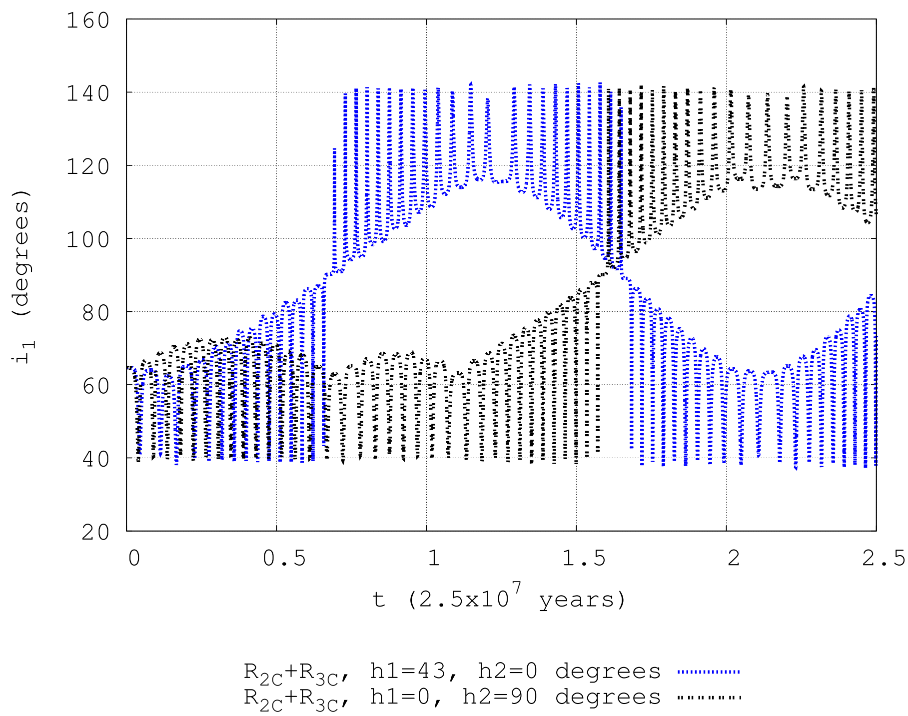

Figure 3. Temporal evolution of the inclination. Initial conditions: $a_1 = 6$ AU, $a_2 = 100$ AU, $e_1 = 0.001$, $e_2 = 0.6$, $i_1 = 64.7°$, $i_2 = 0.3°$, $g_1 = 45°$, $g_2 = 0°$.

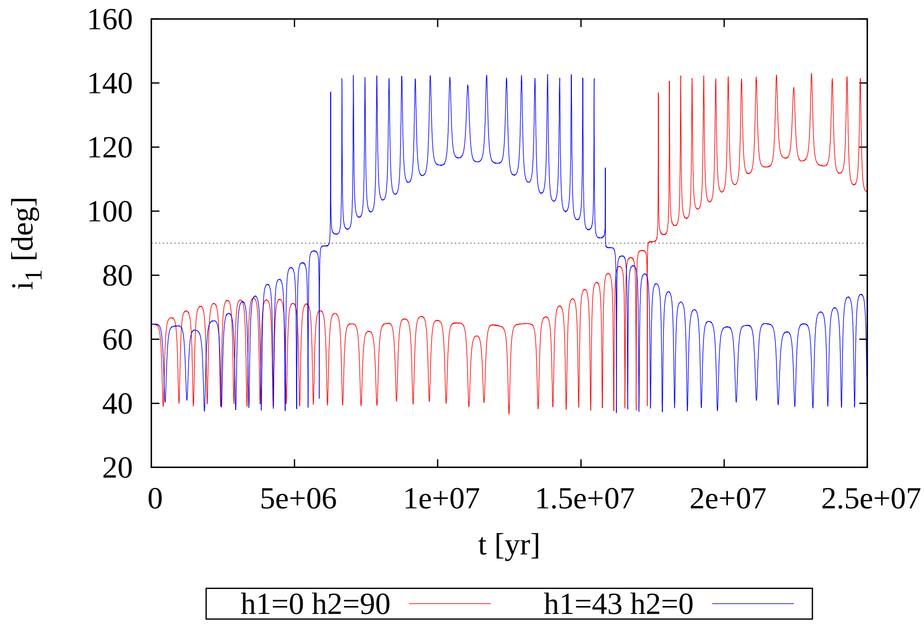

Figure 4. **Initial conditions:** $a_1 = 6$ **AU,** $a_2 = 100$ **AU,** $e_1 = 0.001$**,** $e_2 = 0.6$**,** $i_1 = 64.7°$**,** $i_2 = 0.3°$**,** $g_1 = 45°$**,** $g_2 = 0°$**. (a)** $i_1 \times t$ **numerical simulation of three-body problem. (b)** $1 - e_1 \times t$ **numerical simulation of three-body problem.**

