

# Mapping Swing-By Trajectories in the Triple Asteroid 2001SN<sub>263</sub>

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One of the most interesting asteroids to be visited is the so called 2001SN<sub>263</sub>. It is a triple system and its components have diameters around 2.8 km, 1.2 km and 0.5 km. A mission to this asteroid is under study by several institutions in Brazil nowadays. There are many aspects to be considered in a mission of this type. Since the idea is to study the three bodies that belong to this system, close approaches will occur between the spacecraft and the two smaller bodies during the observation of those bodies, what makes the study of the effects of those close approaches a very important topic, because those modifications may generate situations where the spacecraft escapes from the system. On the other side, those modifications of the orbit can be used to help the required maneuvers that need to be made in the spacecraft in order to make a more complete observation of the system. This type of study is the main goal of the present paper. After that, a second study is made, considering the use of tethers in order to combine the collection of samples from the components of the asteroid with a change in the energy of the spacecraft due to the "Tethered Assisted Swing-By Maneuver". In this type of maneuver the rotation of the spacecraft generated by the tether can give variations of energy around four orders of magnitude larger than the ones that comes from the close approach. After studying this maneuver for the 2001SN<sub>263</sub> system a short exploration of the Jupiter-Adrastea system is made, since it offers very high energy variations, enough to capture a spacecraft coming from the Earth, in particular a nano-satellite, around Jupiter.

## Nomenclature

$C$	=	angular momentum
$E$	=	energy
$l$	=	length of the tether
$r_1$	=	distance from the spacecraft to the first primary
$r_2$	=	distance from the spacecraft to the second primary
$R_p$	=	periapsis distance
$x,y,z$	=	coordinates of the spacecraft
$V_{inf}$	=	velocity of approach of the spacecraft
$V_p$	=	velocity of the spacecraft at the periapsis
$V_2$	=	velocity of smaller body of the close approach
$\delta$	=	rotation of the spacecraft
$\mu$	=	mass parameter
$\Psi$	=	angle of approach of the spacecraft
$\omega$	=	complex variable for the state of the spacecraft
$\Omega$	=	potential

## I. Introduction

The so called "asteroid belt" is a region of the interplanetary space between the orbits of Jupiter and Mars where there is a large number of celestial bodies that are much smaller than the planets and the moons of the Solar System. Those bodies are small, but they are very important, since they are expected to keep information about the original material that formed the Solar System. In this way, missions to those bodies are very interesting for

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scientific studies. To perform this task, a spacecraft has to be sent to one of those systems to make closer observations. The present paper assumes that the spacecraft is orbiting the main body when it makes close approaches with one of the two smaller bodies, in order to observe it. Those close approaches necessarily change the orbit of the spacecraft, and it is important to map those changes, to use them as propulsion for desired maneuvers as well as to avoid situations that are against the goals of the mission, in particular trajectories that end in escapes of the spacecraft from the system. Both smaller bodies will be considered in this study. The close approaches are simulated for a large range of initial conditions and then the orbits are classified according to the effects caused in the trajectories of the spacecraft, like increasing or reducing the two-body energy Spacecraft-Main body, modification in the sense of the orbit (retrograde to direct or vice-versa), capture or escape from the system, etc. The restricted planar circular three-body problem is the mathematical model used, combined with Lemaître regularization to avoid possible numerical problems near the singularities caused by the primaries. This method increases the accuracy of the calculations, that is important, since all the bodies involved are small and so the trajectories are allowed to pass close to the center of the bodies.

The variables used to identify every single trajectory are: 1)  $V_{inf}$ , the velocity of the spacecraft when approaching the smaller body, with respect to this body; 2) the angle between the periapsis of the trajectory and the line connecting the primaries, that is called the angle of approach; 3)  $R_p$ , the periapsis distance. The equations of motion are integrated in positive and negative times, until the spacecraft reaches a large distance from the smaller body such that its gravity can be neglected. At those points, energy and angular momentum are computed before and after the passage and the sixteen classes of orbits are defined based on the variations of the energy (positive or negative, before and after the close approach) and angular momentum (positive or negative, before and after the close approach). The results are shown in letter-plots that specify what happened with the orbit due to the close approach, both in terms of energy and direction of the orbit. Particular attention is given to orbits that result in escape from the system, because this type of trajectory needs to be avoided to keep the spacecraft observing the system. Another important family of orbits are the ones that can be used to transfer the spacecraft from an orbit that passes close to one of the smaller body to an orbit that passes close to the other smaller body, because such passages can be used to observe one body and to give energy to transfer the spacecraft to an orbit that can observe the other body.

This type of trajectory has many applications in the literature and the most common goal is to save fuel in space missions. More details can be found in Broucke<sup>1</sup>, that makes a general overview of this technique. Real missions can be found in Carvell<sup>2</sup>, which considered a three-dimensional maneuver around Jupiter to send the spacecraft to an orbital plane perpendicular to the original one; Casalino et al.<sup>3</sup>, which used low thrust together with close approaches; D'Amario et al.<sup>4</sup>, which considered the Galileo mission. A particular important mission was the grand tour of the Solar System made by the Voyager spacecraft, that is well described in Minovich<sup>5</sup> and Kohlhase and Penzo<sup>6</sup>. Some variants of this problem is also available in the literature, like the inclusion of an impulse during the passage<sup>7</sup>; elliptic motion for the primaries<sup>8</sup>; the motion of a cloud of particles<sup>9,10</sup> or the search for orbits to the Sun passing by the inner planets of the Solar system<sup>11</sup>. Graphical methods were also developed in Strange and Longuski<sup>12</sup> and MacConaghy et al.<sup>13</sup>.

