A CLOSE APPROACH BETWEEN A PLANET AND A PARTICLE: SUN-JUPITER SYSTEM

JORGE KENNETY and ANTONIO F B A PRADO
National Institute for Space Research -INPE
Technology Faculty-FATEC-SJC
Av dos Astronautas 1758, SJC-SP
BRAZIL
jkennety@yahoo.com.br
prado@dem.inpe.br

Abstract: - This paper presents an investigation of a close approach between a planet and a particle. It is assumed that the dynamical system is given by two main bodies that are in circular orbits around their center of mass and the particle that is moving under the gravitational attraction of the two primaries. This method has been under study for a long time by several authors, where the dynamical system given by the "patched-conics" is used and the motion is assumed to be planar. A series of two-body problems is used to generate analytical equations that describe the problem. Two solutions are considered for the Swing-By (clock-wise and counter-clock-wise orbit), to take into account the possibility that the particles crosses the line Sun-Planet between the primaries. The goal is to study the orbital change (energy, angular momentum, orbital elements) of the particle after some maneuvers with the planet desired and to know those values after the close approach. In particular, we are looking for geometries that allows multiple Swing-Bys without correction maneuvers. Finally, numerical simulations are performed for the Sun-Jupiter system.

Key-Words: - Swing-By, orbital maneuver, astrodynamics, celestial mechanics, orbital dynamics.

1 Introduction

In aerospace engineer, the spacecraft trajectories can be controlled by thrusters and several other physical forces. The determination of these trajectories in the solar system, considering gravitational effects, is performed by several techniques. This paper will use the Swing-By maneuver (gravity-assist) to analyze missions involving celestial bodies and spacecrafts (particles) or celestial bodies and a cloud of particles. The maneuver uses a close approach with a celestial body to modify the energy, angular momentum and velocity of the spacecraft with respect to the Sun. The dynamical system given by the "patched-conics" is used and the motion is assumed to be planar. An important example of gravitational assist occurred in December 1973, with the encounter of the Pioneer 10 spacecraft with the planet Jupiter. The description of this encounter are shown in the detailed ephemerides prepared by NASA's Jet Propulsion Laboratory and Ames Research Center [1]. In this problem, solar gravity may be ignored, assuming that its effect will not modify the results [2]. The literature shows several applications of the Swing-By technique: the study of

Swing-By trajectories around Jupiter [3]; the design of missions using Swing-Bys [4]; optimization of multiple swing-bys around the Moon [5]; the numerical study of the Swing-By in three dimensions [6], the study of a close approach considering a planet and a cloud of particles [7]; a classification of trajectories making a Swing-By with the Moon [8]; the study of transfer orbits using those close approaches to gain energy [9][10][11]. This technique can be combined with gravitational capture (se references [12] and [13] for more details) to generate economical trajectories to the Moon.

It is known that, when in the neighborhood of a planet, a spacecraft in a orbit around the Sun experiences perturbations which depend on the relative velocity between the spacecraft and the planet and the distance separating the two at the point of the closest approach. If only the gravitational field of the planet affected the motion of the spacecraft, the vehicle would make it is approach along a hyperbolic path.

In this paper the maneuver is assumed to be performed in the Sun-Jupiter System. The motion

of the spacecraft near the close encounter with the planet will be studied. The spacecraft leaves the point A, passes by the point P (Fig. 1) and goes to the point B. Theses points are chosen in a such way that the influence of the Sun at those two points can be neglected and, consequently, the energy can be assumed to remain constant after B and before A. Thus, a series of two-body problems is used to generate analytical equations that describe the problem. In particular, the energy and the angular momentum of the spacecraft before and after this close encounter are calculated, to detect the changes in the trajectory during the close approach.

