Zero-temperature transition and correlation-length exponent of the frustrated XY model on a honeycomb lattice

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Phase coherence and vortex order in the fully frustrated *XY* model on a two-dimensional honeycomb lattice are studied by extensive Monte Carlo simulations using the parallel tempering method and finite-size scaling. No evidence is found for an equilibrium order-disorder or a spin/vortex-glass transition, suggested in previous simulation works. Instead, the scaling analysis of correlations of phase and vortex variables in the full equilibrated system is consistent with a phase transition where the critical temperature vanishes and the correlation lengths diverge as a power law with decreasing temperatures and corresponding critical exponents ν_{ph} and ν_v. This behavior and the near agreement of the critical exponents suggest a zero-temperature transition scenario where phase and vortex variables remain coupled on large length scales.

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I. INTRODUCTION

Two-dimensional frustrated *XY* models,[1,2] in which vortices form a dense incommensurate lattice, have attracted considerable interest as a possible two-dimensional vortex glass without quenched disorder[3–7] or a structural glass of supercooled liquids.[8,9] In superconducting systems with pinning described by *XY* models, frustration can be introduced by applying an external magnetic field as in periodic arrays of Josephson junctions,[10–12] superconducting wire networks,[13–15] and superconducting thin films with a periodic pattern of nanoholes.[16] The frustration parameter f sets the average density of vortices in the lattice of pinning centers and can be tuned by varying the strength of the external field.[2] Depending on the structure of the lattice of pinning centers and the value of f, a commensurate vortex lattice is favored in the ground state, which leads to discrete symmetries in addition to the continuous symmetry of the phase variables characterizing the superconducting order parameter. In this case, the phase transitions and resistive behavior of the system are reasonably well understood for simple low-order commensurate phases such as $f = 1/2$ on a square lattice[2] and $f = 1/3$ on a triangular lattice[17] of pinning centers. However, when the vortex lattice is incommensurate with the pinning centers, both the nature of the equilibrium phase transition and of the low-temperature state in the thermodynamic limit is less clear. This is the case of Josephson-junction arrays[10,12] and superconducting wire networks[13,14] on a square lattice, described by the frustrated *XY* model with irrational f, which has been extensively studied by various methods.[3,4,6–8,18,19] Another interesting physical realization but much less investigated so far should occur in Josephson-junction arrays and superconducting wire networks[15] on a honeycomb lattice, and superconducting films with a triangular pattern of nanoholes,[16] which can be described by the fully frustrated *XY* model ($f = 1/2$) on a honeycomb lattice.

In early Monte Carlo (MC) simulations of the fully frustrated *XY* model on a honeycomb lattice,[17] a phase transition at a nonzero temperature in the Koterlitz-Thouless (KT) universality class was suggested, based on the saturation of the specific-heat peak with increasing system sizes and the apparent jump in the helicity modulus. On the other hand, calculations of the the spin-glass order parameter by MC simulations[5] suggested that instead of an order-disorder transition, a spin-glass transition should take place approximately at the same temperature. In contrast, from MC simulations of a similar model in the vortex representation,[20] it was argued that only a crossover region rather than an equilibrium phase transition should occur at any nonzero temperature, since the energy of domain-wall excitations which disorder the ground state was found to approach a finite constant for increasing system sizes. However, the free energy of domain-wall excitations in the frustrated *XY* model obtained analytically in the phase representation[21] was found to increase linearly with the system length, but with an extremely small numerical coefficient. As a consequence, although vortex ordering could be possible in the thermodynamic limit, domain-wall excitations would only be negligible for system sizes which are much beyond the ones studied numerically or even experimentally. It is unclear whether the behavior observed in the earlier numerical works[5,17] could be a signature of such vortex ordering or the effect of slow dynamics which prevents observing the true equilibrium behavior. Given these conflicting results, it should be of interest to further investigate the fully frustrated *XY* model using a MC method which can ensure full equilibration and a scaling analysis of both phase and vortex correlations.

In this work we study phase coherence and vortex order in the fully frustrated *XY* model on a honeycomb lattice, using the parallel-tempering method (exchange MC method)[22,23] to obtain equilibrium configurations of the system. This method has been shown to reduce significantly the long equilibration times in glassy systems.[22,24] To study the equilibrium phase transitions we use numerical data in the temperature regime in which full equilibration can be insured and employ a scaling analysis of the correlation lengths of the phase and vortex variables to extrapolate to the low-temperature and large-system limits. No evidence of an equilibrium order-disorder phase transition or even a spin/vortex-glass transition is found at nonzero temperatures. Our results, however, are consistent with an equilibrium zero-temperature transition, where the critical temperature vanishes ($T_c = 0$) and the correlation lengths diverge as a power law with decreasing temperatures and corresponding critical exponents ν_{ph} and ν_v. This transition has important consequences for the resistivity behavior and current-voltage scaling[6,31,32] at low temperatures in the superconducting systems described by the frustrated *XY* model. Moreover, the near agreement of the critical exponents estimated numerically suggests a $T_c = 0$ transition scenario, where phase and vortex variables remain coupled on large length scales. This is in contrast with the $T_c = 0$ transition in the frustrated *XY* model on a square lattice at irrational frustration, where a decoupled scenario was found recently.[7]

