ANALYSIS OF PERFORMANCE OF CONTROL METHODS IN ATTITUDE CONTROL SYSTEM OF A RIGID-FLEXIBLE SATELLITE

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Abstract. *Nowadays, attitude control systems of satellites with rigid and flexible components are demanding more and more better performance resulting in the development of several methods control. For that reason, control designs methods presently available, including parameters and states estimation, robust and adaptive control, linear and nonlinear theory needs more investigation to know their capability and limitations. In this paper, the analysis of performance of Control Methods, Linear Quadratic Regulator and Linear Quadratic Gaussian are investigated in the performance of the Attitude Control System of a Rigid-Flexible Satellite.*

Keywords: *Attitude Control System, LQR, LQG, Performance.*

1. INTRODUCTION

Several applications in industries involve mechanical structures that during its operations are subject to flexibility. Because of this there is a need of techniques to control these structures. The control applications flexible structures include: robotic manipulators (Fenili, 2004), control of aircraft wings and control structures such as buildings and land towers (Kruck, 2002), artificial satellites (Cubillos, 2008), flexible space structures (Chae and Park, 2003) and solar sails (Cubillos, 2011). Large flexible structures were considered for space applications since the early 1960s. Modi (1999) reinforces the importance of employment in the use of flexible space structures and missions more complex, noting that without them the conquest of space would not be possible.

The study of the Attitude Control System (ACS) of space structures with flexible antennas and/or panel and robotic manipulators then becomes more complex when the dimensions of such structures increase due to the necessity to consider a large number of vibration modes in its model in order to improve the model fidelity. In the last years several aspects associated to the control of Rigid-Flexible Satellite (RFS) have been studied. In Joshi, (1989) treats in details the problem of a simple RFS, where the main tasks of the Attitude Control System (ACS) are: i) fine-pointing of some of the appendages to different targets, ii) rotating some of the appendages to track specified periodic scanning profiles, and iii) changing the orientation of some of the appendages through large angles.

The ACS must stabilize and orient the satellite during its mission, counteracting external disturbances torques and forces. Depending on the complexity of the satellite mission the ACS design methods can be based on linear or on nonlinear dynamics. The space engineer needs to use design procedure, which will guaranty that the ACS performance meets the mission requirements when the flexible space structure change (Won, 1999).

Cubillos et al. (2010) said that the success of a space mission depends on the performance of the attitude control system. In which to deal with the variations of the parameters and flexible elements requires special care. Several control techniques such as Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG), Quantitative feedback theory (QFT), H-Infinity ($H\infty$) and μ-synthesis were used in the quest to ensure the robust stability of the project. Future space missions will increasingly involve new techniques, architectures, technologies and sophistication of computers using more sophisticated control algorithms.

In this paper one applies the Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) methodologies to compare the performances in the ACS of a RFS. The mathematical model of RFS is constituted of a "rigid body" of cubic form, and two flexible panels. This model is obtained using the assumed mode method and the Lagrange equation.

2. MATHEMATICAL MODELING

The mathematical satellite model used is constituted of a "rigid body" of form cubic, two flexible panels; the "Fig. 1" shows the exposition. The center of mass of the satellite is considered with the point 0 origin of the system of coordinates (X, Y, Z), that coincides with its main axis of inertia. The elastic appendixes with the beam format are connected in the central body, being treated as a punctual mass in its free extremity. The length of the panel is represented by L, its m mass and v(x, t) its elastic displacement in relation to the axis Z. The moments of inertia of the rigid body of the satellite in relation to the mass center it is J_0. The moment of inertia of the panel in relation to its own mass center is given by J_p. The angle of rotation of the satellite around the axis Y is θ.