Finally, some numerical simulations are performed with several initial conditions. Two solutions are considered for the Swing-By: when the maneuver is performed behind the planet (solution 1) and when the maneuver is performed in front the planet (solution 2), to take into account the possibility that the particles crosses the line Sun-Planet between the primaries. The goal is to study the orbital change (energy, angular momentum, orbital elements) of the particle after some maneuvers with the desired planet and to know those values after the close approach in order to decrease the fuel expense in space missions. In particular, we are looking for geometries that allows multiple Swing-Bys without corrections maneuvers, where it is possible to study the lunar or planetary environment by having a spacecraft travelling in different areas without fuel consumption.

2 Mathematical model

The patched-conic approximation offers an efficient method for describing interplanetary orbits. By partitioning the overall orbit into a series of two-body orbits, it greatly simplifies mission analysis. The baseline of the patched conic approximation is that, in any space domain, the trajectory of a spacecraft is determined by the gravitational field that dominates the motion. The patched conic theory assumes that the dynamical system is given by two main bodies that are in circular orbits around their center of mass and the particle that is moving under the gravitational attraction of the two primaries. So, in this approach, this problem can be studied assuming a system formed by three bodies: the Sun as the main massive primary (M_1), a planet as secondary mass (M_2), that is orbiting the M_1 body, and a particle with infinitesimal mass (M_3) that remains orbiting the primary and makes a close approach with M_2. When M_3 enters in the sphere of influence of M_2, the orbital motion of M_3 around M_1

is modified. It is like having a series of single impulses with zero-cost to modify the orbit of the spacecraft. Fig. 1 explains the geometry involved in the close approach.
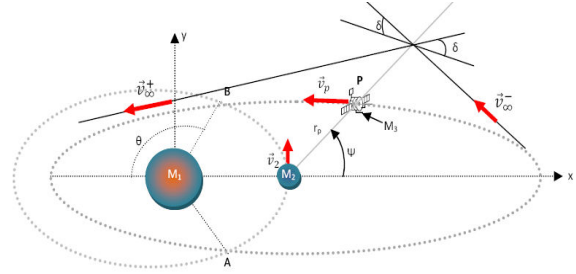

Fig. 1- **Swing-By variables**

Based in Fig.1 a set of variables can be used to identify one Swing-By trajectory,: $\vec{v}_2$ (velocity of M_3 with respect to M_1), $\vec{v}_\infty^-$ and $\vec{v}_\infty^+$ (velocity of M_3 with respect to M_2, before and after the maneuver in the referential frame), $\vec{v}_i$ and $\vec{v}_0$ (velocity of M_3 with respect to M_1, before and after the maneuver in referential frame), δ (half angle of the curvature), r_{ap} (the distance from the spacecraft to the center of M_2 at the closest approach) and ψ (angle of approach).

The velocity and orbital elements of M_3 are changed when it has a close approach with M_2. The orbital elements and energy before the encounter with the planet are obtained from the equations

$$a = \frac{r_a + r_p}{2} \tag{1}$$

$$e = 1 - \frac{r_p}{a} \tag{2}$$

$$E = -\frac{\mu_s}{2a} \tag{3}$$

$$C = \sqrt{\mu_s a(1 - e^2)} \tag{4}$$

where a =semi-major axis, e = eccentricity, E=energy, μ_s=Gm$_s$=1,33.10^{11}km^3/s^2.

It is possible to determine the velocity of the particle with respect to the Sun in the moment of the crossing with the planet's orbit and the true anomaly of that point.

$$|v_i| = \sqrt{\mu_s\left(\frac{2}{r_{sp}} - \frac{1}{a}\right)} \tag{5}$$

$$\theta = \cos^{-1}\left[\frac{1}{e}\left(\frac{a(1 - e^2)}{r_{sp}} - 1\right)\right] \tag{6}$$

where the parameter r_{sp} is the distance between the Sun and the planet. Eq. (6) given us two solutions (θ_A and θ_B). In this study we will consider the angle

