ROBUSTNESS ASSESSMENT BETWEEN SIGMA POINT AND EXTENDED KALMAN FILTER FOR ORBIT DETERMINATION

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Abstract: *A comparison between the extended Kalman filter (EKF) and the nonlinear sigma point Kalman filter (SPKF) for a real time satellite orbit determination problem, using GPS measurements is presented. Such comparison is based on testing the filters robustness for degraded initial conditions. The main subjects for the comparison between the estimators are convergence speed and computational implementation complexity. Based on the analysis of such criteria, the advantages and drawbacks of each estimator are presented. In this work, the orbit of an artificial satellite is determined using real data from a space borne Global Positioning System (GPS) receiver. This is a fully nonlinear problem, with respect to both the dynamics and measurements equations, in which the disturbing forces are not easily modeled. The problem of orbit determination consists essentially of estimating values that completely specify the body trajectory in the space, processing a set of measurements related to this body. In this orbit determination problem the focus is to analyze each filter convergence behavior in situations where the initial conditions are inaccurate, introducing since small up to larger errors in the initial accurate position conditions. Concomitantly another aim is to know how such inaccuracies affect the estimators performance.*

Keywords: *Sigma Point Kalman Filter, Extended Kalman Filter, Robustness Assessment, Orbit Determination.*

1 Introduction

In this work, the extended Kalman filter (EKF) is compared with the nonlinear sigma point Kalman filter (SPKF) for a real time satellite orbit determination problem, using GPS measurements. The comparison is based on the assessing the robustness of the filters for purposely degraded initial conditions. The main subjects for the comparison between the estimators are convergence speed, divergence occurrence, flaws and statistical shortcomings. Based on the analysis of such criteria, the advantages and drawbacks of each estimator are presented.

Here, the orbit of an artificial satellite is determined using real data from the Global Positioning System (GPS) receivers. In orbit determination of artificial satellites, both the dynamic system and the measurements equations are of nonlinear nature. Therefore one deals here with a fully nonlinear problem in which the acting forces as well as measurements are not easily modeled. The orbit determination problem consists of estimating variables that completely specify the body trajectory in the space, processing a set of information (pseudo-range measurements) related to this body. As far as this work is concerned, the more accurate GPS phase measurements are not used here, because the aim is not the search for accuracy, but a comparison of performance under different error levels of initial conditions. Besides using carrier phase measurements, the ambiguity resolution algorithm or any other artifacts to overcome such hindrance could eventually mask the results, misleading the conclusions.

A spaceborne GPS receiver is a powerful means to determine orbits of artificial Earth satellites by providing many redundant measurements which ultimately yields high degree of the observability to the problem. The Topex/Poseidon (T/P) satellite is a nice example of using GPS for space positioning. Through an onboard GPS receiver, the pseudo-ranges (error corrupted distance from satellite to each of the tracked GPS satellites) can be measured and can be used to estimate the full orbital state.

The EKF is probably the most widely used real time estimation algorithm for nonlinear systems [1]. It is a nonlinear version of the Kalman filter that generates reference trajectories which are updated at each measurement processing, at the corresponding instant. However, the experience from the estimation community has shown that the EKF is difficult to implement, requires some skill to get tuned, depends very much on the closeness of the initial conditions to the truth values, and is only reliable for systems that are nearly linear on the time scale of the filter working updates. Many of these difficulties arise from the linearizations needed by the EKF method. Specifically for the orbit estimation problem, under inaccurate initial conditions and scattered measurements, the EKF implementation can lead to unstable or diverging solutions. Therefore, there is a strong need for a method that is probably more accurate than linearization, but that does not be liable to neither the implementation nor additional computational costs of other higher order filtering schemes. To overcome this limitation, the unscented transformation was developed as a technique to propagate mean and covariance information through nonlinear transformations. The SPKF is a new estimator that claims to yield equivalent or better performance than the EKF and elegantly extends to nonlinear systems, without the linearization steps [2, 3, 4]. This algorithm is a new approach to generalize the Kalman filter for nonlinear process and observation models.

In this orbit determination problem the focus is to analyze each filter convergence behavior in situations where the initial conditions are purposely degraded, introducing since small up to larger errors in the known initial orbit elements. Another aim is to characterize how such inaccuracies affect the performance of the estimators. Therefore, the EKF (the most widely used estimation algorithm) as well as the SPKF (supposedly the most appropriate estimation algorithm for nonlinear problems) performance evaluation in the real time orbit determination problem is due and justified.

