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MINIMUM FUEL MULTI-IMPULSIVE ORBITAL MANEUVERS USING GENETIC ALGORITHMS

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This work aims to calculate transfer trajectories between two coplanar orbits using several impulses, trying to find solutions that minimize the costs related to the fuel consumption required to apply those impulses. The algorithm used here uses a time-free approach and a genetic algorithm as a method for solving the problem. Evolutionary optimization is used to solve the Lambert´s Problem associated with those transfers and searches for the best trajectories within the various possibilities for solving the problem. After that, a numerical algorithm to solve the same transfers is used, but now considering a low thrust maneuver. This type of propulsion system provides large savings in the consumption, at the expense of more complex and longer maneuvers.

INTRODUCTION

Genetic algorithms are heuristic methods that search for optimum solutions. It has applications in several fields. Functions, within a specific space domain, are determined by applying methods based on the theory of the Darwinian evolution. In this situation, a set of possible solutions of the problem considered here can be considered as a population. Then, applying methods of crossover and mutation among individuals, the population evolves and tends to produce better individuals that represent better the solutions of the problem. The key point in this approach is to specify a measurement that can define the best individuals.

Several missions can benefit from the optimization algorithm used in this work. The main ones are: transfer with free time (to change the orbit of the space vehicle without restrictions to the time required for the execution of the maneuver); "Rendezvous" (when one desires that the space vehicle reaches and remains on the side of another space vehicle); "Flyby" (when it is desired to intercept another celestial body, however without the objective of remaining next to it); "Swing-By" (when a close approach with a celestial body is used to gain or lose energy, velocity and angular momentum). Reference [1] shows more details on those problems.

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DESCRIPTION OF THE PROBLEM

To solve the optimal transfer problem described here, the Lambert's problem formulation is used. This problem is very popular in celestial mechanics and can be formulated as follows: Considering only keplerian motion, which means that the gravitational force follows a power law of the inverse of the square of the distance (Newtonian formulation), find an orbit that connects two points P_1 and P_2, with the time (Δt) for this transfer specified. In the literature, several researchers have used this problem in orbital maneuvers, like shown in References [2] to [6].

The parameters of the transfer orbit can be defined by:

- ν_1, the true anomaly of the departure point P_1 on the initial orbit.. $\nu_1 \in [0,2\pi]$.
- $\Delta\nu$, the angular length of the transfer. $\Delta\nu \in [0,2\pi]$.
- $a_t = \frac{a_{min}}{4y(1-y)}$, semi-major axis of the transfer orbit. For each pair of departure and arrival points, a minimum value a_{min} exists for a_t. Two transfer orbits can be found for the same value of a_t.
- ν_1 and $\Delta\nu$ determine the positions of the points P_1 and P_2, that can be related to the radius vectors $\vec{r}_1$ and $\vec{r}_2$.
- Any permitted value of the parameter y determine univocally one transfer orbit. These parameters are, from the point of view of the genetic optimizer, the genes of the members of the population.

The genetic algorithm searches for the best solution among a large number of possible solutions, represented by vectors in the solution space. To find the best solution is equivalent of looking for some extreme value (minimum or maximum) in the solution space.

The velocities at the thrust-points, before and after firing the engine, are computed and it provides the total velocity impulse, which is the measurement of the individual fitness. The evolutionary process will select individuals with the genes corresponding to the maneuvers with lower consumption. Figure 1 shows an instantaneous scenario of the problem.

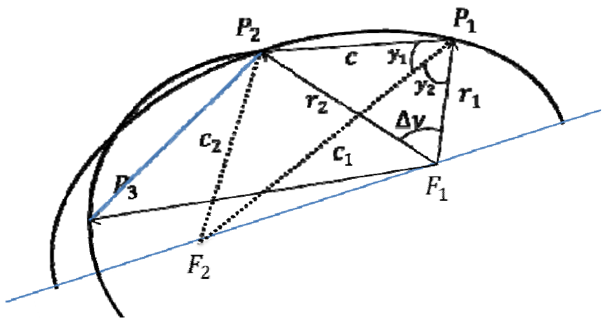

Figure 1. The instantaneous geometry of problem.

Note that ν_1 is the true anomaly of the point P_1 on the initial orbit; ν_2 is the true anomaly of the point P_2 on the final orbit; $\Delta\nu$ is the angular length of the transfer; the orientation of the transfer orbit is defined by the angle ω between its axis and the axis of the initial orbit; c is the distance between P_1 and P_2 (Equation 5); F_i are the focus of the ellipse.

The parameter a_t is usually replaced by a different parameter y, that has the advantage of being a variable that assumes values between 0 and 1, which provides a_t by means of the following relationship:

$$a_t = \frac{a_{min}}{4y(1-y)} \tag{1}$$

THE GENETICS ALGORITHM

Genetic algorithms are iterative schemes where, in each iteration, the population is modified, using the best features of the elements of the previous generations. They are subjected to five basic operations to produce better results:

- Creation: a procedure of making a population, randomly;
- Reproduction: a process where each string is copied, considering the values of the adaptive function;
- Crossover: a process where the combination of two chromosomes generates a new descendant;
- Mutation: a process where there is an occasional random modification (that has low probability) of the value of one element of the chain;
- Epidemic: a process where part of the population is exterminated, with a consequent entrance of new individuals in the population.

The reproduction is a process that is assigned to the elements that have the highest value of the quantity chosen to be the measurement of the fitness of the individual, and thus has a higher probability to contribute to the next generation, creating at least one descendant. The higher the values of this objective function, the higher are the chances that the individual will survive in the environment and that it will reproduce its genetic material to the next generations.

