

# Close Approaches for a Cloud of Particles with the Moon

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**Abstract:** - The goal of this research is to study close approaches between the Moon and a cloud of particles. The system is formed by two main bodies (the Earth and the Moon) and it is assumed that they are in circular orbits. The cloud of particles is moving under their gravitational attraction. The motion is assumed to be planar for all the particles and the dynamics given by the “patched-conic” approximation is used, which means that a series of two-body problems are used to generate analytical equations that describe the problem. The main objective is to understand the change of the orbit of this cloud of particles after the close approach with the Moon. It is assumed that all the particles that belong to the cloud have semi-major axis  $a \pm \Delta a$  and eccentricity  $e \pm \Delta e$  before the close approach with the Moon, such that the particles are inside a circle (in the semi-major axis-eccentricity plane) with center in the center of mass of the cloud. It is desired to know those values after the close approach.

**Key-Words:** - Astrodynamics, Orbital maneuvers, Swing-By, Gravity assisted maneuvers, Orbital motion.

## 1 Introduction

In the study of space missions, it is very important to consider the problem of performing optimization in fuel expenditure when making orbital maneuvers. References [1] to [5] show some of the techniques that can be used to solve this problem.

One of the alternative techniques that can be used with this objective is the close approach between a spacecraft and a planet. This is a very popular technique used to decrease fuel expenditure in space missions. This maneuver modifies the velocity, energy and angular momentum of a spacecraft. There are many important applications well known, like the Voyager I and II that used successive close encounters with the giant planets to make a long journey to the outer Solar System; the Ulysses mission that used a close approach with Jupiter to change its orbital plane to observe the poles of the Sun, etc.

In the present paper we study the close approach between a planet and a cloud of particles. It is assumed that the dynamical system is formed by two main bodies (the Earth and the Moon) that are in circular orbits around their center of mass and a cloud of particles that is moving under the gravitational attraction of the two primaries. The motion is assumed to be planar for all the particles and the dynamics given by the “patched-conic” approximation is used, which means that a series of two-body problems are used to generate analytical equations that describe the problem. The standard canonical system of units is used and it implies that

the unit of distance is the distance between the two primaries and the unit of time is chosen such that the period of the orbit of the two primaries is  $2\pi$ .

The goal is to study the change of the orbit of this cloud of particles after the close approach with the planet. It is assumed that all the particles that belong to the cloud have semi-major axis  $a \pm \Delta a$  and eccentricity  $e \pm \Delta e$  before the close approach with the Moon, in the form shown in the plots. It is desired to know those values after the close approach.

Among the several sets of initial conditions that can be used to identify uniquely one swing-by trajectory, a modified version of the set used in the papers written by [6], [7] and [8] is used here. It is composed by the following three variables: 1)  $V_p$ , the velocity of the spacecraft at periape of the orbit around the secondary body; 2) The angle  $\psi$ , that is defined as the angle between the line M1-M2 (the two primaries) and the direction of the periape of the trajectory of the spacecraft around M2; 3)  $r_p$ , the distance from the spacecraft to the center of M2 in the moment of the closest approach to M2 (periape distance). The values of  $V_p$  and  $\psi$  are obtained from the initial orbit of the spacecraft around the Sun using the “patched-conics” approximation and  $r_p$  is a free parameter that is varied to obtain the results.

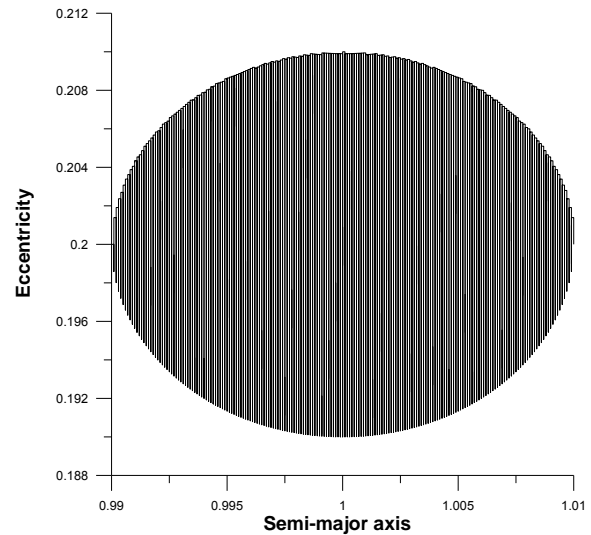
## 2 Review of the Literature for the Swing-By