θ_A. The next procedure is to calculate the angle between the inertial velocity of the particle and the velocity of the planet:

$$\gamma = \tan^{-1}\left[\left(\frac{e \sin\theta}{1 + e \cos\theta}\right)\right] \quad (7)$$

and the magnitude of the particle velocity with respect to the planet in the moment that the approach starts,

$$v_\infty = \sqrt{v_i^2 + v_2^2 - 2v_i v_2 \cos\gamma} \quad (8)$$

This paper considers two solutions assuming a close approach behind the planet (rotation of the velocity vector in the counter-clock-wise sense- ψ_1) and a close approach in front of planet (clock-wise sense- ψ_2) for the spacecraft around the Sun (Fig. 2). These two values are obtained from

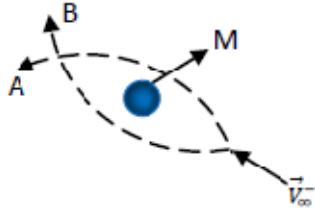$$\begin{aligned}\psi_1 &= 180^0 + \beta + \delta \\ \psi_2 &= 360^0 + \beta - \delta\end{aligned} \quad (9)$$

Fig. 2- Possible rotations of the velocity vector

where

$$\beta = \cos^{-1}\left[-\frac{v_i^2 - v_2^2 - v_\infty^{-2}}{2 \, v_2 \, v_\infty^-}\right]$$

$$\delta = \sin^{-1}\left[\frac{1}{1 + \frac{r_p v_\infty^2}{\mu_p}}\right] \quad (10)$$

μ_p is the gravitational constant of the planet. The next step is to determine the variations in energy and angular momentum from the equations [4]:

$$\Delta v = \vec{v}_0 - \vec{v}_i = 2|\vec{v}_\infty| \sin\delta \quad (11)$$
$$\Delta E = E_+ - E_- = -\vec{v}_2 \, \vec{v}_\infty \sin\delta \, \sin\psi \quad (12)$$
$$\Delta C = \frac{\Delta E}{\omega} \quad (13)$$

where ω is the angular velocity between the primaries, δ is the angle of deflection and E_-, E_+ are the energy before and after maneuver. After calculating the variations in energy and angular

momentum, the orbits are classified by: elliptic direct (negative energy and positive angular momentum), elliptic retrograde (negative energy and angular momentum), hyperbolic direct (positive energy and angular momentum) and hyperbolic retrograde (positive energy and negative angular momentum).

Finally, to obtain the semi-major axis and the eccentricity after the Swing-By, it is possible to use the equations

$$a = -\frac{\mu}{2E} \quad (14)$$

$$e = \sqrt{1 - \frac{C^2}{\mu.a}} \quad (15)$$

3 Simulations and Numerical Results

In this study some simulations will be performed to analyze the orbital variation of the spacecraft subject to a close approach with Jupiter under the "patched conics" model. It is assumed that the spacecraft is in orbit around the Sun with a given semi-major axis and eccentricity where the periapse distance (r_p) and the apoapse (r_a) of the orbit are assumed to be known. All the physical elements of the planets can be seen in Table 1. The patched conic model was implemented using the software Fortran. The energy, angular momentum and orbital elements have been analyzed for multiple Swing-Bys without correction maneuvers. The simulations are stopped when the energy (Eq.12) becomes positive.

Table 1- Physical elements of the Planet

Planet	Equatorial Radius (km)	Average distance to the Sun (10^6 km)	Orbital velocity (km/s)	μ=Gm (10^6 km^3/s^2)
Jupiter	*71370*	*778*	*13,1*	*126,0*
μ_{sol} =1,33 . 10^{11} km^3/s^2				

All simulations are performed with the following characteristics:
i)   The close approach will be at the point A (Fig. 1);
ii)   The Sun (or the other perturbations) does not affect the motion of the particle;
iii)   The energy can be assumed to remain constant after B and before A;

iv) The energy and angular momentum will be analyzed after and before the maneuver for several situations;

v) Solution 1 will be performed with the first maneuver behind the planet (considering ψ_1 as the angle of approach) and solution 2 considers the situation where the first maneuver is performed in front the planet (considering ψ_2 as the angle of approach).