II. MODEL AND MONTE CARLO SIMULATION

We consider a uniformly frustrated *XY* model described by the Hamiltonian[2]

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - A_{ij}), \tag{1}$$

where θ_i is a phase variable defined at the sites i of a two-dimensional honeycomb lattice (Fig. 1), representing the local angle of the *XY* spin with an arbitrary fixed direction. $J > 0$ is a uniform ferromagnetic coupling and A_{ij} is a gauge-invariant quantity constrained to be $\sum_{ij} A_{ij} = 2\pi f$ around each plaquette of the lattice. The parameter f controls the frustration of the system. For the fully frustrated case considered here, $f = 1/2$. In the calculations we choose a gauge where $A_{ij} = 2\pi f n_i/2$ on the bonds along the horizontal rows numbered by the integer n_i and $A_{ij} = 0$ on the vertical bonds of the lattice.

As a model of an array of superconducting grains coupled by Josephson junctions, the phase θ_i in Eq. (1) corresponds to the phase of the superconducting order parameter of the grains, J is the Josephson coupling between grains, and A_{ij} is the line integral of the vector potential $\vec{A}$ due to an external magnetic field $\vec{B} = \nabla \times \vec{A}$ applied perpendicular to the array. The magnetic flux in each plaquette in units of the flux quantum $\Phi_o = hc/2e$ can be written as $2\pi f$ with the frustration parameter f corresponding to the number of flux quantum per plaquette.

In the Monte Carlo simulations we use the parallel tempering method[22] to obtain equilibrium configurations. In this method, many replicas of the system with different temperatures are simulated simultaneously and the corresponding configurations are allowed to be exchanged with a probability satisfying detailed balance. The exchange process allows the configurations of the system to explore the temperature space, being cooled down and warmed up, and the system can escape more easily from metastable minima at low temperatures. Full equilibration can be insured within reasonable computer time in systems of sufficiently small sizes.[22] Without the replica exchange step, the method reduces to conventional MC simulations at different temperatures. We performed MC simulations using the heat-bath algorithm for each replica, simultaneously and independently, for a few MC passes. Periodic boundary conditions were used on lattices with rhombic geometry (Fig. 1) of side L, containing $N = 2L^2$ sites. Exchange of pairs of replica configurations at temperatures T_i and T_j and energies E_i and E_j is attempted with probability $\min(1, \exp(-\Delta))$, where $\Delta = (1/T_i - 1/T_j)(E_j - E_i)$, using the Metropolis scheme. The equilibration time to reach thermal equilibrium can be measured as the average number of MC passes required for each replica to travel over the whole temperature range. We used typically 4×10^6 MC passes for equilibration with up to 120 replicas and 1.6×10^7 MC passes for calculations of average quantities.

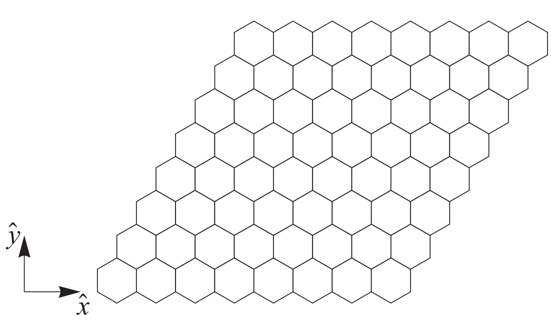

FIG. 1. Honeycomb lattice and the coordinate axes.

III. CORRELATION LENGTH AND SCALING ANALYSIS

For the study of phase coherence, we consider the overlap order parameter[33] of the phase variables defined as $q_{ph}(j) = \exp(i\theta_j^1 - i\theta_j^2)$, where 1 and 2 denote two thermally independent copies of the system, with the same parameters J and f in the model of Eq. (1). At high temperatures, where each copy is thermally disordered, the correlation function

$$C_{ph}(r) = \frac{1}{N} \sum_j \langle q_{ph}(j) q_{ph}(j+r) \rangle \tag{2}$$

is short ranged, decaying exponentially with r, while at low temperatures it is long ranged if an ordered phase or a glassy phase exits. The latter possibility was suggested in Ref. 5 based on MC simulations and the glass phase was characterized by the behavior of the Edwards-Anderson order parameter q_{EA}, which corresponds here to $\langle \sum_j q_{ph}(j)/N \rangle$.

