ANALYSIS OF JUPITER'S THIRD-BODY PERTURBATION EFFECTS ON OPTIMAL ASTEROID DEFLECTION MANEUVERS Rodolfo Batista Negri[1], J.P. Sanchez[2], A.F.B.A. Prado[1], [1]National Institute for Space Research, São José dos Campos, Brazil; rodolfo.negri@inpe.br; [2]Cranfield University, Cranfield, United Kingdom

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Introduction: This paper aims to study the influence of a perturbing body in an impulsive asteroid deflection. A first attempt was made by reference [1], which showed that the presence of a perturbing body could severely change the outcome of a deflection when compared with the analytical estimate made in reference [2]. The analysis is done by applying a four-body problem, in order to test the two bodies that we believe perturb most such dynamics, namely Earth and Jupiter. For this purpose, it is applied a BCR4BP (Bi-circular Restricted Four-Body Problem), which consists in the Earth, Jupiter, Sun and the Asteroid. Additionaly, the analytical estimate presented in reference [3] is applied to be compared with the numerical results, considered here to be the real ones.

The asteroid chosen to be deflected is inspired in the 2019 PDC. Its aphelion is varied to test different MOIDs (minimum orbit intersection distance) with the perturbing bodies. We have made simulations considering an intercept time of the target up to 50 years before the collision with Earth, in which a ΔV in the direction of the asteroid's velocity is applied. We choose to apply the impulse in the asteroid's velocity direction because it is the dominant component in an optimal deflection approach, at least when considering two-body approximations [4, 3]. Because our goal is to study the effects of a perturbing body, our analysis is focused in the miss distance rather than an optimal ΔV, as usually found in the literature.

Bi-Circurlar Restricted Four Body Problem (BCR4BP): In the BCR4BP approach, a synodic reference frame, in which M_1 and M_2 are fixed, is used and the equations of motion are written using the canonical system of units (the same is true for the analytical estimate) [5]: the unit of distance is the distance between M_1 and M_2, the unit of time is defined such that the period of the synodic frame is unitary and the canonical masses of M_1 and M_2 are, respectively, $1-\mu$ and $\mu = M_2/(M_1+M_2)$.

In this model, M_1 and M_2 represent the Sun and Jupiter, respectively, as shown in Fig. 1. The Earth is assumed to be in a circular orbit around the center of mass of the Sun and Jupiter and on the same orbital plane. The equations of motion are [6]:

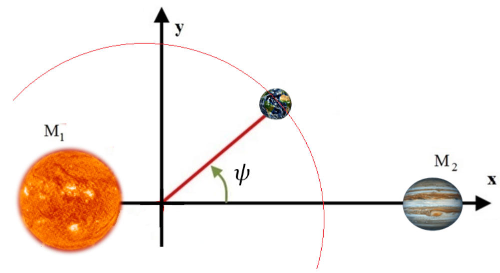

Figure 1: Representation of the BCR4BP.

$$\ddot{x} - 2\dot{y} = x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} - \frac{\mu_e}{r_3^3}(x - R_e\cos\psi) + \frac{\mu_e}{R_e^2}\cos\psi \tag{1a}$$

$$\ddot{y} + 2\dot{x} = y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} - \frac{\mu_e}{r_3^3}(y - R_e\sin\psi) - \frac{\mu_e}{R_e^2}\sin\psi \tag{1b}$$

$$\ddot{z} = -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3} - \frac{\mu_e}{r_3^3}z, \tag{1c}$$

where r_1, r_2 and r_3 are the distances between the asteroid and the Sun, Jupiter and Earth, respectively. The angle ψ is the phase angle between Jupiter and the Earth, as shown in Fig. 1, μ_e is the mass of the Earth in canonical units and R_e is the distance between the Earth and the center of the reference frame.

Analytical Estimate: The deviation at the MOID after a deflection is calculated by reference [3] as:

$$\vec{\Delta} = \begin{bmatrix}\delta s_r & \delta s_\theta & \delta s_h\end{bmatrix}^{\mathbf{T}}, \tag{2}$$

where δs_r, δs_θ and δs_h are the displacements in the radial, transversal, and perpendicular-to-the-orbit-plane directions, respectively. These components can be calculated by proximal motion equations as functions of variations in the orbital elements [3].