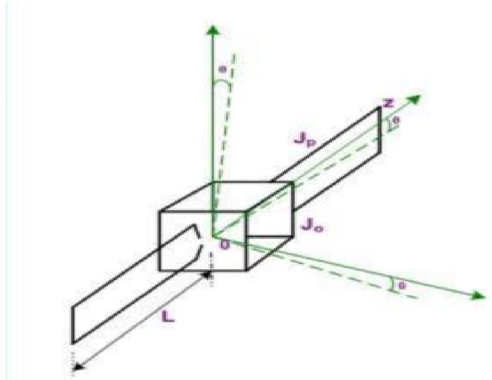

Figure 1. The satellite model

The equations of motion in which motion of rotation of the satellite and the elastic displacement of the panels are considered, the Lagrange equation (Goldstein, 2002):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}+\frac{\partial M}{\partial q_i}=Fx_i \tag{1}$$

where L is the Lagrange, M it is the dissipation energy associated to the deformation of the panel, q_i it represents each one of the generalized coordinates of the problem and F_{xi} the external forces associated to each of those coordinates. The discretization of the system is done using assumed mode method (Junkins and Kim, 1996).

$$v(x,t)=\sum_{i=1}^{n}\phi_i(x)q_i(t) \tag{2}$$

where n represents the number of manners to be adopted in the discretization $\Phi i(x)$ it represents each one of the own modes of the system. So $L=T\text{-}V$ (kinetics energy - potential energy) is equal to:

$$L=\frac{1}{2}J_0\dot{\theta}^2+\left[\rho A\int_0^L\left[\begin{array}{l}\left(\sum_{i=1}^{n}\phi_i\dot{q}_i\right)^2\\+2\left(\sum_{i=1}^{n}\phi_i\dot{q}_i\right)x\dot{\theta}\\+(x\dot{\theta})^2\\+\left(\dot{\theta}\sum_{i=1}^{n}\phi_i\dot{q}_i\right)^2\end{array}\right]\right]-\left(\sum_{i=1}^{n}\phi_i q_i\right)^2\cdot K \tag{3}$$

where K is constant elastic of the panels. After some manipulation, finally, the two equations of the motion found through the assumed mode method that represent to the dynamics of the motion of rotation of the satellite and of the elastic displacement of the panels, respectively:

$$\begin{aligned}&\ddot{\theta}J_1+\alpha_i.b.\ddot{q}_i+b.q_i^2\ddot{\theta}=\tau\\&\ddot{q}_i+\alpha_i\ddot{\theta}-\dot{\theta}^2q_i+d.\dot{q}_i+c.q_i=0\end{aligned} \tag{4}$$

Where in Eq.(4) b,c,d are constants; J_0 is the moment of inertia of the rigid body, J_p is the moment of inertia of the panels and the moment of total inertia of the system is J_1. The linear equation of motion in state space modal form, for one mode, has possesses this form:

$$\dot{Y} = AY + B\tau$$

$$\begin{vmatrix} \dot{Y}_1 \\ \dot{Y}_2 \\ \dot{Y}_3 \\ \dot{Y}_4 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{\alpha_1.a.c}{-1+\alpha_1^2.a} & -\dfrac{\alpha_1.a.d}{-1+\alpha_1^2.a} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{c}{-1+\alpha_1^2.a} & \dfrac{d}{-1+\alpha_1^2.a} \end{vmatrix} . \begin{vmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{vmatrix} + \begin{vmatrix} 0 \\ -\dfrac{1}{\left(-1+\alpha_1^2.a\right)J_1} \\ 0 \\ \dfrac{\alpha_1}{\left(-1+\alpha_1^2.a\right)J_1} \end{vmatrix} .\tau \tag{5}$$

The Law of Control τ is a simple proportional derivative (PD) type, where the gains K_1 and K_2 are determined through try and error in the simulation represented for:

$$\tau = -K_1\theta - K_2\dot{\theta} \tag{6}$$

3. CONTROL TECHNIQUES

3.1. The LQR Technique

Given a controllable and observable system, exists a linear control law ***u***, such that minimizes the deterministic cost (Ridgely and Banda, 1986) (Maciejowski, 1989).

$$J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt \tag{7}$$

where the matrices *Q* and *R* defined semi-positive *R* positive defined, respectively. The dynamic of the system is represented by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{8}$$

And the control law is defined by

$$u = -K_r(t)x(t) \tag{9}$$

where *K(t)* is given by

$$K_r = R^{-1}B^T P(t) \tag{10}$$

with P(t) solution of Riccati equation

$$-\dot{P} = A^T P + PA + Q - PBR^{-1}B^T P \tag{11}$$

In the stationary case, the Riccati equation is equal to zero. The LQR approach assumes that the system dynamic is perfect, in other words, the disturbances doesn't exist and that all of the states are available to feedback what is not occurring in the reality. But for the first results for analysis and study the model is very important.