The initial conditions can be seen in Table 2 and 3, and it is composed by: r_a (apoapsis distance), r_p (periapsis distance), a (semi-major axis), e (eccentricity), v (velocity), E (energy) and C (angular momentum). They are obtained from the initial orbit of the spacecraft around the Sun. The r_{ap} is the distance of the close approach between the particle and the planet. With the numerical algorithm available, the given initial conditions are varied in any desired range and the number of maneuver and their respective effects in the close approach in the orbit of the spacecraft are studied. The numerical results will be obtained in the point A (see Fig. 1) with rotation of the velocity vector in counter-clock-wise sense (solution ψ_1) and clock-wise sense (solution ψ_2).

Table 2- Initial conditions: Sun-Jupiter System

Simulation	r_a (10^8 km)	r_p (10^8 km)	r_{ap} (10^8 km)	a (10^8 km)	e	v (km/s)
1ᵃ	*10*	*3,5*	*1,5*	*6,75*	*0,48*	*12,03*

3.1 Sun-Jupiter System

In the results, the first column shows the results for the angle of approach ψ_1 (solution 1) and the second column shows the results for the angle of approach ψ_2 (solution 2).

Fig. 3 shows the number of maneuvers as a function of the periapsis distance (r_p), considering $r_a=10^9$ km and the angle of approach (ψ_1) in the elliptic initial orbit of the spacecraft around the Sun. In Fig. 4 the r_a distance was fixed. To understand better the importance of this parameter, we made simulations using several values for r_p and r_a. After performing a large number of simulations using different values for the periapsis and apoapsis distances, it is possible to study the effects of this parameter in the whole maneuver. Those results (Fig. 3) allow us to analyze the increase in the number of maneuvers for the range 3.10^9 km $\leq r_p \leq 7.10^9$ km. This range implies in regions with $0,1764 \leq e \leq 0,5385$ and $6,5.10^8$ km $\leq$ a $\leq 8,5.10^8$ km. This information is essential to analyze the orbital

characteristics of the particle before the Swing-By. It is visible a strong influence of the r_p and r_a in the whole mission. Some of these characteristics will be studied with detail in this paper.
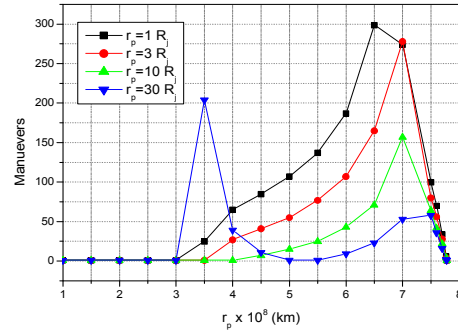

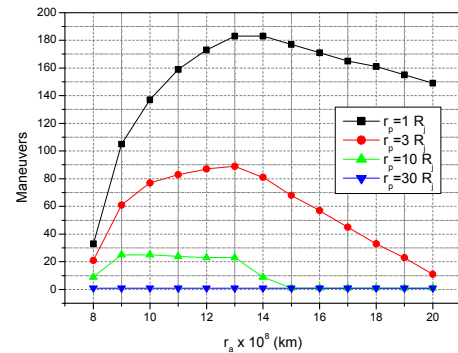Fig. 3- Maneuvers performed vs. periapsis distance (r_p), $r_a=10^9$ km and angle approach ψ_1.

Fig. 4- Maneuvers performed vs. apoapsis distance (r_a), $r_p=5,5.10^8$ km and angle approach ψ_1.