2. The Extended Kalman Filter EKF

The EKF is a nonlinear version of the Kalman filter that generates reference trajectories which are updated at each measurement processing times [1, 5]. Due to the complexity of accurately modeling the nonlinear satellite orbit, the EKF is generally used in works of such nature. The algorithm always provides up to date reference trajectory around the most current available estimate.

Exploiting the assumption that all transformations are quasi-linear, the EKF simply linearizes all nonlinear transformations and to the first order replaces the Kalman filter matrices by their counterpart Jacobian partial derivatives matrices. The EKF consists of cycles of time and measurement updates. Firstly, state and covariance are propagated from one previous instant to a later one, meaning that they are propagated between discrete instants of the system dynamics model. In the sequel, state and covariance are corrected for the later instant corresponding to the measurement time, through the observations model. This method has recursive nature and does not need storage of the measurements previously in large matrices, being therefore well suited for real time processing.

The EKF time update cycle is given by

$$\dot{\bar{\mathbf{x}}}_k = \mathbf{f}(\hat{\mathbf{x}}_{k-1})$$
$$\bar{\mathbf{P}}_k = \boldsymbol{\varphi}_{k,k-1}\hat{\mathbf{P}}_{k-1}\boldsymbol{\varphi}_{k,k-1}^T + \mathbf{Q}_k \quad (1)$$

where **f** is a nonlinear vector function modeling the orbit motion, $\bar{\mathbf{x}}_k$ and $\bar{\mathbf{P}}_k$ are respectively the propagated state and the covariance for t_k; $\boldsymbol{\varphi}$ is the state transition matrix between t_{k-1} and t_k; $\mathbf{Q}_k$ is the dynamics noise matrix. It is required the Jacobian matrix ($\partial\mathbf{f}/\partial\mathbf{x}$) for the transition matrix computation which can either be simplified or can be very difficult to obtain.

The equations for the EKF measurement update cycle are

$$\begin{aligned}\mathbf{K}_k &= \bar{\mathbf{P}}_k\mathbf{H}_k^T(\mathbf{H}_k\bar{\mathbf{P}}_k\mathbf{H}_k^T + \mathbf{R}_k)^{-1}\\ \hat{\mathbf{P}}_k &= (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\bar{\mathbf{P}}_k\\ \hat{\mathbf{x}}_k &= \bar{\mathbf{x}}_k + \mathbf{K}_k[\mathbf{y}_k - \mathbf{h}_k(\bar{\mathbf{x}}_k)]\end{aligned} \quad (2)$$

where $\mathbf{h}_k$ is a nonlinear vector function modeling the measurements; **H** is the corresponding partial derivative matrix $\left(\frac{\partial\mathbf{h}_k}{\partial\mathbf{x}}\right)$; $\mathbf{K}_k$ is the Kalman gain; $\hat{\mathbf{P}}$ and $\hat{\mathbf{x}}$ are the state vector and the covariance updated for the instant *k*.

There are some limitations for the EKF. For instance the linearized transformations are reliable only if the error propagation can be matched with good approximation by a linear function; linearization can be applied only if the Jacobian matrix exists; and obtaining the analytical Jacobian matrices can be a very difficult and error-prone process. Summarizing, linearization, as applied in the EKF, is widely recognized to be inadequate, however other alternatives incur substantial costs in terms of derivation and computational complexity. The sigma point algorithms via unscented transformation were developed in an attempt to overcome these flaws and offer a feasible and reliable alternative.

3. The Sigma Point Kalman Filter SPKF

If the system dynamics and the observation model are linear, the conventional Kalman filter (KF) is the optimal solution and must be used fearlessly. However because, not rarely, the system dynamics and/or the measurement models are nonlinear, convenient extensions of the KF like the EKF have been used.

The SPKF is a new estimator that allows similar performance than the KF for linear systems and elegantly extends to nonlinear systems, without need of the linearization procedures. This algorithm family is a new approach to generalize the KF for nonlinear process and observation models [3, 4, 6]. A set of weighted samples, the sigma points, is used for computing mean and covariance of a probability distribution. Such algorithms include the unscented Kalman filter (UKF), which is based on the unscented transformation (UT), a nonlinear transformation for mean and covariance.

The SPKF is a technique claimed to lead to a more accurate and easier to implement filter than the EKF or a second order Gaussian filter. The SPKF approach is described, as follows [2]

1. A set of weighted samples is deterministically calculated, based on mean and covariance decomposition of a random variable.