3.2. The LQG Technique

Consider the state estimation problem of the stochastic system given by (Ridgely and Banda, 1986).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Gw(t) \\ y(t) &= Cx(t) + v(t) \end{aligned} \tag{12}$$

Where *w(t)* and *v(t)* are gaussian noises with mean zero having covariances:

$$\begin{aligned} &E\left\{w(t)w^T(t)\right\} = W \;\geq\; 0 \\ &E\left\{v(t)v^T(t)\right\} = V \;>\; 0 \\ &E\left\{w(t)v^T(t)\right\} = 0 \end{aligned} \tag{13}$$

The input *u(t)* represents the control vector and *y(t)* the vector of measured outputs. One refers to *w(t)* and *v(t)* as the system noise and the noise of the measures, respectively. The solution of the LQG problem consists in obtained a feedback control law that minimizes the cost (Maciejowski, 1989).

$$J = \lim E\left\{\int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt\right\} \tag{14}$$

The solution to the LQG problem is prescribed by the separation principle (Kwakernaak, 1972) reduces the problem to two sub-problems. The first sub-problem is the Kalman Filter that is given by a state estimator of the form

$$\dot{\hat{x}}(t) = (A - K_f C)\hat{x}(t) + Bu(t) + K_f y \tag{15}$$

with the control law *u=-Kr* based on the estate estimated vectors $\hat{x} = x_{est}$. The Kalman filter gain is given by

$$K_F = P_K C^T V^{-1} \tag{16}$$

P_K satisfies another algebraic Riccati equation.

$$0 = AP_K + P_K A^T + GWG' - P_K C^T V^{-1} CP_K \tag{17}$$

Once obtained the estimated states, passes to the second sub-problem, which is to get an optimal control law, based on the LQR method. It is very well knows that the optimal LQR and the Kalman filter have very good robustness and performance properties when are designed separately.

4. SIMULATION AND RESULTS

In the simulations the performance of control law are analyzed. The system without control law is first presented, second the system with law control LQR and then the system with law control LQG. The matrix weight Q and R of control methods LQR and LQG are be used one diagonal form, once this makes possible that the components of the state and of the control be turned individually, facilitating the adjustment and the physical interpretation.
The values for the control system simulation are determined in "Table 1".

Table 1. Values for The Control System Simulation

Parameters	**Value**
J_0	720 kg.m^2
J_P	40 kg.m^2
K	320 kg,rad^2/s^2
K_d	0.48 kg,rad^2/s^2
L	2 m
M	20 kg
Q and R	1

The "Fig. 2" shows the satellite without ACS, the whole system converges totally. Its shows the need of control law in the system. The '"Fig. 3" shows the simulation of the attitude control of the satellite model with the control law designed by the LQR here all the states are available to feedback. The "Fig. 4" shows the rigid-flexible satellite ACS using the LQG method using the Kalman Filter to estimate the flexible states and feedback they into the control law.

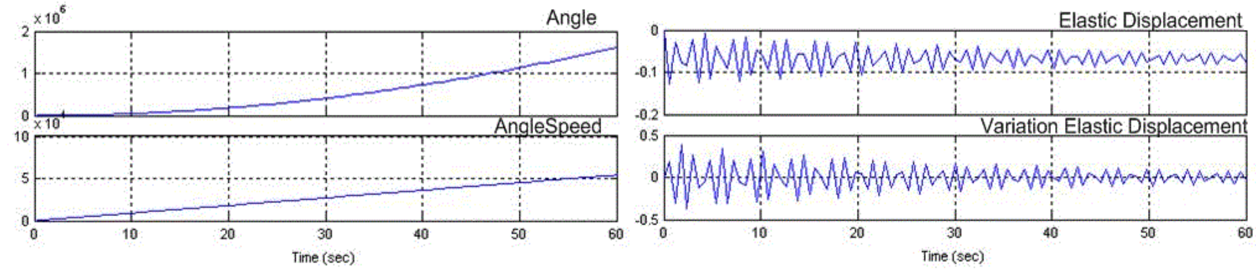

Figure 2. The Satellite without ACS.