The literature shows several applications of the swing-by technique. Some of them can be found in Swenson [9], that studied a mission to Neptune using

swing-by to gain energy to accomplish the mission; [10], that made a similar study for a mission to Pluto; [11], that formulated a mission to study the Earth's geomagnetic tail; [12], [13] and [14], that planned the mission ISEE-3/ICE; [15], that made the first studies for the Voyager mission; [16], that design a mission to flyby the comet Halley; [17], [18] that studied multiple flyby for interplanetary missions; [19] and [20] that design missions with multiple lunar swing-bys; [21], that studied the effects of the atmosphere in a swing-by trajectory; [22], that used a swing-by in Venus to reach Mars; [23], that studied numerically a swing-by in three dimensions, including the effects in the inclination; [24], that considered the possibility of applying an impulse during the passage by the perapsis; [25], that classified trajectories making a swing-by with the Moon. The most usual approach to study this problem is to divide the problem in three phases dominated by the "two-body" celestial mechanics. Other models used to study this problem are the circular restricted three-body problem (like in [26]) and the elliptic restricted three-body problem ([27]).

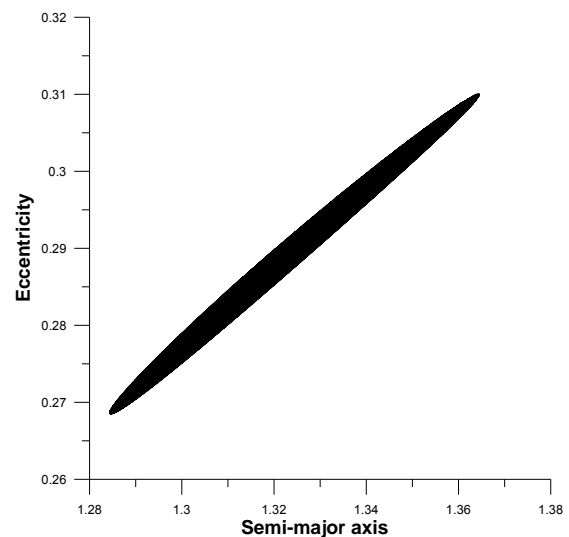
### 3 Orbital Change of a Cloud of Particles

The algorithm described in [28] can be applied to a cloud of particles passing close to the Moon. The idea is to simulate a cloud of particles that have orbital elements given by:  $a \pm da$  and  $e \pm de$ , as shown in the plots for the initial conditions. The goal is to map this cloud of particles to obtain the new distribution of semi-major axis and eccentricities after the swing-by. Figures 1 to 3 show some results. Those figures allow us to get some conclusions. The solution called "Solution 1" has a larger amplitude than the Solution 2 in both orbital elements, but it concentrates the orbital elements in a line, while the so called "Solution 2" generates a distribution close to an ellipse. The area occupied by the points is smaller for "Solution 1". The effect of increasing the periapsis distance is to generate plots with larger amplitudes, but with the points more concentrated, close to a straight line.



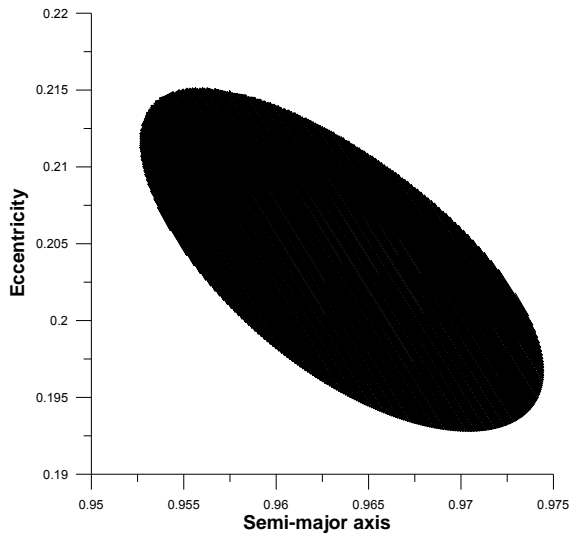
Eccentricity vs. Semi-major axis before the Swing-By