## II. The Aster Mission

The asteroids that exist in the "asteroid belt" are a potential danger for the Earth, due to the always existing risk of collision<sup>14</sup>. From the scientific point of view, most of these asteroids have information related to the original disk of matter that formed the Solar System, which means that a detailed study of those bodies is very important. One of the best candidates to be visited by a spacecraft is the asteroid 2001SN<sub>263</sub>, which is a triple system, first observed in February 2008 from Arecibo, Puerto Rico. The three bodies have radius around 1.30 km, 0.39 km and 0.29 km. In a reference system centered in the most massive body, the second component is in an elliptical orbit with semi-major axis 16.63 km and eccentricity 0.015; and the third component is also in an elliptical orbit with semi-major axis 3.80 km and eccentricity 0.016<sup>15</sup>. The verification of the composition of those bodies, in particular the search for organic components, is one of the most important goals of this mission. It is also of interest the orbit determination of the two satellite bodies and their shapes, since they are expected not to be spherical. Considering those facts, the ASTER project was conceived<sup>16</sup>, which consists in sending a spacecraft to this asteroid. A detailed schedule for the mission is still not available, but the visit is expected to last near one year.

The main goal of the present paper, as explained above, is to study this dynamical system from the point of view of mapping the effects of the close approaches between the spacecraft and the minor bodies that orbits the largest one. Those mappings are useful to find trajectories that can be used by the spacecraft for maneuvers when it is

inside the system and also those trajectories that need to be avoided. After the study of those natural trajectories, some considerations are made regarding the use of tethers in this mission. The idea is to simulate situations where a tether is sent from the spacecraft to the minor body and then it is linked to this body. This tether has an equipment that can be used to get samples from the surface of the body, which can be recovered later to the spacecraft, when the tether is released from the surface of the body. Besides collecting samples, this tether also modify the trajectory of the spacecraft, due to the rotation of the spacecraft around the visited body during the time that they are linked to each other. This is the so called "Tethered Assisted Swing-By Maneuver" and it is described in several papers in the literature, like in the ones by Penzo and Mayer<sup>17</sup>; Lanoix<sup>18</sup>; Puig-Suari, Longuski and Tragesser<sup>19</sup>; Thomson and Stern<sup>20</sup>; Lanoix and Misra<sup>21</sup>. This technique is particularly interesting if the spacecraft is small and light, like the popular nano-satellites, that now are under study to travel to the interplanetary space. In this situation the spacecraft is light and so it does not demand very resistant cables for the maneuvers and they are also not allowed to carry too much fuel onboard, due to the weight restrictions. Those facts makes this type of maneuver very interesting for those spacecrafts. The present paper makes simplifications in this problem in order to obtain estimates of the possible energy variations that can be obtained from a maneuver of this type. It is shown that the variations in energy are much larger than the ones obtained by pure gravity assisted maneuver, around 20000 times larger. The absolute values of those energy variations are still small to be important in terms of helping to capture the spacecraft coming from the Earth, but the values are large enough to help to maneuver the spacecraft from one orbit to another. So, the idea is to combine the collection of the samples by the tether with orbital maneuvers, what makes this topic interesting for further studies.

Looking at the literature searching for papers related to the dynamics of asteroids, that is also a topic related to the present research, there are several important researches. Scheeres<sup>22</sup> analyzed the motions around the asteroid 433 Eros; Scheeres, Ostro, Hudson, Werner<sup>23</sup> considered orbits near the asteroid 4769 Castalia; Scheeres<sup>24</sup> made a detailed study of the evolution of rotating asteroids subject to YORP effect.

### III. Definition of the Problem

The dynamical system has four bodies, assumed to be points of mass: the three members of the triple asteroid and the spacecraft, which has a negligible mass. The two smaller bodies are orbiting the largest one, assumed to be in circular non-coplanar orbits. The assumption of circular motion is made based in the fact that the eccentricities of both bodies are small and have negligible effects in the problem under study here. Another simplification is the assumption that the mass of one of the smaller body does not affect the motion of the spacecraft when it is moving near the other small body. It means that the system composed by the main body, the closest smaller body and the spacecraft is considered a circular restricted three-body problem.

Then, the objective of the present paper is the verification of the two-body energy and the angular momentum of the spacecraft with respect to the larger body of the triple system before and after the close approach by one of the smaller bodies. The modifications of the orbits are shown, and particular attention is given to find orbits resulting in escapes and captures. Those trajectories are shown in detail, as a function of the mass of the body, the periastron distance, the velocity and the angle of approach of the spacecraft.

For the maneuvers assisted by the tether, it is assumed that the spacecraft approaches the minor body when it sends a tether that connect the spacecraft with the celestial body. In this situation the spacecraft turns around the body until the moment that the tether is released. The tether is assumed to keep its length fixed during the maneuver and no limits were imposed to the mass of the spacecraft or the velocity of approach. In other words, it is assumed that the tether is ideal and supports the tension that it is submitted without any distortion or breaking. Those assumptions are not far from reality if a light nano-satellite is considered as the spacecraft performing the maneuver.