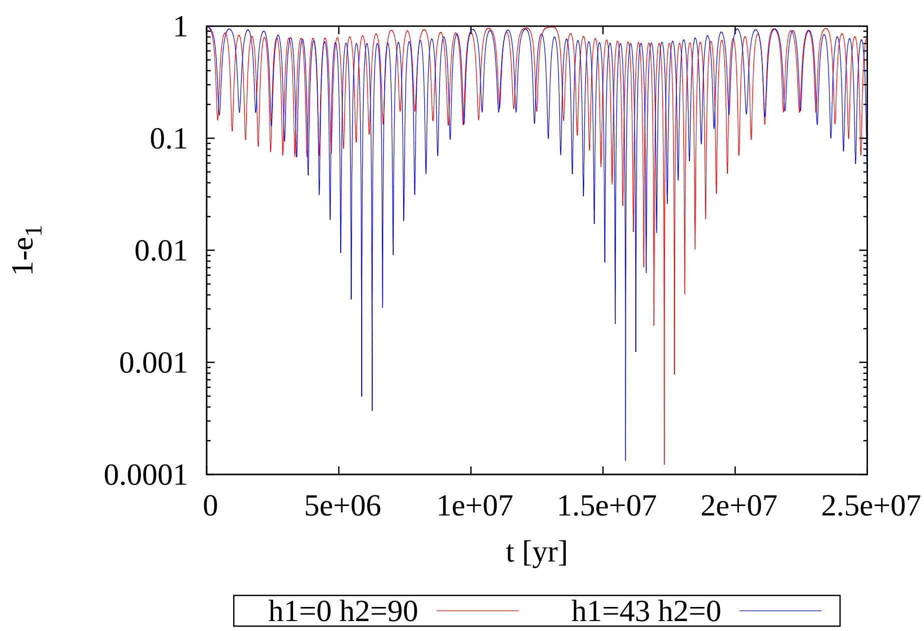

Figure 5. **Initial conditions:** $a_1 = 6$ **AU,** $a_2 = 100$ **AU,** $e_1 = 0.001$**,** $e_2 = 0.6$**,** $i_1 = 64.7°$**,** $i_2 = 0.3°$**,** $g_1 = 45°$**,** $g_2 = 0°$**. (a)** $i_1 \times t$ **numerical simulation of three-body problem. (b)** $1 - e_1 \times t$ **numerical simulation of three-body problem.**

the stellar system. We used the built-Stoer Burlish package and all the objects were considered massive bodies. Figures 4 and 5 were generated from the direct numerical integration of the three-body problem. Figure 4 shows the behavior of the inclination of the planet (m_1) considering the initial conditions of Figure 3. Note that the result from the direct numerical integration is in agreement with Figure 3. The longitude of the ascending node of the planet m_1 and the disturbing star (m_2) should be taken into account because they interfere directly in the phenomenon. The potential given by R_{2C} has three degrees of freedom, and the potential R_{3C} has four degrees of freedom. Figure 5 shows that the eccentricity can reach very high values and their inclination can become higher than 90 degrees.

In Reference[3] the elimination of nodes is used after deriving the equations of motion to simplify the equations and thus to analyze a system with one degree of freedom for the potential R_{2N} and two degrees of freedom for the potential R_{3N}. This simplification misses information with respect to the inclination inversion time and the time of the permanence of the planet in prograde or retrograde orbits. Here we use Eq. (3), given by Reference ,[11] to analyze the influence of the longitude of the ascending node in the dynamical behavior of the planet m_1. We show that the orbits are extensively modified for different values of the nodes. The nodes must be present in the equations of motion due to the characteristics that they represent for the dynamics.

CONCLUSION

We investigate the secular dynamics of a planet that moves around a central star disturbed by a brown dwarf. We developed the disturbing potential in closed form up to the third-order in a small parameter. We analyzed the effects caused by the potential of the second and third order, where the dynamics is strongly modified when compared with the Kozai-Lidov classic problem, where only the potential up to the second-order is considered. We have compared the secular evolution of the systems with and without the third order part of the disturbing potential. The R_{2C} (quadrupole) and R_{3C} (octupole) terms of the disturbing potential are developed without using the elimination of nodes.

Considering the R_2 (quadrupole) and R_3 (octupole) terms, as a function of the variables g_1, g_2, h_1 and h_2, we found that the inclination of the planet migrates from prograde to retrograde orbits. We found that the longitude of the ascending node and the argument of periastron of the inner outer orbit makes a significant change in the time of the first flip between prograde and retrograde orientation of the orbits, and also in the subsequent ones. Consequently, it also affects the period of time that the orbit remains in a given orientation. For the numerical simulations we adapted the Mercury integrator package for the stellar system. The results show that the analytical model is in agreement with the numerical model.

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