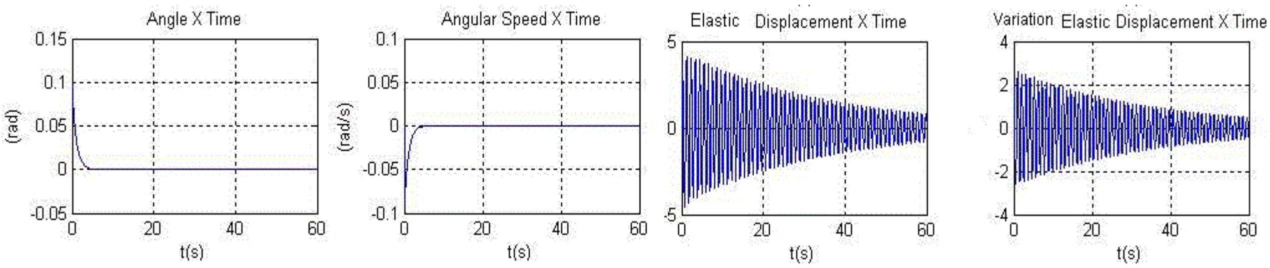

Figure 3. Satellite with ACS-LQR

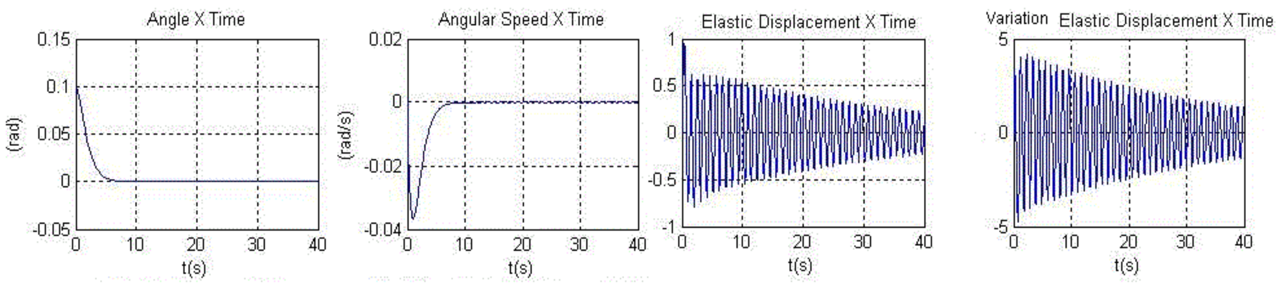

Figure 4. Satellite with ACS- LQG

The performance of the control law with full states into feedback, LQR, is better of the control law LQG. The "Fig. 3", shows a simulation where the control law LQR action where the stabilization of Angle or Angle Speed is close to 2 seconds, however in the "Fig. 4" the performance of method LQG is not same, the stabilization is close to 7 seconds. Criterion practically equal to the displacement of overshoot, in LQR, the overshoot is approximately 0 second. Already in the LQG, the displacement overshoot is close to 4 seconds. Comparing the responses of Elastic Displacement and the Variation Elastic Displacement of the both methods, the panels have a level of vibration more softly in the LQR method than LQG method.

5. CONCLUSIONS

In this paper one investigates the need and the performance of methods of control multivariable in the Attitude Control System (ACS) of Rigid-Flexible Satellites (RFS). The LQR method is more appropriate for systems that have project models reasonably exact and ideal sensors/actuators; and in the preliminary apprenticeship of the project of the control laws. The control law based on LQR method has shown a good performance, but all of the states are considered in the feedback. This feedback is not possible in the practice, because the flexible coordinates are not measured once there are no sensors in the flexible appendages of the satellite. To estimate these coordinates from measures of the rigid coordinates (angle and angular position) and feedback, this technique is known as LQG. The LQG method is more realistic and overcomes some inconveniences of the LQR. However, when the filter is introduced, it is observed that the performance of the control law is degraded. To recovery the good performance obtained in the LQR, it is employed the LQG/LTR method which by simulations have shown to be able to recover the good properties of performance and robustness of the control law projected by the LQR. The next step will be the analysis of performance with the Robust Control.

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