### IV. Mathematical Model and Algorithm

The equations of motion of the spacecraft are given by the planar circular restricted three-body problem. They are, using the canonical system of units<sup>25</sup>, given by:

$$\ddot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} = \frac{\partial \Omega}{\partial x} \quad (1)$$

$$\ddot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} = \frac{\partial \Omega}{\partial y} \quad (2)$$

$$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \quad (3)$$

where  $x, y$  are the position of the spacecraft,  $\mu$  is the mass parameter of the system (the ratio between the mass of the smaller primary and the total mass of the system),  $r_1$  is the distance between the spacecraft and the larger primary and  $r_2$  is the distance between the spacecraft and the smaller primary. This canonical system of units use the distance between the two primaries as unit of distance, the total mass of the system as unit of mass and the unit of time is defined such that the period of the motion of the two primaries is  $2\pi$ . To obtain the energy of the spacecraft it is possible to use:

$$E = \frac{(x + \dot{y})^2 + (\dot{x} - y)^2}{2} - \frac{1-\mu}{r_1} + \frac{\mu}{r_2}, \quad (4)$$

This equation can be used to see if the orbit of the spacecraft is open or closed by looking at the sign of the energy (if positive the orbit is open and, if negative, the orbit is closed). To obtain the angular momentum, that is necessary to know the sense of the orbit, it is possible to use:

$$C = x^2 + y^2 + x\dot{y} - y\dot{x} \quad (5)$$

To increase the accuracy of the numerical integrations, in particular when  $r_1$  or  $r_2$  are small, that is the case due to the small values of the radius of the bodies involved, the Lemaitre regularization is used. Details of this transformation can be found in Szebehely<sup>25</sup>. The final equations are given by:

$$\omega'' + 2i|f'(\omega)|^2 \omega' = \text{grad}_{\omega} \Omega^* \quad (6)$$

with  $\omega = \omega_1 + i\omega_2$  is the new variable for position,  $\omega'$  and  $\omega''$  are the first and second derivatives of  $\omega$  with respect

to the regularized time  $\tau$ ;  $\text{grad}_{\omega} \Omega^*$  represents  $\frac{\partial \Omega^*}{\partial \omega_1} + i \frac{\partial \Omega^*}{\partial \omega_2}$ ,  $\Omega^*$  is the transformed pseudo-potential given by

$\Omega^* = \left( \Omega - \frac{C}{2} \right) |f'(\omega)|^2$ , where  $C = \mu(1-\mu) - 2J$ , and  $f'(\omega)$  denotes  $\frac{\partial f}{\partial \omega}$ . The algorithm used has the following

steps:

- i) Values are attributed to the variables  $R_p, V_{\text{inf}}, \psi$ ;
- ii) The initial coordinates are obtained from  $X_i = R_p \cos(\psi) + (1-\mu)$ ,  $Y_i = R_p \sin(\psi)$ ,  $V_{Xi} = -V_p \sin(\psi)$ ,  $V_{Yi} = +V_p \cos(\psi)$ , where  $V_p$  is the velocity of the spacecraft at the periapsis, which can be obtained from  $V_{\text{inf}}$  with the use of the conservation of energy of the two-body problem spacecraft-smaller body;
- iii) The equations of motion are integrated forward in time until the spacecraft is far from the minor body. At this point the energy ( $E_+$ ) and angular momentum ( $C_+$ ) after the close approach are calculated by Eqs. (4) and (5);
- iv) The equations of motion are then integrated backward in time, again starting at the initial conditions shown above, and the energy ( $E_-$ ) and angular momentum ( $C_-$ ) before the close approach are measured, when the spacecraft is again far from the minor body, again by using Eqs. (4) and (5);
- v) These information allow us to verify when a capture ( $E_- > 0$  and  $E_+ < 0$ ) or an escape ( $E_- < 0$  and  $E_+ > 0$ ) occurs, as well as other facts like the modification of the sense of the orbit (change in the sign of the angular momentum).

## V. Results

The results consist of graphs showing the effects of the passage in the trajectories, in particular the regions of captures and escapes, that are marked in dark gray in the figures. Table 1 shows the definition of the classes of orbits according to the modification obtained in the orbit of the spacecraft, in terms of being open/closed or direct/retrograde. The figures have the velocity of approach, in m/s, in the vertical axis and the angle of approach  $\psi$ , in degrees, in the horizontal axis. The range for the approach angle is 180-360 degrees. One plot is made for each value of the periaapsis distance during the approach. There is symmetry in the plots and so only values of  $\psi$  from 180 to 360 degrees are showed. This is the range of gains in energy, so the trajectories that change the sign of the energy are all escape trajectories. The capture trajectories are in the mirror image of this region, that is not shown here, for values of  $\psi$  from 0 to 180 degrees. This symmetry exists because an orbit with  $\psi = \theta$  differs from an orbit with  $\psi = \theta + 180^\circ$  only by a time reversal. So, there is an equivalence between these two intervals. This equivalence implies in the following substitutions when going from one interval to another:  $I \leftrightarrow C$ ,  $J \leftrightarrow G$ ,  $L \leftrightarrow O$ ,  $B \leftrightarrow E$ ,  $N \leftrightarrow H$ ,  $M \leftrightarrow D$ . The orbits A, F, K and P are not modified. For the periaapsis distance the values 1.1, 5.0 and 10.0 radius of the smaller body were considered. Figure 2 shows the results. The numerical values used in this research are:

1. Distance from the smaller asteroid to the points where we consider that the spacecraft is free from its gravity field: 0.5 canonical units;
2. Distance for the spacecraft to be considered far from the system of asteroids: 2.0 canonical units;
3. The data of the system is shown in Table 2.

**Table 1** - Rules for the assignment of letters to orbits.

Before:	After:	Direct Ellipse	Retrograde Ellipse	Direct Hyperbola	Retrograde Hyperbola
Direct Ellipse		A	E	I	M
Retrograde Ellipse		B	F	J	N
Direct Hyperbola		C	G	K	O
Retrograde Hyperbola		D	H	L	P

Table 2: Physical data for the three bodies of the asteroid.