The correlation length in the finite system $\xi_{ph}(L,T)$ can be obtained as[24,25]

$$\xi_{ph}(L,T) = \frac{1}{2\sin(k_o/2)} \left(\frac{S_{ph}(0)}{S_{ph}(k_o)} - 1 \right)^{1/2}, \tag{3}$$

where $S_{ph}(k)$ is the Fourier transform of $C_{ph}(r)$ and k_o is the smallest nonzero wave vector in the finite system. This expression for the correlation length $\xi(L,T)$, which is both temperature and size dependent, can be obtained from the correlation length $\xi(T)$ in the large system limit, $\xi(T)^2 = -\frac{1}{S(k)} \frac{\partial S(k)}{\partial k^2}|_{k=0}$, as a finite-difference approximation taking also into account the lattice periodicity.[24] Thus, $\xi(L,T)$ tends to $\xi(T)$ for large L, when correlations are short ranged. If there is long-range order, $\xi(L,T) \sim L^{1+d/2}$, while if there is only algebraic order $\xi(L,T) \sim L$. Note that the correlation length defined in Eq. (3), in terms of the overlap order parameter $q_{ph}(j)$, may have a different magnitude from the correlation length defined in terms of a single copy of the system $\exp(i\theta_J^1)$. However, they should have the same leading scaling behavior near the transition.

Analogous expressions are used to determine the correlation length for vortex variables $\xi_v(L,T)$,

$$\xi_v(L,T) = \frac{1}{2\sin(k_o/2)}\left(\frac{S_v(0)}{S_v(k_o)} - 1\right)^{1/2}, \quad (4)$$

in terms of the overlap order parameter $q_v(p) = v_p^1 v_p^2$ of the net vorticity $v_p = n_p - f$. The vorticity n_p is defined as $n_p = \sum_{ij}(\theta_i - \theta_j - A_{ij})/2\pi$, where the summation is taken around the elementary plaquette located at sites p of the dual lattice, formed by plaquette centers, and the gauge-invariant phase difference is restricted to the interval $[-\pi,\pi]$. For the fully frustrated case, $f = 1/2$, the vortex variables $v_p = \pm 1/2$ are Ising-like variables and equivalent to the chirality variables originally introduced by Villain.[1]

Finite-size scaling can be used to extrapolate the behavior of the system to the large-system limit and temperatures near the transition. In the scaling analysis of the correlation length,[24] we consider the dimensionless ratio $\xi(L,T)/L$ which, for a continuous transition, should satisfy the scaling form

$$\xi(L,T)/L = G((T - T_c)L^{1/\nu}), \quad (5)$$

where T_c is the critical temperature and ν is the critical exponent of the power-law divergent correlation length $\xi(T) \propto |T - T_c|^{-\nu}$. The scaling function $G(x)$ has the properties $G(0) = C$, a constant, and $G(x) \to x^{-\nu}$ as $x \to \infty$. This scaling form implies that data for the scaled correlation length $\xi(L,T)/L$ as a function of temperature, for different system sizes L, should come together for decreasing temperatures and cross at the same temperature $T = T_c$. In addition, the data should splay out for different system sizes with slopes determined by the critical exponent ν. If the data satisfy the scaling form of Eq. (5) then we can infer that the correlation length diverges as a power law $\xi(T) \propto |T - T_c|^{-\nu}$ and estimate the critical exponent ν. A very large value of ν would be an indication that the correlation length may diverge exponentially, $\xi(T) \propto e^{c/|T-T_c|^{-\nu'}}$, rather than as a power law. In particular, for the standard (unfrustrated) *XY* model, which is critical at temperatures equal and below T_c, curves of $\xi(L,T)/L$ as function of decreasing temperatures for different system sizes merge[24] near T_c and remain L-independent for $T < T_c$.

IV. RESULTS AND DISCUSSION

We study the behavior of the correlations of phase and vortex variables *in thermal equilibrium*, which implies that we should only use the numerical data obtained from the MC simulations in the temperature range where full equilibration is achieved. To check equilibration, we follow the trajectory in the temperature space of a replica starting at the highest temperature, where the system equilibrates fast even without the replica exchange process.[7] In equilibrium, this replica should be able to explore the whole temperature range. For the system sizes studied, $L \geqslant 24$, equilibration was only possible for temperatures above $T_f \approx 0.11$, despite the improvement of the parallel tempering method. Below T_f, the configurations of the replicas at different temperatures cannot be warmed up and cooled down. Thus T_f can be regarded as a freezing temperature, below which the system remains trapped in metastable configurations within the available time scale of the present simulation. It is interesting to note that T_f agrees with the apparent glass transition temperature $T_g = 0.11$ observed in earlier MC simulations.[5] However, as will be shown in the following, T_f does not correspond to the critical temperature of an equilibrium glass transition. Instead, it should be regarded as the characteristic temperature of a dynamical freezing transition.