**Fig. 1(a) – Swing-by for a cloud of particles with for  $rp = 1.1 R_M$ .**



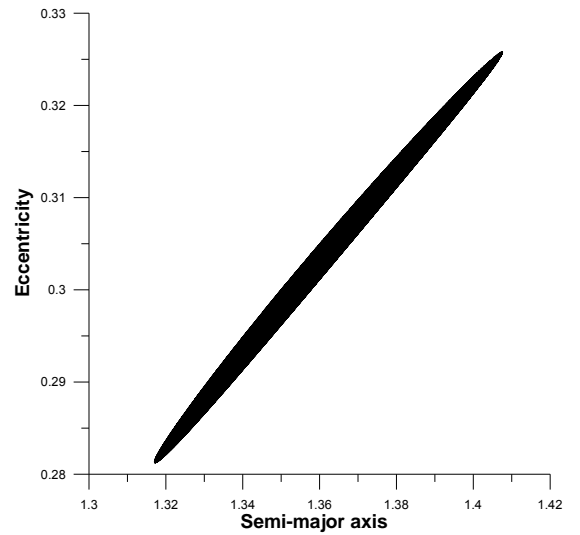
Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

**Fig. 1(b) – Swing-by for a cloud of particles for  $rp = 1.1 R_M$ .**



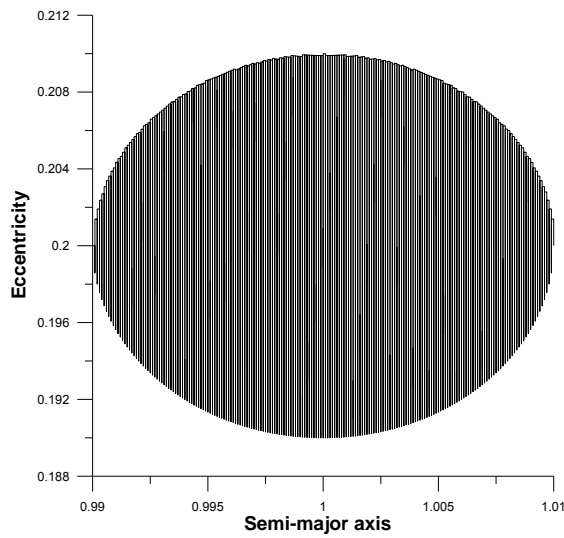
Eccentricity vs. Semi-major axis after  
Swing-By for "Solution 2"

**Fig. 1(c) – Swing-by for a cloud of particles with  
Jupiter for  $r_p = 1.1 R_M$ .**



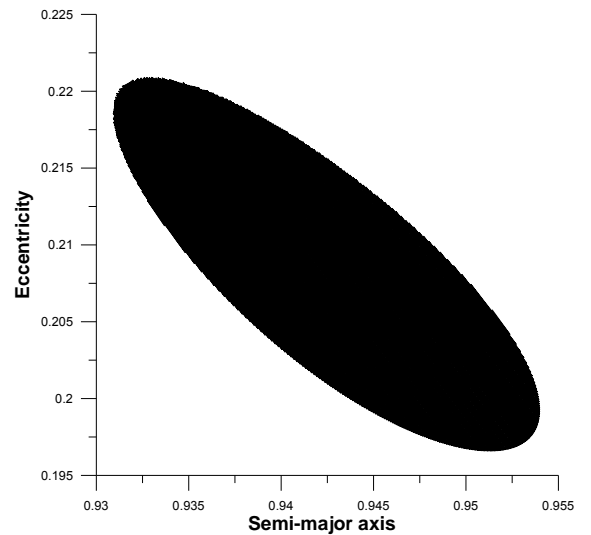
Eccentricity vs. Semi-major axis after  
Swing-By for "Solution 1"