The variations on each of the orbital elements may be approximated through the Gauss planetary equations as, for a tangential to the asteroid's velocity impulse:

$$\delta a \approx \frac{2a^2V}{(1-\mu)}\Delta V, \tag{3a}$$

$$\delta e \approx \frac{2}{V}(e+\cos f_i)\Delta V, \tag{3b}$$

$$\delta \omega \approx \frac{2}{eV}\sin f_i \Delta V, \tag{3c}$$

$$\delta M \approx -\frac{2a\sqrt{1-e^2}}{eaV}\left(1+\frac{e^2R}{a(1-e^2)}\right)\sin f_i\Delta V + \left(\sqrt{\frac{1-\mu}{a^3}}-\sqrt{\frac{1-\mu}{(a+\delta a)^3}}\right)t_i, \tag{3d}$$

where e, a and ω are the asteroid's orbit eccentricity, semi-major axis and argument of the perihelion. The true anomaly at the time t_i, in which the spacecraft intercepts the asteroid and applies the impulse, is f_i. The variations δi and $\delta\Omega$ are zero for the tangential impulse.

The gravitational focusing due to the Earth is also taken into account and the miss distance is obtained as [8]:

$$\delta = \frac{\mu_e}{v_\infty^2}\left(\sqrt{1+\left(\frac{bv_\infty^2}{\mu_e}\right)^2}-1\right), \tag{4}$$

where v_∞ is the geocentric velocity at the infinity and b represents the approach distance, which is obtained as function of $\vec{\Delta}$ (shown in reference [8]).

Method: The semi-major axis, inclination and eccentricity of the asteroid is defined, which are $a = 1.9150$ au, $i = 18°$ and $e = 0.5352$ for the 2019 PDC. The other three Keplerian elements of the asteroid are found such that the orbit of the asteroid intersects exactly the position correspondent to the center of the Earth at the collision instant[1], in a two-body propagation. It is also assumed an initial phase angle ψ_0 between Jupiter and the Earth.

Once all of the orbital elements are obtained, the true anomaly is set a few degrees before the collision[2] and the asteroid state at this point is obtained, which will be the initial state for our simulations. This procedure is necessary to avoid that the gravitational influence of the Earth severely changes the asteroid's orbital elements in the further backward integration of the equations of motion.

[1]Therefore, there is no difference in applying the tangential impulse in favor or against the asteroid's velocity vector [2].

[2]This is arbitrarily chosen in a way that the distance between the asteroid and the Earth is at least greater than the Earth's SOI

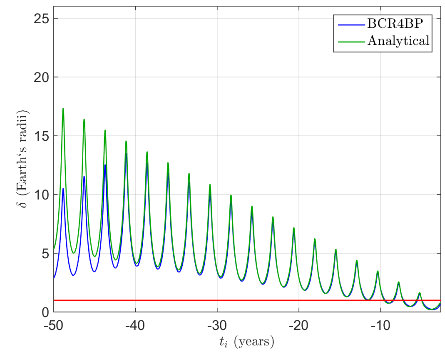

Figure 2: Miss distance as a function of the intercept time before the collision with the Earth for the asteroid 2019 PDC, $\Delta V = 1$ cm/s and $\psi_0 = -180°$.

The equations of motion for each problem are integrated backward in time for different values of intercept time t_i, up to 50 years before the collision, when the ΔV is applied. An 8-7th order Runge-Kutta integrator with Dormand and Prince formulae is chosen to integrate the equations of motion. As observed in other works [4, 3], an impulse tangential to the velocity of the asteroid is the optimal solution for intercept times greater than about the orbital period of the asteroid. Therefore, we choose ΔV to be tangential to the velocity of the asteroid. The chosen ΔV is applied to the asteroid state at the intercept instant and the equations of motion are then integrated forward in time, while keeping track of the relative distance between the asteroid and the Earth. The minimum relative distance between the asteroid and the Earth is defined as the miss distance δ.