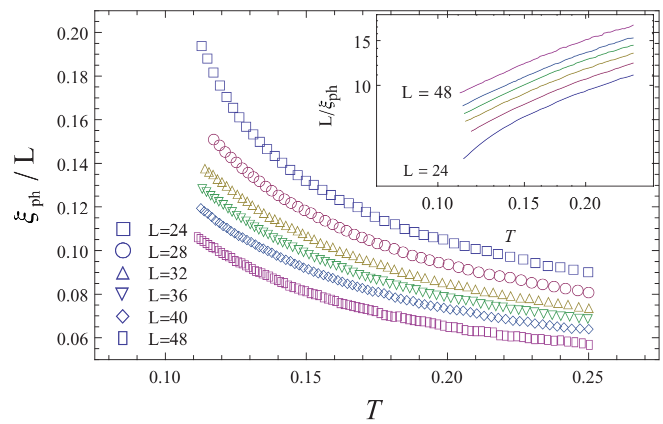

FIG. 2. (Color online) Scaled correlation length of phase variables ξ_{ph}/L for different system sizes L. Inset: Log-log plot of L/ξ_{ph} versus T for the corresponding system sizes L.

Figures 2 and 3 show the temperature dependence of the scaled correlation length in the $\hat{x}$ direction for phase variables, ξ_{ph}/L, and for vortex variables, ξ_v/L, in the temperature range $(T \geqslant T_f)$ where full equilibration was possible and for different system sizes. Both quantities increase faster on lowering the temperature as the system size L increases indicating a divergent length scale for decreasing temperatures. However, for fixed temperature they decrease with L even at the lowest available temperature. Therefore, the curves for the different system sizes do not cross or merge at a common temperature, as would be expected from the scaling form of Eq. (5), if a transition occurs in this temperature range. This is more evident in the insets of Figs. 2 and 3, where lines joining the data for different system sizes are presented as a log-log plot of $L/\xi(L,T)$ versus T.

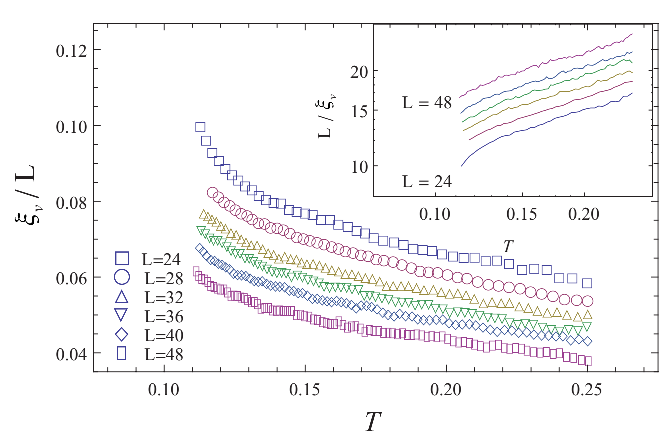

FIG. 3. (Color online) Scaled correlation length of vortex variables ξ_v/L for different system sizes L. Inset: Log-log plot of L/ξ_v versus T for the corresponding system sizes L.

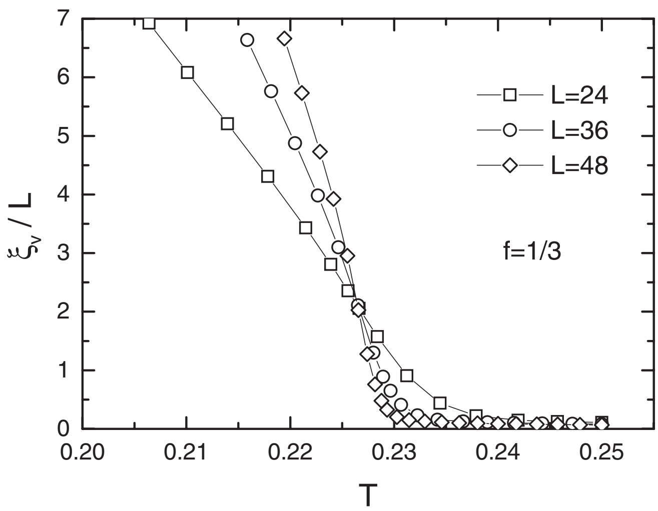

FIG. 4. Scaled correlation length of vortex variables ξ_v/L for different system sizes L, when the frustration parameter is $f = 1/3$.

For the sake of comparison to the behavior of the vortex correlation for the fully frustrated case ($f = 1/2$) in Fig. 3, we show the results of additional calculations when the frustration parameter is $f = 1/3$, in Fig. 4. In this case, a hexagonal vortex pattern commensurate with the honeycomb dual lattice is possible in the ground state and an order-disorder phase transition is expected at finite temperature from symmetry arguments. From the earlier work,[17] this transition takes place at a critical temperature $T_c \sim 0.23$. Indeed, the curves in Fig. 4 cross for different system sizes at a common temperature $T_c = 0.226(1)$, which is compatible with this estimate.