Body	Semi-major axis (km)	Eccentricity	Inclination (deg)	Mass (kg)	Radius (km)
Alpha	297,704,000 (around the Sun)	0.48 (around the Sun)	0.0 (around the Sun)	$917.47 \times 10^{10}$	1.30
Beta	16.63 (around Alpha)	0.015 (around Alpha)	0.0 (around Alpha)	$24.04 \times 10^{10}$	0.39
Gamma	3.80 (around Alpha)	0.016 (around Alpha)	14.0 (around Alpha)	$9.77 \times 10^{10}$	0.29

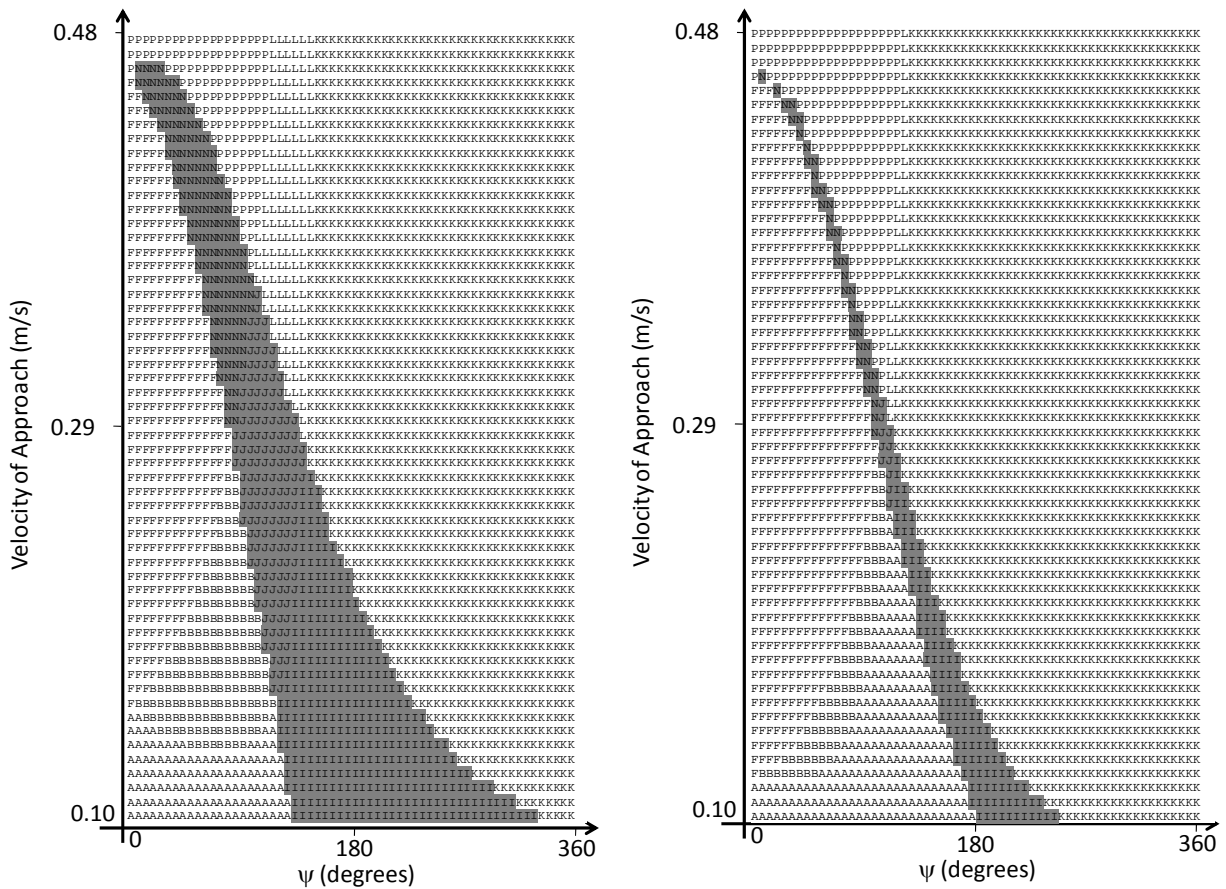


Figure 2a - Results when considering the body Beta for the close approach with for  $R_p = 1.1$  (left) and  $5.0$  (right) radius of Beta. Dark gray indicates escape trajectories.