2. The sigma points are propagated through the real nonlinear function, using only functional estimation, that is, analytical derivatives are not used to generate a posteriori set of sigma points.
3. The later statistics are calculated using propagated sigma points functions and weights. In general, they assume the form of a simple weighted average of the mean and the covariance.

Herein, the UT and the SPKF, i.e., the filter steamming from this transformation, will be described.

3.1 The Unscented Transformation UT

Essentially this is a way of calculating the statistics of a random variable that passes through a nonlinear transformation. The UT approach selects a suitable set of points (sigma points) so that their mean and covariance are $\bar{\mathbf{x}}$ and $\mathbf{P}_{xx}$ [3, 4]. The nonlinear function is applied to each point of the set, in turn, to yield a cloud of transformed points. The statistics of the transformed points (mean $\bar{\mathbf{y}}$ and covariance $\mathbf{P}_{yy}$) can then be calculated to form an estimate of the non linearly transformed mean and covariance.

The sigma points are carefully and deterministically chosen so that they exhibit certain specific properties, that is, they are not drawn at random like common Monte Carlo methods. Besides, they can be weighted in ways that are inconsistent with the distribution interpretation of sample points like in a particle filter [2, 3].

The n-dimensional random variable $\mathbf{x}$, with $\bar{\mathbf{x}}$ mean and $\mathbf{P}_{xx}$ covariance, is approximated by $2n + 1$ weighted points, the so known sigma points, given by

$$\begin{aligned} \boldsymbol{\chi}_0 &= \bar{\mathbf{x}} \\ \boldsymbol{\chi}_i &= \bar{\mathbf{x}} + \left(\sqrt{(n+\kappa)\mathbf{P}_{\mathbf{xx}}}\right)_i \\ \boldsymbol{\chi}_{i+n} &= \bar{\mathbf{x}} - \left(\sqrt{(n+\kappa)\mathbf{P}_{\mathbf{xx}}}\right)_i \end{aligned} \tag{3}$$

in which $\kappa \in \Re$, $\left(\sqrt{(n+\kappa)\mathbf{P}_{\mathbf{xx}}}\right)_i$ is either the i-th row or column of the square root matrix of $(n+\kappa)\mathbf{P}_{\mathbf{xx}}$. The transformation occurs as follows

1. Transform each point through the nonlinear function to yield the set of transformed sigma points

$$y_i = \mathbf{f}\left[\boldsymbol{\chi}_i\right] \tag{4}$$

2. The observations mean is given by the weighted average of the transformed points

$$\bar{\mathbf{y}} = \sum_{i=0}^{2n} W_i \, y_i \tag{5}$$

3. The covariance is the weighted outer product of the transformed points

$$\mathbf{P}_{yy} = \sum_{i=0}^{2n} W_i \left[y_i - \bar{\mathbf{y}} \right]\left[y_i - \bar{\mathbf{y}} \right]^T \tag{6}$$

W_i is the weight associated to the i-th point given by

$$W_0 = \frac{\kappa}{(n+\kappa)}$$
$$W_i = \frac{1}{2(n+\kappa)}, \; i = 1, \ldots, n \tag{7}$$
$$W_{i+n} = \frac{1}{2(n+\kappa)}, \; i = 1, \ldots, n$$

3.2 The Unscented Sigma Point Kalman Filter

Using UT, the following steps are processed in the Kalman filter

1. Predict the new state system and its associated covariance, taking into account the effects of the gaussian white noise process.
2. Predict the expected observation and its residual innovation matrix considering the effects of the observation noise.
3. Predict the cross correlation matrix.

Figure 1 shows these 3 steps of the UT, changing the EKF, in order to lead to the new filter: the SPKF. These steps are put in order in the EKF with the re-structuring of dynamics, state vector and observations models.