For the fully frustrated case, $f = 1/2$, the lack of intersection of the curves of $\xi_{\rm ph}/L$ and ξ_v/L for different system sizes at a common temperature in Figs. 2 and 3 suggests that phase coherence and vortex order, or even glasslike order, can only occur at $T \ll T_f$, which is not accessible in our calculations, or else only at $T = 0$. The latter scenario corresponds to a phase transition where $T_c = 0$ and the correlation length $\xi(T)$ is finite at any nonzero T but diverges when approaching $T = 0$. To verify the possibility of such zero-temperature transition, we first consider the scaling of the total correlation function χ defined as $\chi_{\rm ph} = \sum_r C_{\rm ph}(r)$ and $\chi_v = \sum_r C_v(r)$ for phase and vortex variables, respectively. In the absence of finite-size effects, χ should diverge as[26,27]

$$\chi = \frac{A}{(T - T_c)^\gamma} + B, \tag{6}$$

where A and B are constants and γ is a critical exponent. B represents possible background corrections to scaling. Using data for a large system size, where finite-size effects are negligible, we can then determine T_c from the best numerical fit according to this scaling form. Using a least-squares fit[28] we have obtained the error estimate $\sigma(T_c)$ when fitting $\ln(\chi - B)$ against $\ln(T - T_c)$ to a straight line, assuming different values of T_c. The error estimate is defined as

$$\sigma^2 = \frac{1}{N_p}\sum_{i=1}^{N_p}[y_i - f(x_i)]^2, \tag{7}$$

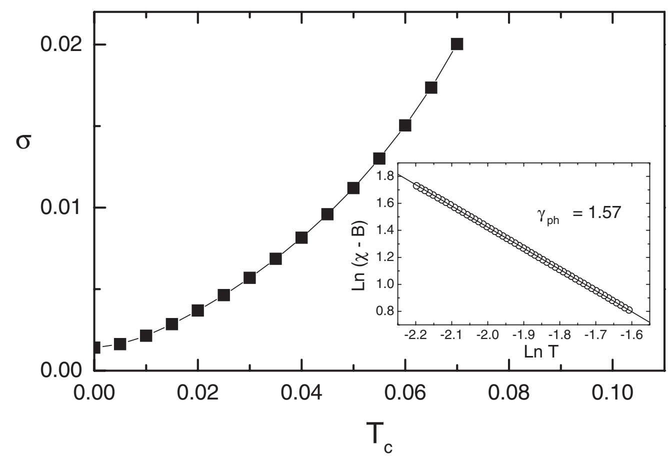

FIG. 5. Variation of the fitting error σ against T_c, for the total phase correlation $\chi_{\rm ph}$ according to Eq. (6), with $B = 2.6$. Inset: log-log fit assuming $T_c = 0$.

where N_p is the number of data points $\{x_i, y_i\}$ and $f(x_i)$ are the values of the fitting function at the corresponding data points. Figures 5 and 6 show the dependence of the error σ on T_c, for phase and vortex variables, respectively, for $L = 40$. In both cases, $T_c = 0$ gives the lowest error, which suggests that a zero-temperature transition is possible. The insets in the figures show the corresponding best fits, which provide estimates of the critical exponents $\gamma_{\rm ph} = 1.57(1)$ and $\gamma_v = 1.14(1)$.

The $T_c = 0$ scenario can be further verified from the finite-size scaling analysis of the correlation length $\xi(L,T)$. In this case, the data for $\xi(L,T)$ should satisfy the finite-size scaling form of Eq. (5) with $T_c = 0$. The best data collapse is obtained by adjusting a single parameter, ν, providing an estimate of this critical exponent. We use the following procedure, which is a simplified version of more general methods of measuring data collapse.[29,30] The standard finite-size scaling expansion near T_c,

$$G(x) = a_o + a_1 x + a_2 x^2 + \cdots, \tag{8}$$

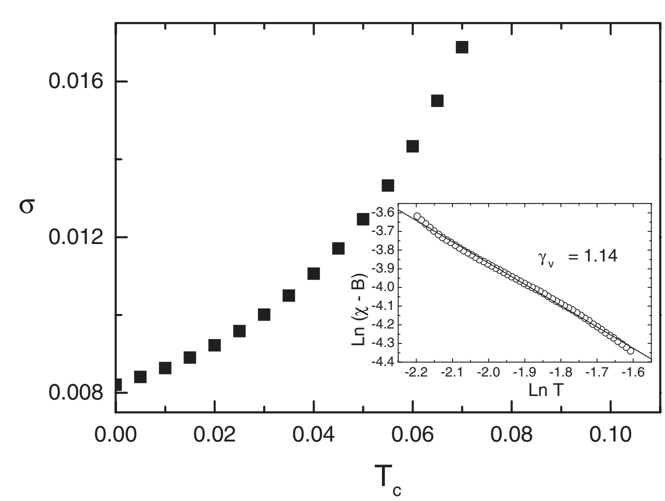

FIG. 6. Variation of the fitting error σ against T_c, for the total vortex correlation χ_v according to Eq. (6), with $B = 0.04$. Inset: log-log fit assuming $T_c = 0$.