**Fig. 2(b) – Swing-by for a cloud of particles for  $r_p = 1.5 R_M$ .**



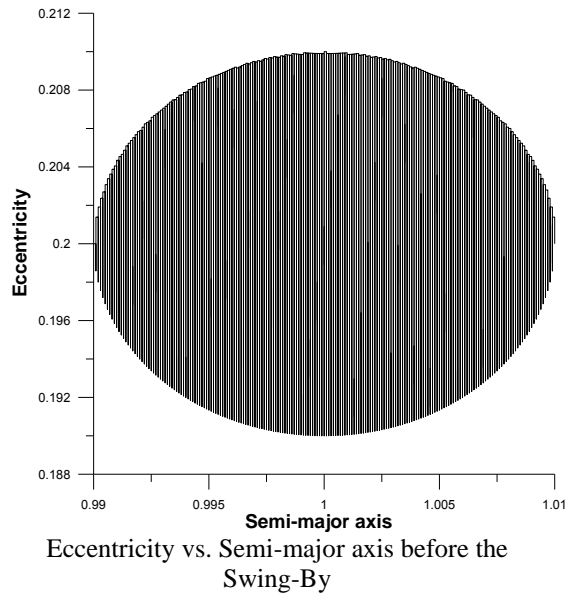
Eccentricity vs. Semi-major axis before  
the Swing-By

**Fig. 2(a) – Swing-by for a cloud of  
particles with for  $r_p = 1.5 R_M$**

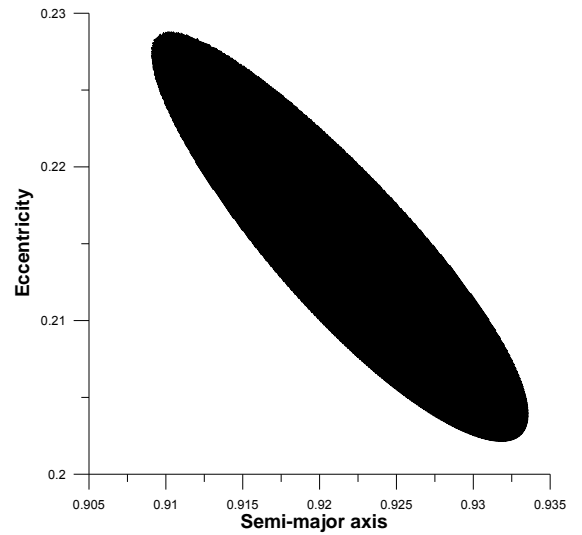


Eccentricity vs. Semi-major axis after  
Swing-By for "Solution 2"

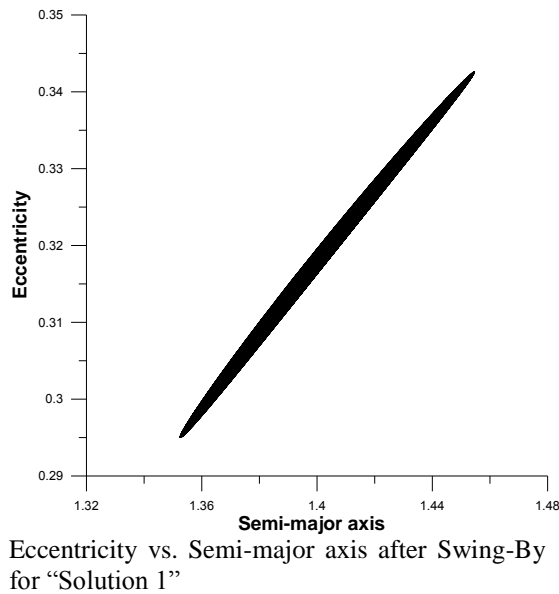
**Fig. 2(c) – Swing-by for a cloud of particles with  
Jupiter for  $r_p = 1.5 R_M$ .**