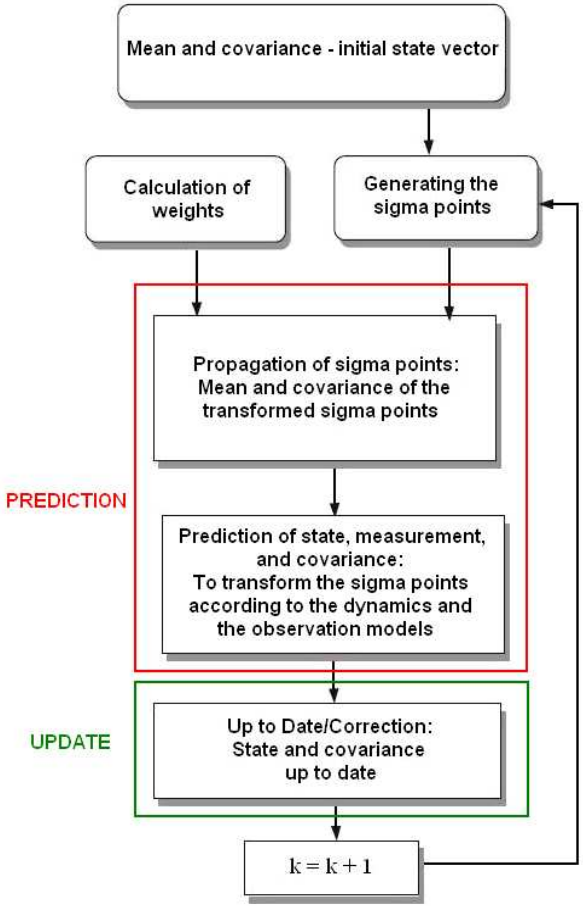

Figure 1. UT introduced in the EKF, leading to SPKF

4. The Orbit Determination

The instantaneous orbit determination using GPS satellites is basically a geometric method. In this method, the observer knows the set of GPS satellites position in a reference frame, obtaining its own position in the same reference frame. Figure 2 presents the basic parameters for user position determination. In Fig. 2, $\mathbf{R}_{GPS_i}$ is the position of *i*-th GPS satellite in the reference system; $\boldsymbol{\rho}_i$ is the pseudorange; and $\mathbf{r}_u$ is the user position in the reference system.

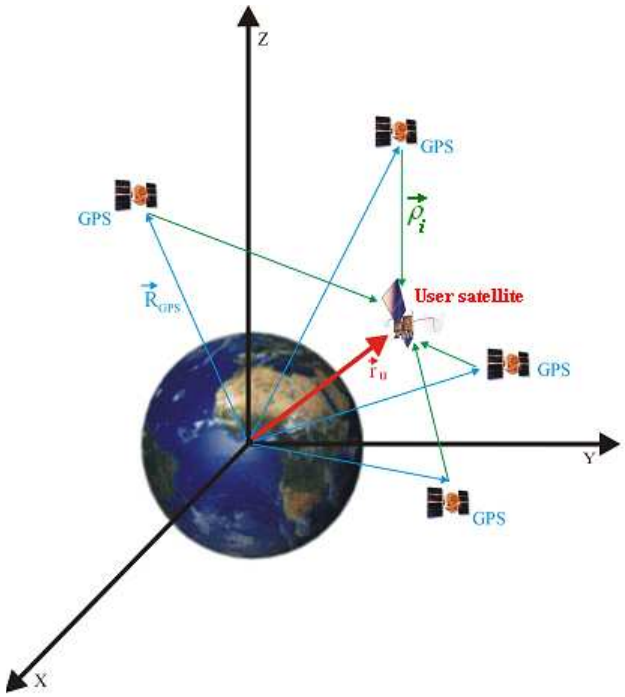

Figure 2. The Geometric Method

However sequential orbit determination makes use of the orbit motion model to predict between measurement times and measurement model to update the orbit by processing of GPS measurements. This gives rise to recursive and real time Kalman filter estimator for the orbit determination [5].

4.1 The Filter Dynamic Model

In the case of orbit determination via GPS, the ordinary differential equations that represent the dynamic model are in its simplest form given traditionally as follows

$$\dot{\mathbf{r}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = -\mu \frac{\mathbf{r}}{r^3} + \mathbf{a} + \mathbf{w}_v \qquad (8)$$

with variables given in the inertial reference frame. In the equations above, **r** is the vector of the position components vector (x, y, z); **v** is velocity vector; **a** represents the modeled perturbing accelerations; and $\mathbf{w}_v$ is the white noise vector with covariance **Q**. The GPS receiver clock offset was not taken into account, so as not to obscure the conclusions drawn in this paper due to introduction of clock offset models in the filters. Indeed, the receiver clock offset was beforehand

obtained and used to correct the GPS measurements, so that the measurements are free from the receiver clock offset error.

4.2 The Force Model

The main disturbing forces of gravitational nature that affect the orbit of an Earth's artificial satellite are: the non uniform distribution of Earth's mass; ocean and terrestrial tides; and the gravitational attraction of the Sun and the Moon. There are also the non gravitational effects, such as: Earth atmospheric drag; direct and reflected solar radiation pressure; electric drag; emissivity effects; relativistic effects; and meteorites impacts.