Figures 5 and 6 show the evolution of the amplitude of the energy, angular momentum, semi-major axis, eccentricity and velocity. The results show two types of maneuvers: the ones performed behind Jupiter (solution 1) and the ones performed in front of Jupiter (solution 2). The energy variation for these solutions is enough to increase the orbital elements of the particle. The moment that the particle escapes of the orbit can be easily seen in Fig. 6, in the plot of semi-major axis vs maneuver. The change of energy and angular momentum causes the existence of hyperbolic orbits ($\Delta E > 0$ and $\Delta C > 0$) and the semi-major axis variation leads to eccentricities that reaches a maximum value.
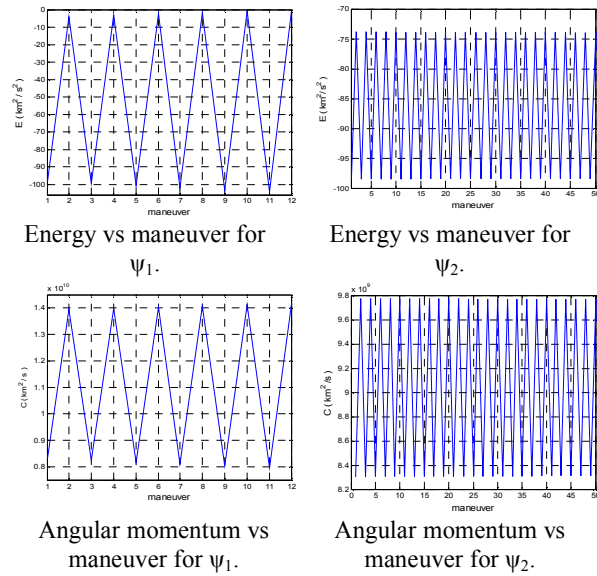
Energy vs maneuver for ψ_1.

Energy vs maneuver for ψ_2.

Angular momentum vs maneuver for ψ_1.

Angular momentum vs maneuver for ψ_2.

Fig. 5- Energy and angular momentum of the spacecraft after the Swing-Bys for $r_a=10^9$ km, $r_p=3,5.10^8$ km, $r_{ap}=1,5$ Rj, considering solution 1 (ψ_1) and solution 2 (ψ_2).

Semi-major axis vs maneuver for ψ_1.

Semi-major axis vs maneuver for ψ_2.

Eccentricity vs maneuver for ψ_1.

Eccentricity vs maneuver for ψ_2.

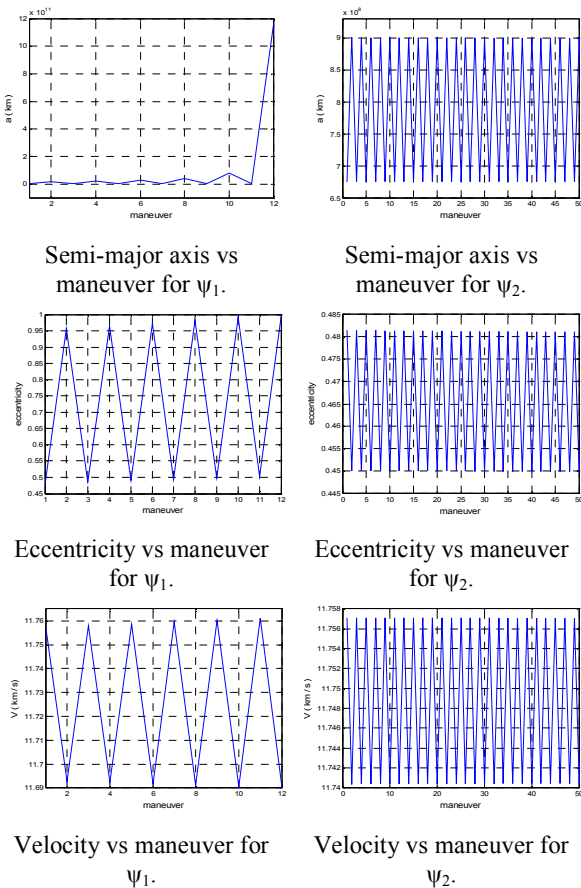Velocity vs maneuver for ψ_1.

Velocity vs maneuver for ψ_2.

Fig. 6- Semi-major axis, eccentricity and velocity of the spacecraft after the Swing-Bys for $r_a=10^9$ km, $r_p=3,5.10^8$ km, $r_{ap}=1,5$ Rj, considering solution 1 (ψ_1) and solution 2 (ψ_2).