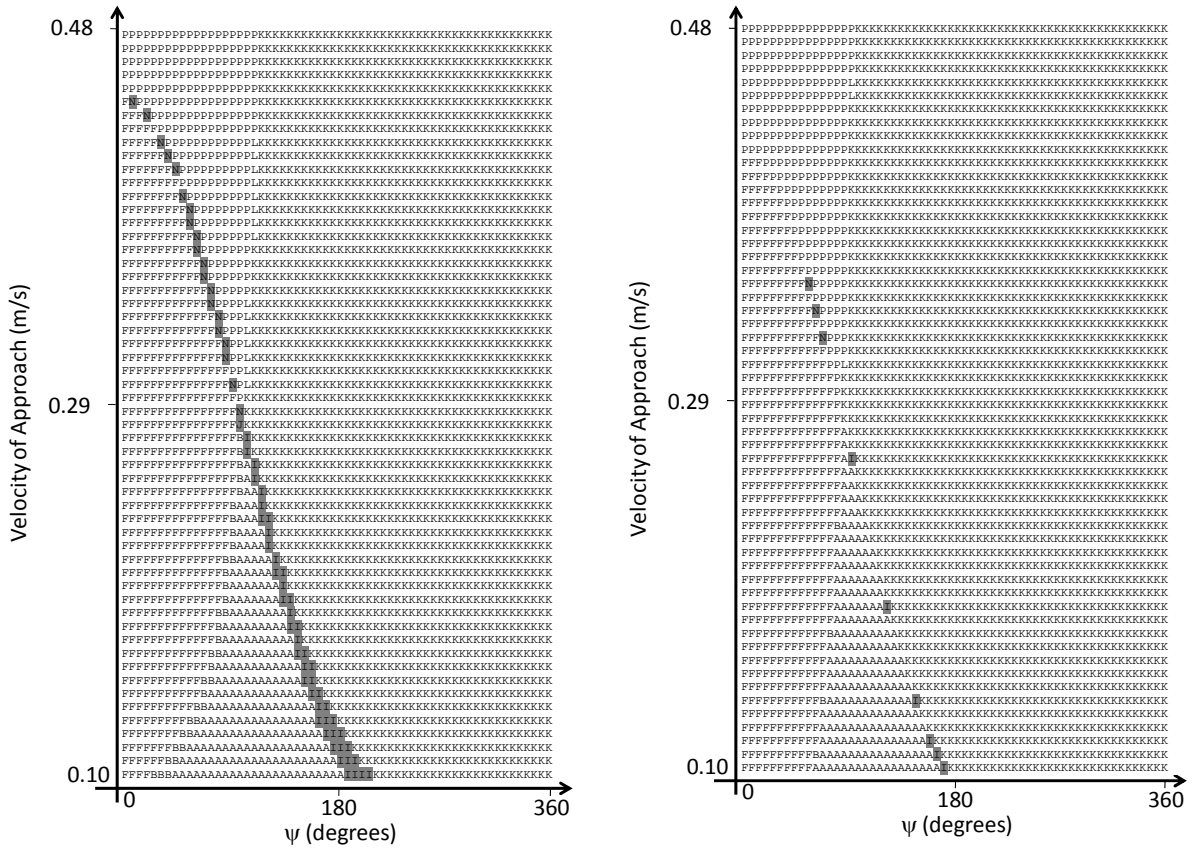


Figure 2b - Results when considering the body Beta for the close approach with for  $R_p = 10$  (left) and 20 (right) radius of Beta. Dark gray indicates escape trajectories.

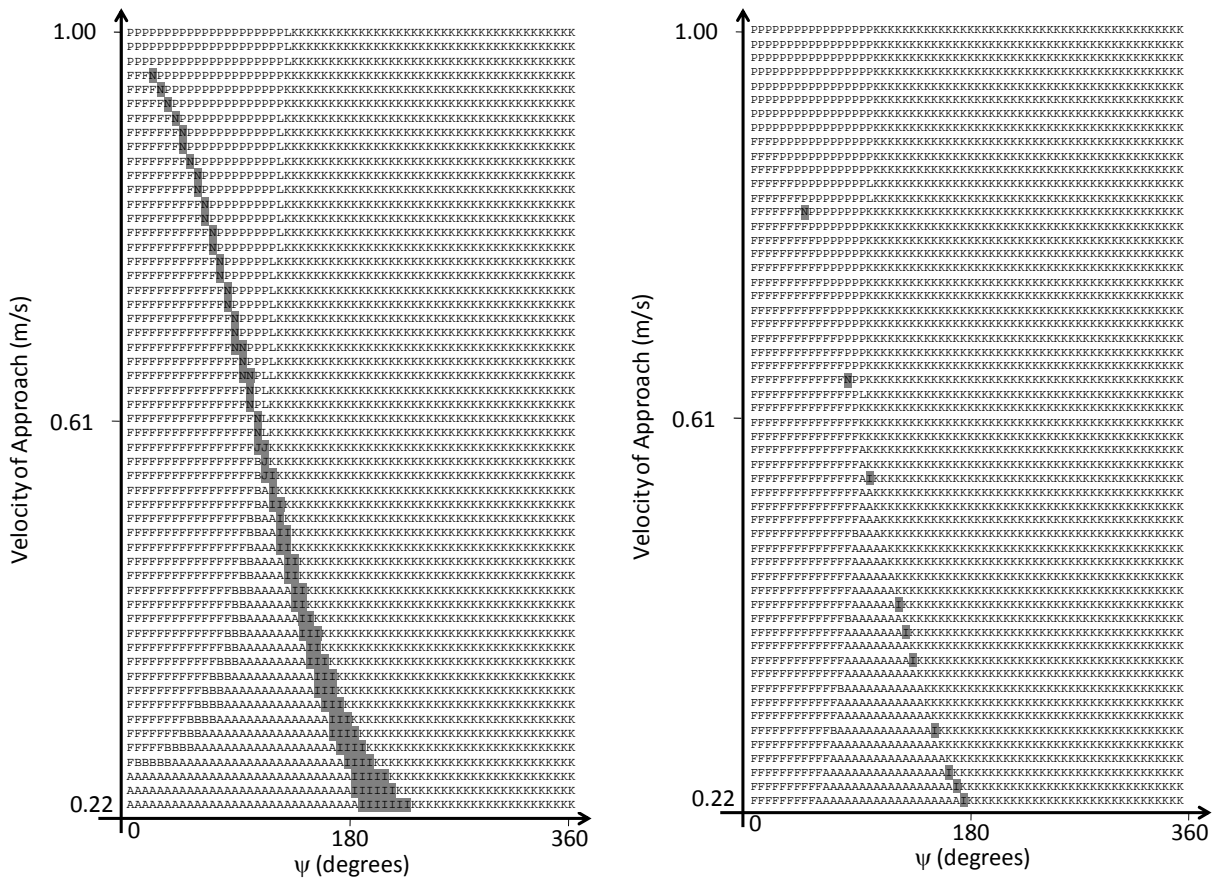
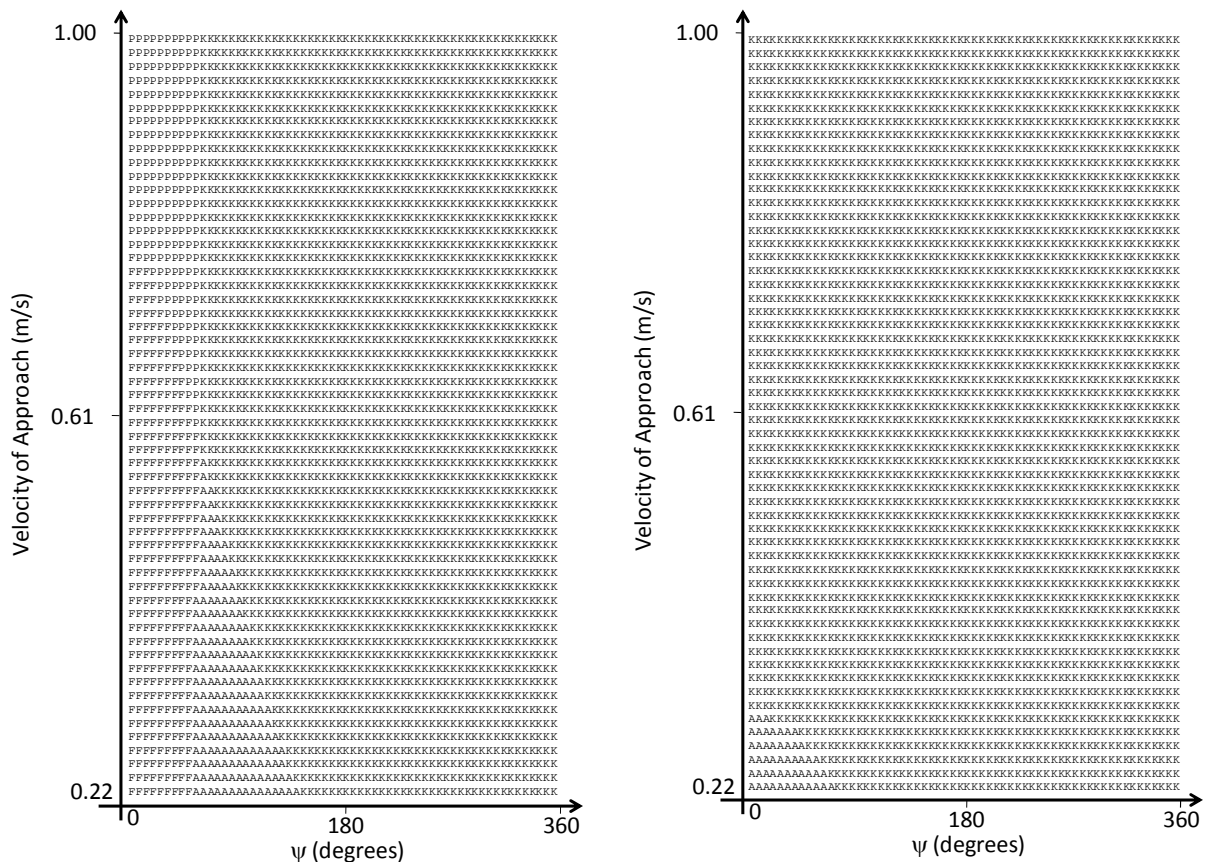