The disturbing effects are in general included according to the physical situation presented and to the accuracy that is intended for the orbit determination. Here we include only a minimum set of perturbations which enable us to assess the performance of both filters, namely geopotential and third body point mass effect of Sun and Moon.

The Earth is not a perfect sphere with homogeneous mass distribution, and cannot be considered as a material point. Such irregularities disturb the orbit of an artificial satellite and the keplerian elements that describe the orbit do not behave ideally. The geopotential function can be given by [8]

$$U(\mathrm{r},\phi,\lambda)=\frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_{\mathrm{T}}}{\mathrm{r}}\right)^{n}P_{nm}(\sin\phi)(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda) \tag{9}$$

where μ is Earth gravitational constant; R_{T} is mean Earth radius; r is the spacecraft radial distance; ϕ is the geocentric latitude; λ is the longitude on Earth fixed coordinates system; C_{nm} and S_{nm} are the harmonic spherical coefficients of degree n and order m; P_{nm} are the associated Legendre functions. The constants μ, R_{T}, and the coefficients C_{nm}, and S_{nm} determine a particular gravitational potential model.

Another gravitational perturbation source is that due to the Sun and Moon attraction. They are more meaningful at farther distance from Earth. As the orbital variations are of the same type, be the Sun or the Moon the attractive body, they are normally studied without distinguishing the third body. The luni-solar gravitational attraction mainly acts on node and perigee causing precession of the orbit and on the orbital plane. The general three-body problem model is here simplified to the circular restricted three-body problem, where the orbital motion of a third body (satellite), which mass can be neglected, around two other massive bodies is studied. The motion equations that provide the third body accelerations can be expressed as [9]

$$\ddot{\mathbf{r}}_3=-Gm_1\frac{\mathbf{r}_{13}}{\mathrm{r}_{13}^3}-Gm_2\frac{\mathbf{r}_{23}}{\mathrm{r}_{23}^3} \tag{10}$$

where $\mathbf{r}_{13}=\mathbf{r}_3-\mathbf{r}_1$, $\mathbf{r}_{23}=\mathbf{r}_3-\mathbf{r}_2$, and $\mathbf{r}_i$, $i=1,2,3$ correspond to the i-th body distance vector to the system center of mass.

4.3 The Observations Model

The nonlinear equation of the observation model is

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, t) + \mathbf{v}_k \tag{11}$$

where, at time t_k, $\mathbf{y}_k$ is the vector of m observations; $\mathbf{h}_k(\mathbf{x}_k)$ is the nonlinear function of state $\mathbf{x}_k$, with dimension m; and $\mathbf{v}_k$ is the observation errors vector, with dimension m and covariance $\mathbf{R}_k$. For the present application, one only uses the ion-free pseudo-range measurements from the Topex/Poseidon GPS receiver. Also, the receiver clock offset was computed before and used to correct the pseudo-range measurements. Additionally, the nonlinear pseudo-range measurement was modeled according to [7].

5. Results

The tests and the analysis for the extended EKF and the sigma point SPKF algorithms are presented. To validate and to analyze the methods, real GPS data from the T/P (Topex/Poseidon) satellite are used. The filter estimated position and velocity are compared with T/P precise orbit ephemeris (POE) from JPL/NASA. The test conditions consider real ion-free pseudo-range data, collected by the GPS receiver onboard T/P, on November 19th, 1993, at 60s sampling rate, presenting on average between 5 to 6 GPS satellites tracked. The GPS data were previously preprocessed to remove the outliers so they can not mislead the filters or mask different data rejection policies of each filter. The tests have covered a long (almost a day) period of orbit determination.

The force model includes perturbations due to geopotential up to order and degree (30x30), with harmonic coefficients from JGM-2 model, and the Sun-Moon gravitational attraction [10, 11]. The pseudo-range measurements were corrected to the first order with respect to ionosphere.

The obtained results are evaluated through error in position components, which represents the difference between the POE reference and the estimated position components, and are after translated to radial, normal, and along-track (RNT) components of T/P orbit fixed system.

As already pointed, this work is not a search for results accuracy. It aims at the comparison of performance between SPKF and EKF estimators under different degradation levels of initial conditions. There are peculiar interest for speed convergence, and divergence occurrence. In order to analyze these subjects, the approach consists of introducing increasing levels of uncertainties in the accurately known (from POE) initial position.