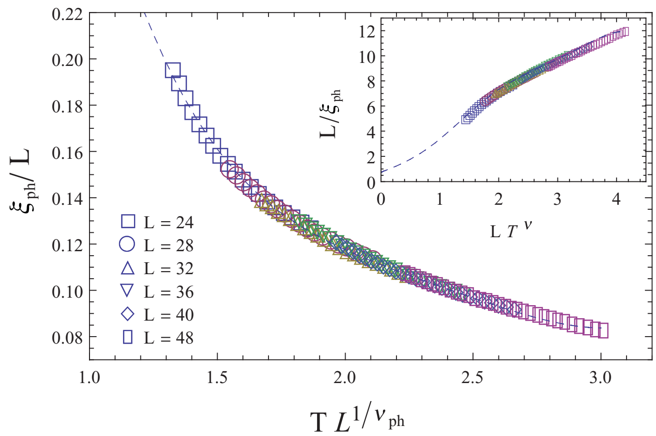

FIG. 7. (Color online) Scaling plot according to Eq. (5), assuming $T_c = 0$, for the phase correlation length ξ_{ph}, giving the estimate $\nu_{ph} = 1.29(15)$. Dashed line is a fit to the data with Eq. (8). Inset: Replot of data collapse in terms of the variable $LT^{\nu_{ph}}$.

truncated to low order $[O(x^5)]$, is used as a smooth interpolation function, which is convenient both for numerical data fitting and for providing a measure of the data collapse as the error defined by Eq. (7). Then we determine ν from the best least-squares fit of $\xi(L,T)/L$ to this function with $x = TL^{1/\nu}$. Data for the largest temperatures are successively removed from the data collapse if this leads to a decrease of the fitting error, since in this case such data are presumably outside the asymptotic scaling regime where Eq. (5) applies. The best data collapse is obtained for temperatures $T < 0.16$. Figure 7 shows that indeed the data for the phase variables ξ_{ph} satisfy the scaling form with an exponent $\nu_{ph} = 1.29(15)$. Similarly, for the vortex variables, we obtain from the data collapse in Fig. 8 the critical exponent $\nu_v = 1.03(25)$, although the data collapse is not as good. These exponents agree with each other within the estimated errors. It should be noted, however, that the scaling behavior in Eq. (5) has only been verified here in a limited range of temperatures since there is no available data below T_f, due to the lack of equilibration. This can be seen more clearly by replotting the data collapse as $L/\xi(L,T)$ versus $y \equiv LT^{\nu}$, as presented in the insets of Figs. 7 and 8. Indeed, $L/\xi(L,T)$ tends to a linear behavior for large y as expected but the finite limit for $y \to 0$ cannot be explicitly verified.

The zero-temperature transition, which was inferred here from the behavior of correlations at finite temperatures as discussed above, is also consistent with the ground-state properties of the model. As first pointed out by Shih and Stroud,[17] the ground state is highly degenerate with an infinite number of different vortex configurations. Therefore, correlations at zero temperature, obtained by averaging over the different configurations, can decay to zero for large distances. One possibility is that such decay obeys a power law. The triangular antiferromagnetic Ising model,[34] for example, displays such behavior. Another possibility is that correlations decay exponentially as in the antiferromagnetic Ising model on a kagome lattice.[35] The $T_c = 0$ scenario for the present model favors a power-law decay of vortex correlations. In fact, the correlation function exponent η_v associated with the transition can be obtained from the scaling law $\gamma = \nu(2-\eta)$ and the above estimates of γ_v and ν_v, giving $\eta_v = 0.90$, which is greater than zero implying a decay of the correlation as $C_v(r) \propto r^{-\eta_v}$, at zero temperature. To verify this behavior more quantitatively, we have also performed additional calculations of the finite-size dependence of the total correlation functions χ_{ph} and χ_v near $T = 0$. However, because full equilibration was only possible for temperatures above the dynamical freezing transition T_f with the present Monte Carlo method, we have to rely on the approximation of using low-energy states close to the ground state to perform the configuration averages. The low-energy states were obtained using the simulated-annealing method[28] by gradual annealing from initial temperatures $T \gtrsim T_f$. Figure 9 shows a log-log plot of the finite-size behavior of χ_{ph} and χ_v obtained by averaging over 800 low-energy states within a range of 0.16% above the known ground-state energy[17] $E_g = -1.2071$. If