In the solution 2 (Fig. 5), there is a minimal amplitude variation of energy that leads to the accomplishment of several maneuvers. This solution is important when we want the particle to remain for a long time in an elliptic orbit. Fig. 7 shows the energy variation as a function of the angle of approach that leads to several maneuvers due to the energy gain and energy loss involved. Near the values of 100 km^2/s^2 (for angle ψ_1) and -97,5 km^2/s^2 (for angle ψ_2) there are maneuvers with energy loss. In the vicinity of -2 km^2/s^2 (ψ_1) and -75 km^2/s^2 (ψ_2), there are maneuvers with energy gain. For those cases, when ψ_1 is around 405 degrees, there is the existence of parabolic orbits, because the angular momentum is positive and the eccentricity is equal to 1,0. An overview of the simulations for the maximum values of the amplitude can be seen in Table 3.
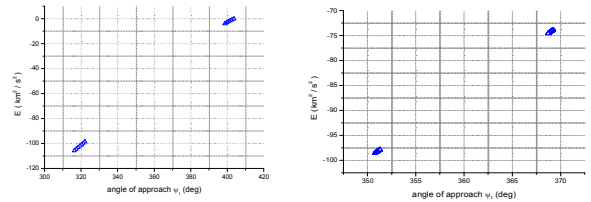

Fig. 7- Energy variation vs. angle of approach of the spacecraft for several orbits after the Swing-Bys for $r_a=10^9$ km, $r_p=3,5.\ 10^8$ km, $r_{ap}=1,5$ Rj.

Table 3- The maximum value of amplitude: energy, momentum, semi-major axis, eccentricity and velocity after the maneuver (Sun-Jupiter System).

simulation	Δa $(10^8$ km)	Δe	ΔE (km^2/s^2)	ΔC $(10^9$- $km^2/s)$	Δv (km/s)
Solution 1 (ψ_1)					
1ª	5,5	0,51	102	0,57	0,07
Solution 2 (ψ_2)					
1ª	2,7	0,03	21	0,15	0,020

The second simulation was performed considering the distance of close approach $r_{ap} = 30$ Rj (radius of Jupiter). These results can be seen in Fig. 8, where it shows that there are a strong dependence with r_{ap} and the initial conditions. Also in Fig. 8, for solution 1, it is visible the large orbital change after the first Swing-By maneuver.
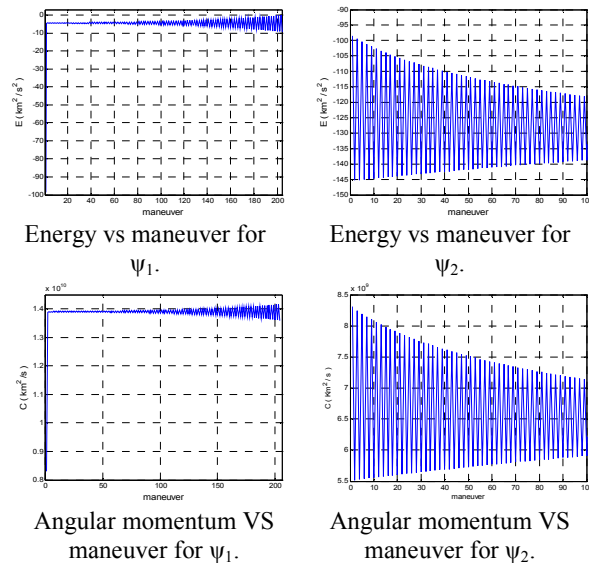
Energy vs maneuver for ψ_1.

Energy vs maneuver for ψ_2.

Angular momentum VS maneuver for ψ_1.

Angular momentum VS maneuver for ψ_2.

Fig. 8- Energy and angular momentum of the spacecraft orbits after the Swing-Bys for $r_a=10^9$ km, $r_p=3,5.10^8$ km, $r_{ap}=30$ Rj, considering solution 1 (ψ_1) and solution 2 (ψ_2).