The errors are introduced ranging from small up to larger values, varying from 0.1 km to 1,000 km, in power of 10, and convergence and divergence behavior in both algorithms results are investigated. First one generates for each filter (EKF or SPKF) a tunned filter run with precise known initial conditions and use the results as reference solutions. In this case both have similar performance and agree very well, i.e., error in position and pseudo-range residuals statistics are equivalent, which means that if the initial conditions are accurate, SPKF and EKF show similar convergence patterns as soon as the estimation process starts, and either one could be used.

Thus the analysis is based on comparing with such a reference solution the errors in position (translated to the orbital RNT components) and the difference in predicted pseudo-range residuals obtained from SPKF and EKF algorithms.

Table 1 shows the analysis for the predicted pseudo-range residuals convergence, which is measured in terms of time span of data processed. The convergence is assumed when the residuals achieve the same statistics of the reference solution residuals. When a small 0.1 km error is introduced, convergence occurs instantaneously after the estimation process starts, for both SPKF and EKF algorithms. For 1 and 10 km errors in initial conditions, SPKF converges again immediately, and long before EKF for the two cases: 2 and 2.5 hours, for 1 and 10 km errors, respectively. For a 100 km error, SPKF needs 2 hours of estimation to reach the convergence zone, and EKF, 12 hours. And, when the largest error of 1,000 km is added, EKF is not able to converge at all, while SPKF still converges after 8 hours.

Table 1. Pseudo-range residual convergence speed

Errors (km)	**SPKF** convergence time (h)	**EKF** convergence time (h)
0.1	0	0
1	0	2
10	0	2.5
100	2	12
1000	8	no convergence observed

Table 2 shows the convergence analysis for the position RNT components error, which is again measured in terms of data time of processing. When a small 0.1 km initial position error is introduced, convergence occurs instantaneously after the estimation process starts, for both SPKF and EKF algorithms, in normal and transverse components, and after 0.5 hours in the radial one. For the 1,10, and 100 km errors in initial conditions cases, SPKF converges always before EKF for the three components, as can be checked in Tab.2. And, when the largest error of 1,000 km is tried, EKF does not converge at all, while SPKF still converges although taking longer time (almost half a day).

Table 2. Error in position convergence speed

errors (km)	**SPKF** convergence time (hours)			**EKF** convergence time (hours)		
	R	N	T	R	N	T
0.1	0.5	0	0	0.5	0	0
1	1	0	1	2.5	1	2
10	3.5	0	1	4.5	1	2
100	3.5	1.5	2	8	16	14
1000	5	11	7	no convergence observed	No convergence observed	no convergence observed

Another statistical check is done, in order to confirm that the algorithms effectively reached convergence. The reference pseudo-range residuals statistics are available in the first line of Tab. 3. They are used as the reference values for SPKF and EKF test cases, which statistics are only computed after convergence time. From Tab. 3 it becomes evident that the estimators really reached convergence, since their statistical values remain nearly the same the reference ones.

Table 3. Pseudo-range residuals mean and standard deviation

Errors (km)	**SPKF** pseudo-range mean ± std dev (m)	**EKF** pseudo-range mean ± std dev (m)
0 (reference)	**-1.248 ± 25.638**	**-1.238 ± 25.614**
0.1	-1.160 ± 25.846	-1.154 ± 28.014
1	-1.037 ± 26.870	-1.604 ± 25.721
10	-1.218 ± 27.410	-0.985 ± 25.779
100	-1.318 ± 25.616	-0.994 ± 36.403
1000	-1.133 ± 29.220	no convergence observed

In order to depict such findings, Fig. 3 illustrates the reference residuals (accurate initial conditions) behavior, and the 1,000 km error case behavior for both the EKF and the SPKF estimators. It clearly indicates clues of EKF divergence for such a very bad initial condition.