Results: Figure 2 shows the miss distance in Earth's radii, for each value of intercept time t_i, obtained by the BCR4BP (blue line) and the analytical estimate (green line) applied to the asteroid 2019 PDC. The Earth's radius value is depicted as a red line. We choose a $\Delta V = 1$ cm/s and the phase angle between the Earth and Jupiter at the collision instant is $\psi_0 = -180°$. The analytical estimate and the numerical simulation agree quite well for intercept times up to 40 years before the collision. However, for intercept times greater than 40 years, the analytical solution overestimates the deflection, and this error is larger than 5 Earth's radii in the largest δ predicted by the analytical solution.

Figures 3 and 4 show the results for asteroids that keep the perihelion and inclination of the 2019 PDC, but change the aphelion r_a to 4.5 au and

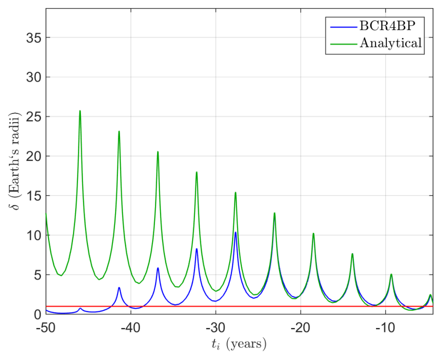

Figure 3: Miss distance as a function of the intercept time before the collision with the Earth for $r_a = 4.5$ au, $\Delta V = 1$ cm/s and $\psi_0 = 0°$.

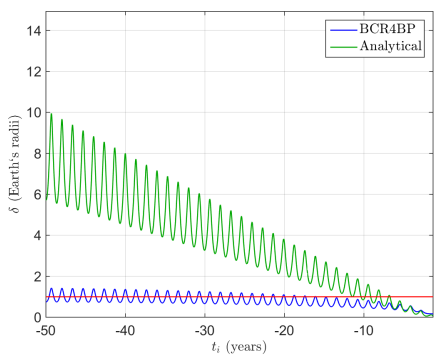

Figure 4: Miss distance as a function of the intercept time before the collision with the Earth for $r_a = 1.5$ au, $\Delta V = 1$ cm/s and $\psi_0 = 0°$.

1.5 au, respectively. In the first of these figures we see that the results obtained with the BCR4BP diverges very much from the analytical estimate. This great difference occurs roughly around $t_i = -30$ years and, in contrary to what the analytical solution indicates, the deflection is attenuated as the time increases, which results in collisions for $t_i < -40$ years. Figure 4 exhibits an even more drastic behavior. The chosen deflection is unable to make the asteroid to miss the Earth for intercept times below 20 years. Additionally, the miss distance has a much lower increase with t_i than expected by the analytical estimate, resulting in larger errors.

If we plot the distance between the undeflected asteroid of $r_a = 1.5$ au and the Earth (r_E) and Jupiter (r_J), as shown in Fig. 5 normalized by the Hill's sphere radius of each body, we see that there is a close approach between the asteroid and the Earth when $t \approx -5$ years. Our studies show that

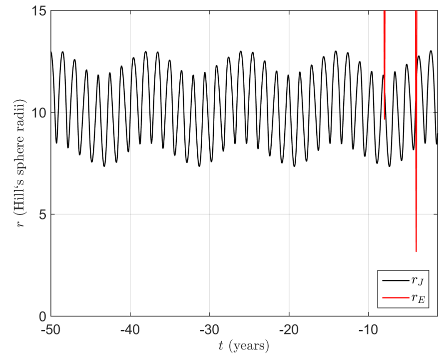

Figure 5: Distance between the undeflected asteroid, with $r_a = 1.5$, and the Earth (r_E) and Jupiter (r_J) as a function of the time before the collision, $\psi_0 = 0°$.

these close approaches, even when larger than the body's SOI (or Hill's sphere), is precisely the main cause of the larger errors in the analytical estimate rather than the perturbations throughout the orbit. A complete description of the reasons and possibly corrections in the analytical estimate for these cases is currently under study and should be published soon.

Conclusion: Our results show that the predicted deflection obtained by the analytical estimate may diverge very much when other bodies are considered. We also present an evidence that this behavior is mostly caused by a close approach with one of these bodies, even if it is larger than the body's SOI. A study clarifying this point is under way and should be published soon.

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