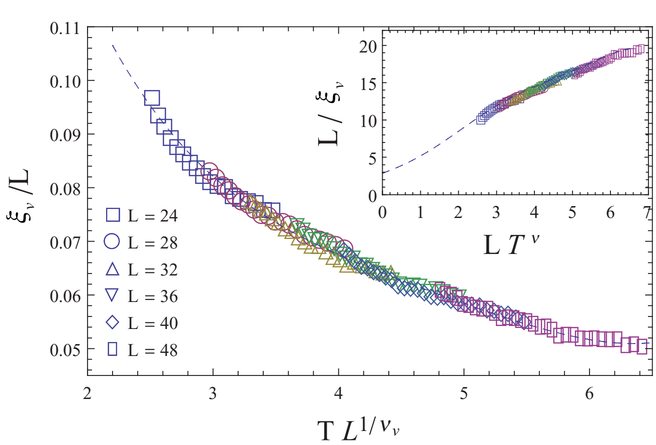

FIG. 8. (Color online) Scaling plot according to Eq. (5), assuming $T_c = 0$, for the vortex correlation length ξ_v, giving the estimate $\nu_v = 1.03(25)$. Dashed line is a fit to the data with Eq. (8). Inset: Replot of the data collapse in terms of the variable LT^{ν_v}.

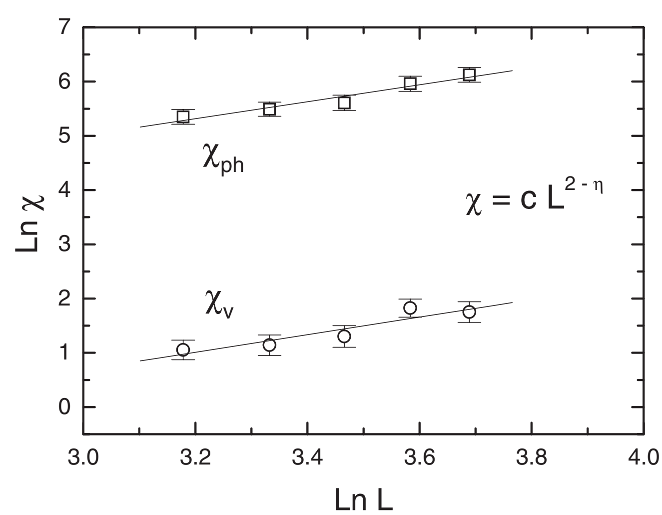

FIG. 9. Finite-size behavior of the total correlation function of phase χ_{ph} and vortex χ_v variables obtained from low-energy states, using simulated annealing. Lines correspond to log-log fits, which give the estimates $\eta_{ph} = 0.4(2)$ and $\eta_v = 0.4(3)$.

correlations decay as a power law in the ground state, then the finite-size behavior $\chi \propto L^{2-\eta}$ is expected near $T = 0$, for sufficiently large systems. This behavior is consistent with the data in Fig. 9 and from a log-log fit we estimate $\eta_{\rm ph} = 0.4(2)$ and $\eta_v = 0.4(3)$, which are compatible within the estimated errors with the corresponding values obtained from γ and ν, using the finite-temperature scaling analysis.

A particular feature of the frustrated *XY* model on the honeycomb lattice is the structure of the dual lattice, where the chiralities are defined, which has the form of a triangular lattice. Since the chiralities are Ising-like variables with antiferromagnetic interactions, if interactions further than nearest neighbors are neglected, there is a geometric frustration effect which may appear similar to the case of the triangular antiferromagnetic Ising model. The exact solution of this model[34] shows that there is no phase transition at nonzero temperatures ($T_c = 0$), which is the same scenario we find in the present case. However, while the correlation length for this Ising model diverges exponentially,[36] $\xi \propto e^{2/T}$, corresponding to $\nu \to \infty$ in Eq. (5), in the present case we find $\nu_v \sim 1$, suggesting a power-law correlation and correspondingly different ground-state properties.

The near agreement of our estimates of the critical exponents for vortex variables ν_v and phase variables $\nu_{\rm ph}$, assuming power-law divergent correlations, suggests that the $T_c = 0$ critical behavior may be described by a single divergent length scale. In a zero-temperature transition scenario where there is no decoupling, phase and vortex variables remain coupled on large length scales and one expects that the corresponding correlation lengths should diverge with a common critical exponent $\nu_{\rm ph} = \nu_v = \nu$. It is interesting to note that this behavior is in contrast with that found for the two-dimensional *XY* model at irrational frustration on a square lattice,[7] where a decoupled zero-temperature transition[37] was found with significant different exponents $\nu_v \gg \nu_{\rm ph}$.