Figure 2c - Results when considering the body Gamma for the close approach with for  $R_p = 1.1$  (left) and  $5.0$  (right) radius of Gamma. Dark gray indicates escape trajectories.





**Figure 2d - Results when considering the body Gamma for the close approach with for  $R_p = 10$  (left) and  $20$  (right) radius of Gamma.**

The results have some interesting aspects. First of all it is clear that the two bodies generate very different results, since their masses are substantially different. The body Beta gives larger effects for the natural close approach maneuvers, since it has more mass. The letter-plots are dominated by the letter K, concentrated in the top-right parts of the plots. This letter represents a direct hyperbola before and after the passage. This domination is expected, since this is a situation with small modification of the trajectory, which is compatible with the small masses of the bodies involved. At the top-left part of the plots there are regions of letter P, that means a retrograde hyperbola before and after the passage. Another family of open orbits all the time is the family L, which is composed by hyperbolic orbits that are retrograde before the passage and direct after that. Closed orbits all the time also exist, and they are mainly concentrated in the letters F (retrograde ellipses before and after the passage) and the more interesting cases of letters B, where the elliptical orbits remain closed, but change the sense of the motion from retrograde to direct. The orbits represented by letter A are composed by elliptical orbits that also remains closed, but changes the sense of the motion from direct to retrograde. Those orbits can be used by a maneuver that uses fuel to reduce the velocity of approach to put the spacecraft in the correct configuration to change the sense of the orbit and then another maneuver may accelerate it again. The net result is a saving in the fuel for the whole maneuver. The more interesting orbits are the ones that ends in escape, in the region shown in the paper, which is equivalent to captures in the symmetric region not shown here. They occur, in the results shown here, for the letters I (a direct elliptical orbit that becomes a direct hyperbolic orbit), J (a retrograde elliptical orbit that becomes a direct hyperbolic orbit), N (a retrograde elliptical orbit that becomes a retrograde hyperbolic orbit). They follow the expected results, being more numerous for the body Beta than for Gamma and

with increasing occurrence when the spacecraft gets closer to the celestial body. In is clear that, for Beta, after a periapsis distance of 5.0 radius of Beta, those types of orbits occurs only for very specific initial conditions and, after a periapsis distance of 10.0 radius of Beta, there is almost no more escapes. Regarding the body Gamma, those limits are even smaller, with almost no escapes for a periapsis distance of 5.0 radius of Gamma or larger.