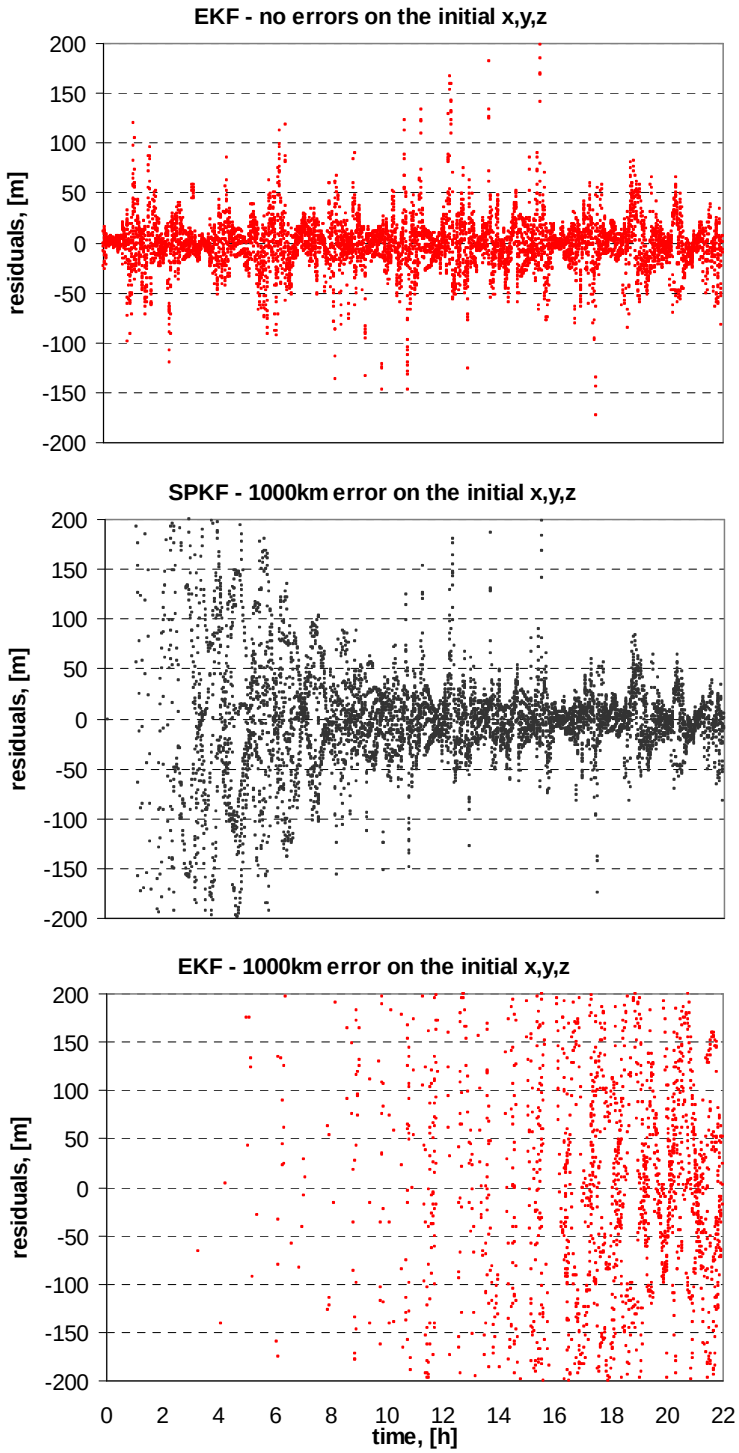

Figure 3. Pseudo-range residuals convergence and divergence behaviors

Proceeding the investigation, Tab. 4 shows RMS position error, where the reference values are again listed in the first row (yellowed). Again, SPKF and EKF resulting RMS errors are only computed after assumed convergence time. For Tab. 4 it is also clear that the estimators really reached convergence, since their RMS values remain nearly close to the reference ones.

Table 4. Total RMS error in position after convergence

errors (km)	**SPKF** error (m)	**EKF** error (m)
0 (reference)	**21.835**	**21.628**
0.1	21.656	21.090
1	21.376	19.520
10	18.941	20.264
100	18.708	20.074
1000	22.279	no convergence observed

Figure 4 shows the errors in the RNT components for the SPKF and EKF reference cases (accurate initial conditions, left side) and the larger 1,000 km error case results for the EKF and the SPKF estimators (1,000km initial error, right side). It indicates signs of the EKF divergence for such a very bad initial condition while SPKF reaches the convergence zone, although much later than the left side results.