4 Conclusion

This study was made to show the evolution of the amplitudes, the strong influence of the angle of approach and the choice of the periapsis position. A method to calculate the variations in semi-major axis, velocity, energy and angular momentum for the Swing-By was developed, based in the "patched-conic" approximation. Some numerical simulations are calculated for the Sun-Jupiter system, considering several initial conditions. A set of analytical equations is used to describe the Swing-By in two dimensions and to evaluate the variation in the orbital elements of the orbit of a spacecraft that is passing by the planet. Then, it is possible to compare two solutions to make an orbital maneuver, considering two angle of approaches: ψ_1 and ψ_2. The results showed that:

- The periapsis of the close approach has a strong influence in the maneuver and it can be used to obtain lower fuel consumption;
- The two solutions considered (ψ_1 and ψ_2), depending on the geometry of the encounter, have different behavior;
- Considering other cases, one has an increase in energy in the majority of the cases, generating hyperbolic orbits after the passage;
- One of them has a decrease in energy due to the passage in most of the cases (generating only elliptic orbits);
- The Solution 1 (ψ_1) has larger amplitudes than the Solution 2(ψ_2);

- It is possible to find useful sequences of Swing-Bys to study the space around a specific body.

References:
[1] Allen, J. A. V. Gravitational assist in celestial mechanics—a tutorial, *American Association of Physics Teachers*.Vol. 5, pp.448-4515, May 2003.
[2] Battin, Richard H. *An introduction to the mathematics and models of astrodynamics, American Institute of Aeronautics and Astronautics*, New York, New York, 1987.
[3] Broucke, R.A. & A.F.B.A. Prado (1993). "Jupiter Swing-By Trajectories Passing Near the Earth", *Advances in the Astronautical Sciences*, Vol. 82, No 2, pp. 1159-1176.
[4] Broucke, R. A. The Celestial mechanics of gravity assist, *In:AIAA/AAS Astrodynamics Conference*, Minneapolis, MN, August, 1988. (AIAA paper 88-4220).
[5] Dunham, D. & S. Davis (1985). "Optimization of a Multiple Lunar- Swing by Trajectory Sequence," *Journal of Astronautical Sciences*, Vol. 33, No. 3, pp. 275-288.
[6] Felipe, G. & A.F.B.A. Prado (2001). "Numerical study of three-dimensional close approach". *SBA Controle e automação,* Vol. 12, No. 3, pp. 197-220.
[7] Gomes, V. M., Prado, A.F.B.A. Swing-By Maneuvers for a Cloud of Particles with Planets of the Solar System. *WSEAS Transactions on applied and theoretical mechanic*s, Vol. 3, November, pp. 869-878, 2008.
[8] Prado, A.F.B.A. & R.A. Broucke (1995). "A Classification of Swing-By Trajectories using The Moon". *Applied Mechanics Reviews*, Vol. 48, No. 11, Part 2, November, pp. 138-142.
[9] Prado, A. F. B. A., Broucke, R. A. Transfer orbits in the Earth-Moon system using a regularized model. *Journal of Guidance, Control and Dyn*amics, v. 19, n.4, p.929-933, 1996.
[10] Prado AFBA, Broucke R Transfer Orbits in Restricted Problem, *Journal of Guidance Control and Dynamics*, v. 18, n. 3, p. 593-598, May-Jun. 1995
[11] Sukhanov AA, Prado AFBA, Constant tangential low-thrust trajectories near an oblate planet, *Journal of Guidance Control and Dynamics*, v. 24, n. 4, p. 723-731, Jul-Aug. 2001.
[12] Prado, AFBA, Numerical and analytical study of the gravitational capture in the bicircular problem, *Advances in Space Research*, v. 36, n. 3, p. 578-584, 2005.
[13] Prado AFBA Numerical study and analytic estimation of forces acting in ballistic gravitational capture, *Journal of Guidance Control and Dynamics*, v. 25, n. 2, p. 368-375, Mar-Apr. 2002.