An additional important feature of the zero-temperature transition found from the above analysis is the temperature dependence of the relaxation time for phase and vortex equilibrium fluctuations. If $T_c = 0$ then the divergence of the relaxation time τ as $T \to T_c$ is determined by thermal activation.[31] Thus, we expect that τ should increase exponentially with the inverse of temperature, corresponding to a dynamical exponent $z \to \infty$ in the usual power-law assumption $\tau \propto |T - T_c|^{-z\nu}$. To verify this behavior, we have in addition calculated the relaxation time τ for different temperatures from the autocorrelation function of phase and vortex fluctuations, $C_{\rm ph}(t)$ and $C_v(t)$, as

$$\tau_{\rm ph,v} = \frac{1}{C_{\rm ph,v}(0)} \int_0^\infty dt\; C_{\rm ph,v}(t). \qquad (9)$$

In these calculations, the starting configurations were taken from the equilibrium configurations obtained with the parallel tempering method and the subsequent time dependence was obtained from standard MC simulations at each fixed temperature. The results shown on the log-linear plot in Fig. 10 are indeed consistent with an activated behavior of $\tau_{\rm ph}$ and τ_v. The straight lines in the plot indicate that the data can be fitted to an Arrhenius behavior, with temperature-independent energy barriers $E_{\rm ph} = 1.00$ and $E_v = 1.02$. In general, the energy barrier can be temperature dependent, scaling with the correlation length[31] as $E \propto \xi^{\Psi}$. The observed Arrhenius behavior indicates that these additional exponents are $\psi_{\rm ph} \approx 0$, $\psi_v \approx 0$. The behavior for the relaxation time $\tau_{\rm ph}$ is particularly important when the frustrated *XY* model is applied to superconductors since it determines the temperature dependence of the linear resistivity,[6,31,32] $\rho_L \propto 1/\tau_{\rm ph}$. As a consequence, ρ_L should be finite at any nonzero temperature but decrease exponentially as temperature vanishes.

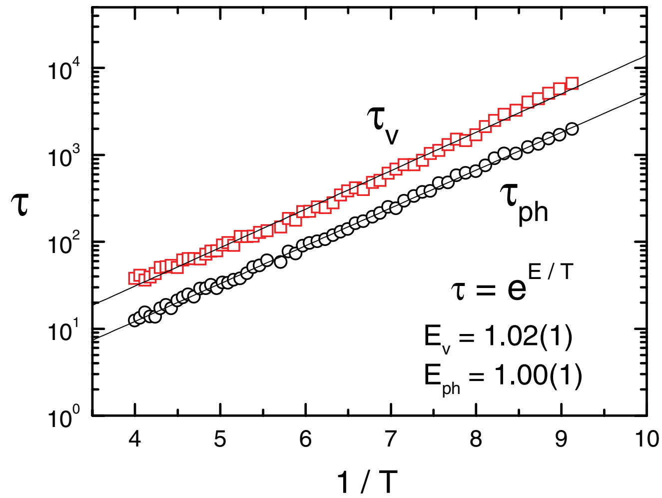

FIG. 10. (Color online) Temperature dependence of the relaxation times of phase fluctuations $\tau_{\rm ph}$ and vortex fluctuations τ_v for system size $L = 32$.

V. CONCLUSIONS

We have investigated the critical behavior of the fully frustrated *XY* model on a two-dimensional honeycomb lattice by Monte Carlo methods and a scaling analysis of the phase and vortex correlations. No evidence of an equilibrium phase transition is found at nonzero temperatures, in agreement with the conclusion of Ref. 20 for a similar model. The absence of vortex ordering at finite temperatures is also in agreement with the estimates of Ref. 21, which finds that this can only be observed for very large system systems ($L > 10^5$). However, our finite-size scaling analysis is consistent with a zero-temperature transition, where the critical temperature vanishes and phase and vortex correlation lengths, $\xi_{\rm ph}$ and ξ_v, diverge for decreasing temperatures as a power law with a common critical exponent, suggesting a coupled $T_c = 0$ transition scenario. Since both correlation lengths remain finite in the temperature range $T \geqslant 0.11$, where a KT transition[17] or a spin-glass transition[5] was proposed to take place in earlier MC simulations, our results also indicate that these apparent transitions should be attributed to slow dynamics effects and not to an equilibrium phase transition. Whether a vortex ordering transition can, nevertheless, occur for very large system sizes ($L > 10^5$), as predicted in Ref. 21, cannot be tested by our numerical simulation, which needs small system sizes in order to ensure full equilibration. The main features of the correlation-length behavior obtained for the present model are the same as those found in other frustrated

XY models where the phase-coherence temperature vanishes, such as the two-dimensional gauge glass model[38] and the *XY* model with irrational frustration,[7] but with different critical exponents. When applied to superconductors, the divergent correlation length ξ_{ph} in these models determine both the linear an nonlinear resistivity behavior, leading to a thermally activated linear resistivity and nonlinear current-voltage scaling at low temperatures.[6,31,32] Similar behavior should be observed for Josephson-junction arrays on a honeycomb lattice and superconducting films with a triangular pattern of nanoholes[16,39] in a perpendicular magnetic field corresponding to half flux quantum per plaquette, since both systems can be modeled by a frustrated *XY* model on a honeycomb lattice. The predicted thermal activated behavior for the linear resistivity seems to be already been observed in the latter system.[16]

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