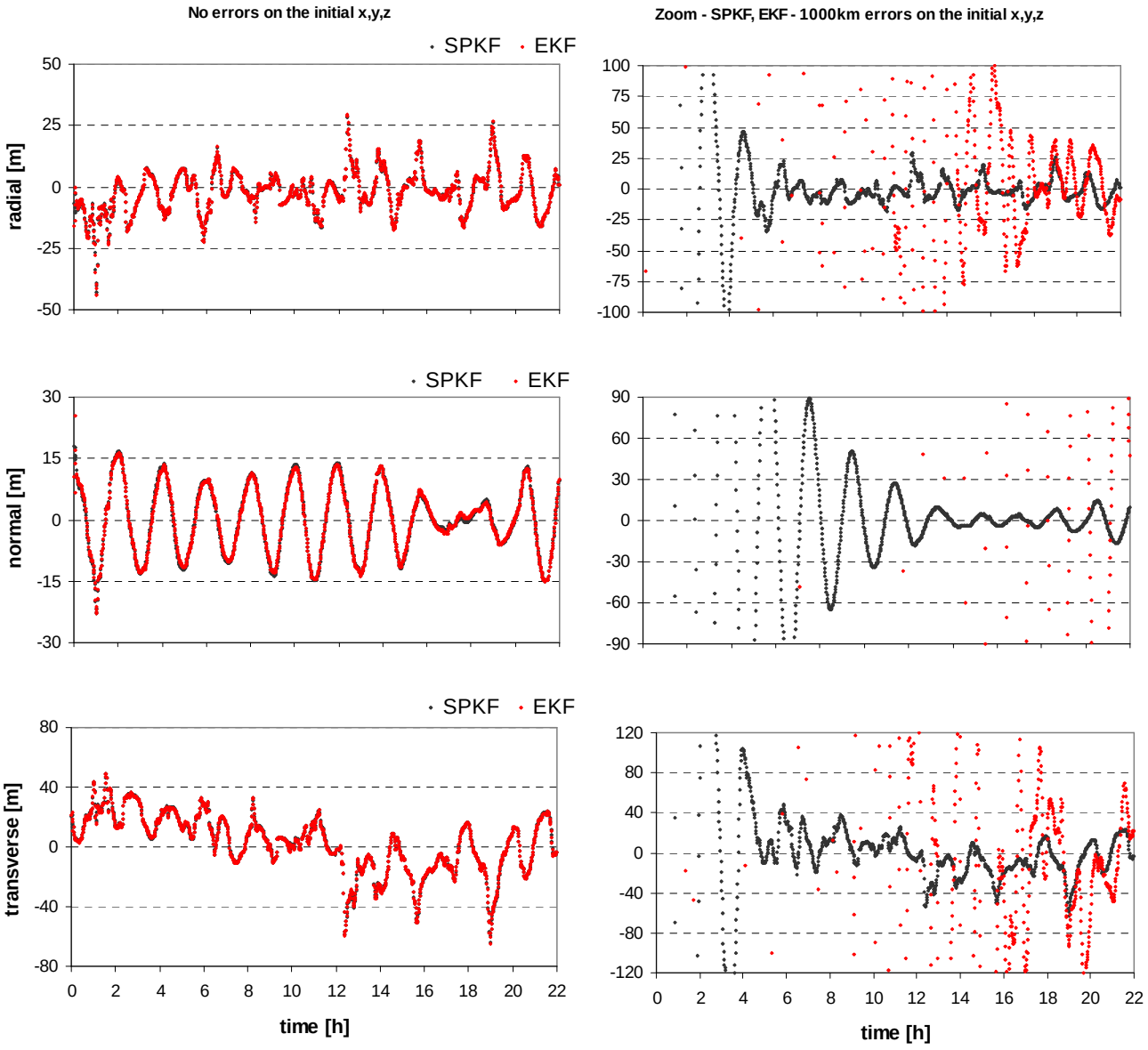

Figure 4. Errors in RNT position components convergence and divergence behaviors

6. Conclusions

The robustness to erroneous initial conditions of two nonlinear estimators, namely the EKF (Extended Kalman Filter) and SPKF (Sigma Point Kalman Filter) was assessed for a real time satellite orbit determination problem using real GPS measurements. Almost one day of GPS receiver measurements of Topex/Poseidon satellite at 60s sampling rate were processed. The emphasis was to characterize each filter convergence behavior in situations where the initial conditions are inaccurate, poor, and degraded, by introducing different levels of errors in the initial position.

Results showed that when small errors in the initial orbit estimates are present, SPKF and EKF yield similar performance compared to the reference solution behavior, with no errors in the initial conditions. As expected, increasing errors decreases the filters performance. As larger is the initial error more difficult is for EKF and SPKF to reach convergence. When SPKF is compared with EKF, in all cases of larger initial errors, the SPKF always attains convergence first. The rupture threshold for this application in particular was at 1,000 km error case, where the EKF could not converge but SPKF still achieved convergence after a while. Therefore it is to be said that SPKF is more robust than EKF for degraded initial errors, in this orbit determination application.

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