**Fig. 3(a) – Swing-by for a cloud of particles with for  $rp = 2.0 R_M$**

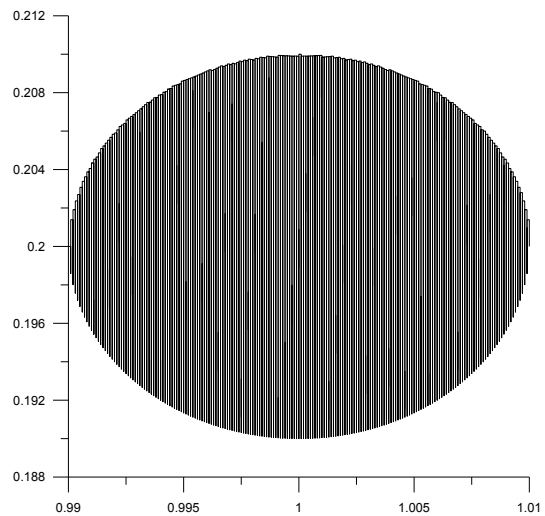


**Fig. 3(c) – Swing-by for a cloud of particles with Jupiter for  $rp = 2.0 R_M$ .**

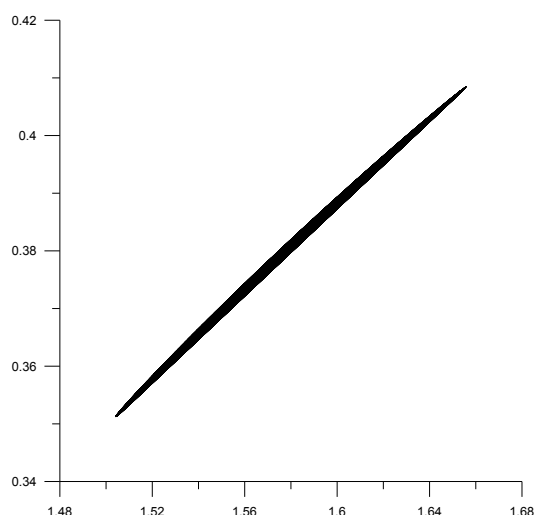


**Fig. 3(b) – Swing-by for a cloud of particles for  $rp = 2.0 R_M$ .**

To understand better the importance of the paerapsis distance, we also made simulations using the value of  $rp = 5.0 R_J$ . The results are shown below.

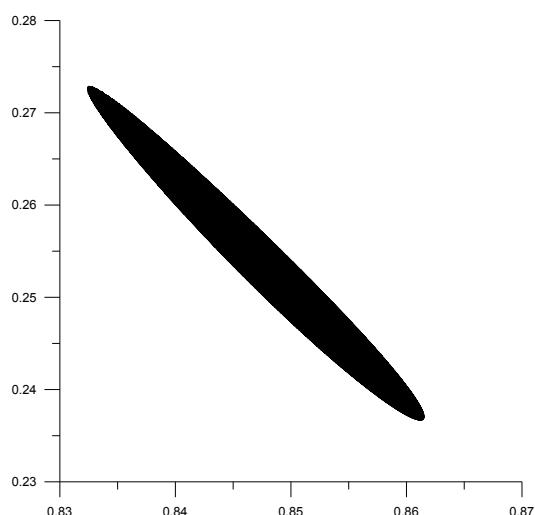


**Fig. 4(a) – Swing-by for a cloud of particles for  $rp = 5.0 R_J$ .**



Eccentricity vs. Semi-major axis after  
Swing-By for "Solution 1"

**Fig. 4(b) – Swing-by for a cloud of particles for  $r_p = 5.0 R_J$ .**



Eccentricity vs. Semi-major axis after  
Swing-By for "Solution 2"

**Fig. 4(c) – Swing-by for a cloud of particles for  $r_p = 5.0 R_J$ .**

## 5 Conclusion

The results showed before can give some conclusions, as already explained in the text. "Solution 1" has amplitude that is larger than Solution 2 in semi-major axis and eccentricity, but there is a concentration of the orbital elements in a line. "Solution 2" generates an elliptical distribution

of the orbital parameters. The area generated by the trajectories is larger for "Solution 2". The effect of increasing the periapse distance is to increase the amplitudes and to concentrate the points. In general, those results can be used to understand better the effects of the periapsis distance when a passage of a cloud of particles near a massive celestial body occurs.

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