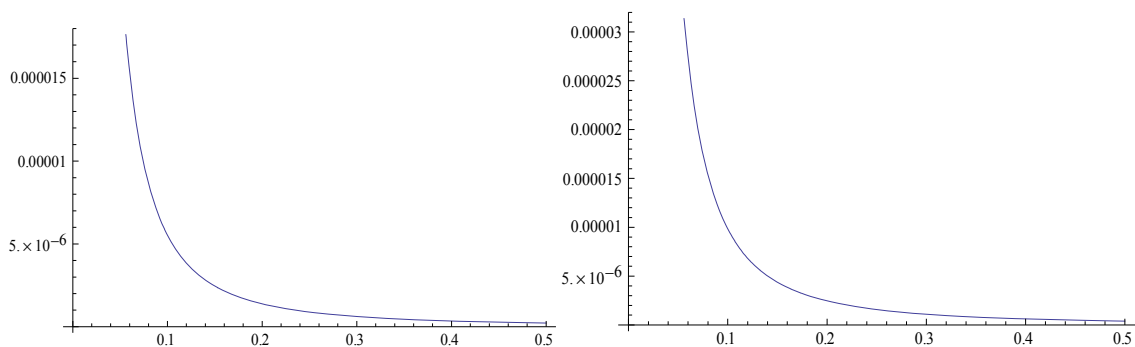
In both cases, the regions of escapes go from around  $\psi = 180^\circ$  to  $300^\circ$ , depending on the periapsis distance and the body considered. The ranges in terms of velocity of approach goes from around 0.20 to 0.50 m/s for Beta and to 0.90 m/s for Gamma. When using higher values for the periapsis distance there is a decrease in the regions representing the escape trajectories. It happens because the effects of the close approach are getting smaller, so the escape regions tend to disappear. After the value of periapsis distance of 5.0 radius of the smaller body, there are few points of escape trajectories. The general conclusion is that the smaller bodies of the triple asteroid are not efficient in helping to capture the spacecraft when it is coming from the Earth, but they are large enough to allow maneuvers around the system and also to be of some concern when verifying if the trajectories followed by the spacecraft will not end up in escaping the system.

## VI. Tethered Gravity Assisted Maneuver

Next, the situation where a tether is used in the mission is briefly studied. The goal is to verify how much energy can be obtained from this type of maneuver, in order to use it to make orbital maneuvers for the spacecraft. The main reason to get more energy variation from the tethered assisted maneuver is the small rotation angle generated

by a close approach with a celestial body with small mass. This angle can be obtained<sup>1</sup> from  $sen(\delta) = \left(1 + \frac{r_p V_\infty^2}{\mu_2}\right)^{-1}$ .

Figure 3 shows half of this rotation angle (deg) as a function of the velocity of approach (m/s) for both bodies Beta and Gamma. It is clear the small values of the variation obtained.



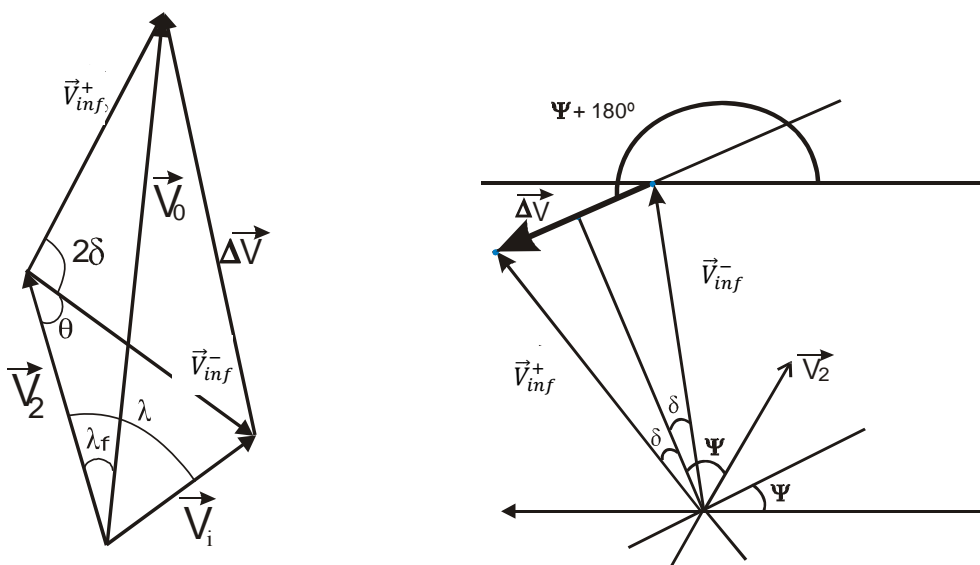
**Figure 3 - Half of the rotation angle due to the close approach (in degrees) as a function of the velocity of approach (in m/s).**

For a first study, it is possible to use a simplified model, in order to obtain estimates of the energy gains that can be obtained. We assume that the overall mission can be divided into three stages. In the first part the spacecraft is moving around the Sun, coming from the Earth, when it approaches the asteroid system. At this moment the spacecraft has a velocity  $\vec{V}_i$  with respect to the central body of the asteroid system. The velocity of the smaller body with respect to this same central body is  $\vec{V}_2$ . So, it is possible to find the velocity  $\vec{V}_{inf}$  of the spacecraft with respect to the smaller body when the spacecraft is approaching the system by using the equation  $\vec{V}_{inf} = \vec{V}_i - \vec{V}_2$ . At this point the spacecraft is attached to the smaller asteroid and the rotation starts. The effects of the mass of the smaller body are neglected, so the rotation has a constant angular velocity and a fixed radius, equals to the length of the tether. The angle of rotation ( $2\delta$ ) is given by:

$$2\delta = \frac{\Delta t * V_{inf}}{l} \quad (7)$$

where  $2\delta$  is the rotation angle,  $V_{inf}$  is the velocity of approach,  $\Delta t$  is the duration of the rotation, which corresponds to the time that the spacecraft remains attached to the smaller body, and  $l$  is the length of the tether. After that the spacecraft is released and the velocity with respect to the smaller body  $\vec{V}_{inf}^+$  after the rotation can be obtained. Using this velocity we can calculate the inertial velocity of the spacecraft when leaving the smaller body. It is given by  $\vec{V}_o = \vec{V}_{inf}^+ + \vec{V}_2$ . Figure 4 shows the diagram of the velocities. From that figure, and considering basic trigonometry, it is clear that:

$$\Delta V = |\Delta \vec{V}| = 2|\vec{V}_{inf}| \sin(\delta) = 2V_{inf} \sin\left(\frac{\Delta t * V_{inf}}{2l}\right) \quad (8)$$

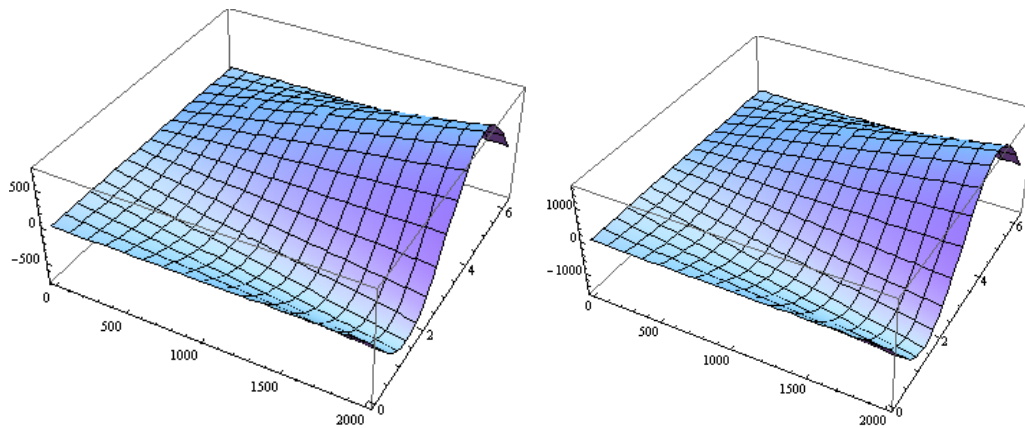


**Figure 4 - Sum of velocity vectors involved in a tethered gravity assisted maneuver.**

The variation of energy can be obtained in a similar way of the variation of the pure gravity assisted maneuver<sup>1</sup>. The final result is:

$$\Delta E = -2V_2V_{inf} \text{sen}\left(\frac{\Delta t * V_{inf}}{2l}\right) \text{sen}(\psi) \quad (9)$$

where  $\psi$  is now the angle between the line connecting the primaries and the middle point of the curvature of the spacecraft around the smaller body. The duration of the rotation can be used as a control of the variation in energy for the maneuver, which means that the dependency of the rotation angle with  $V_{inf}$  can be eliminated by an adequate choice of  $\Delta t$  as a function of the length of the tether. In this way, the angle of rotation can be seen as a free parameter. In that sense, two plots are made to show the maximum variation of energy scenario (when  $2\delta = \pi$  and the variation of energy is maximized with respect to this variable). Figure 5 shows the energy variation ( $m^2/s^2$ ) in the vertical axis as a function of the velocity of approach (m/s) in the left axis and angle of approach (rad) in the right axis, for this value of the rotation angle. For different values it is necessary to multiply the energy gains by the sin of half of the rotation angle. The results show that Gamma offers more energy gains, since its velocity around the main body is larger than the same velocity of Beta. It is a direct consequence of Eq. 9, that shows the proportionality of energy gains end  $V_2$ .



**Figure 5 - Energy variation ( $\text{m}^2/\text{s}^2$ ), in the vertical axis, as a function of the velocity of approach (m/s), in the left axis, and angle of approach (rad), in the right axis, for the maximum rotation angle around Beta (left) and Gamma (right).**

## VII. Studying the Jupiter-Adrastea System

An immediate step that follows the discovered that Gamma gives higher variations of energy due its higher velocity around the main body, is the search for other systems of primaries that allows very large variations of energy. A quick search in the Solar System shows that the moon of Jupiter Adrastea orbits Jupiter at a very high velocity, in the order of km/s. Then, some simulations verified the energy gains of a Tethered Gravity Assisted Maneuver around this moon in order to capture a spacecraft coming from the Earth for a mission that will orbit Jupiter. The results are shown in Fig. 6. It is visible that very high velocities of approach can allow captures, in the order of 20 to 70 km/s. It means that this maneuver can be used alone to capture a spacecraft coming from the Earth to orbit Jupiter. This is particularly interesting if spacecrafts of the sizes of nano-satellites are involved, because they are small and light, so it is less difficult to find a material to make the tether that is strong enough to perform the maneuver. The nano-satellites are also supposed to be very small, so they are not able to carry large amounts of fuel for the insertion maneuver. Other systems like this exist in the Solar System, so this topic deserves some more detailed studies. It is noted that a large angle of rotation delivers more energy variation, so the area of capture is larger when the rotation is  $180^\circ$  degrees when compared to when it is  $120^\circ$  degrees.



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