The procedure starts with a random population of up to 800 individuals. The initial population is generated randomly, and it considers its characteristics of distances and angles according to the constraints of each variable.

The random variables for the implementation of the algorithm are $\vec{x} = (\Delta\theta_i,\ R_i,\ Y_i)$, where $\theta_i = \nu_i - \omega$ is the true anomaly of P_i, referred to the transfer orbit, R_i determines the radius vector (position) in each thrust, and Y_i are the angles between $(\widehat{F_1, P_1, P_2})$.

The first step towards the application of genetic algorithms to any problem is to find how the chromosomal gene should be and which mapping represents better the search space. Each gene is a real number between 0 and 1. The value of the corresponding parameter is $X_i = X_i^{\min} + u_i(X_i^{\max} - X_i^{\min})$, where $X_i^{\min}$ and $X_i^{\max}$ are the minimum and maximum values; therefore they are the boundary conditions. Several papers considered this technique in orbital maneuvers, as shown in references [7] to [9].

Individual fitness (Objective Function)

The fitness (the objective function) is the value attached to the individual that evaluates how well is the solution represented by him. The fitness of each individual is represented by the total velocity impulse ΔV required to perform the orbital transfer. The total impulse is given by the sum of the single impulses ΔV_i provided in each thrust point in order to pass from an orbital arc to the next one. It corresponds to the velocity difference at the relevant thrust point. In this case, the fitness of each individual can be computed using data that define the problem $(a_1, e_1, a_2, e_2, \Delta\omega)$ and the three genes $(\nu_1, \Delta\nu, y)$ that characterize the individual. One obtains, in sequence:

the true anomaly of the arrival point

$$\nu_i = \nu_{i-1} + \Delta\nu \tag{2}$$

the radii of the departure and arrival points

$$r_1 = \frac{a_1(1-e_1^2)}{1+e_1\cos\nu_1} \tag{3}$$

$$r_2 = \frac{a_2(1-e_2^2)}{1+e_2\cos\nu_2} \tag{4}$$

the cord, i.e., the distance between P_1 and P_2

$$c = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\Delta\nu} \tag{5}$$

the semi-major axis of the transfer orbit

$$a_{\min} = \frac{r_1 + r_2 + c}{4} \tag{6}$$

the distances c_1 and c_2 of P_1 and P_2 from the vacant focus F*

$$c_i = 2a - r_i \tag{7}$$

the angles that appear in Figure 1 $(\gamma = \gamma_1 + \gamma_2)$

$$\gamma = \arccos\left(\frac{r_1^2-r_2^2+c^2}{2r_1c}\right) \tag{8}$$

$$\gamma_1 = \arccos\left(\frac{c_1^2-c_2^2+c^2}{2c_1c}\right) \tag{9}$$

the eccentricity of the transfer orbit

$$e_t = \frac{\sqrt{c_1^2+r_1^2-2c_1r_1\cos\gamma_2}}{2a_t} \tag{10}$$

the true anomaly θ_1 of P_1 on the transfer orbit

$$r_1 = \frac{a_t(1-e_t^2)}{1+e_t\cos\theta_1} \tag{11}$$

$$\theta_1 = \arccos\left(\frac{a_t(1-e_t^2)-r_1}{r_1e_t}\right) \tag{12}$$

the argument of perigee for the transfer orbit

$$\omega = \nu_1 - \theta_1 \tag{13}$$

which is the angle between the perigees of the transfer and the initial orbits.

After the geometry of the maneuver has been defined, one calculates the radial and tangential components of the spacecraft velocity before and after both impulses, what permit the computation of the total ΔV, which has been assumed to be the measurement of the individual fitness. Non-dimensional variables are used in the procedure. They are shown below.

$$r = \frac{\tilde{r}}{\tilde{a}_1} \tag{14}$$

$$v = \frac{\tilde{v}}{\sqrt{\frac{\mu}{\tilde{a}_1}}} \tag{15}$$

$$\Delta t = \sqrt{\frac{\tilde{a}_1^3}{\mu}}. \tag{16}$$

The distance and velocity units for the normalized variables are the semi-major axis of the initial orbit and the velocity on a circular orbit with the same energy as the initial one (Equations 14 and 15). The reference time is shown in Equation (16).

NUMERICAL SOLUTIONS

To apply the genetic algorithms in orbital maneuvers, an impulsive hypothesis is used for the engine of the spacecraft. It means that the velocity is assumed to change in zero time and a sequence of keplerian orbits represents the motion of the spacecraft. More details about this type of maneuver can be seen in References [10] to [14].

Then, the optimization method using genetic algorithms was used and several missions were simulated. Some of them are shown in more details, with initial radii $r_o = 1$ and final radius $r_f = 2$ and $r_f = 3$. The results are shown in Table 1 and Figures 2 to 4.

Table 1- Maneuvers between coplanar circular orbits showing the values of the ΔV.

nº	Simulation (r_o=1)	Values of the impulses				
		ΔV_1	ΔV_2	ΔV_3	ΔV_4	$\Delta V_T = \sum_{i=1}^{4} \Delta V_i$
1	$r_f = 2$	0.047578	0.000000	0.200723	0.170924	0.419226
2	$r_f = 3$	0.013201	0.000000	0.235550	0.227344	0.476096
3	$r_f = 1.2$	0.035366	0.017818	0.068514	0.074579	0.196277
4	$r_f = 1.5$	0.036376	0.108244	0.029295	0.170630	0.344545

In particular, optimal rendezvous between two coplanar orbits, with maneuvers using up to 4 burns, were used to generate those results. Rendezvous is a maneuver where a space vehicle needs to encounter a second one that is in a different orbit. Some more information regarding rendezvous maneuver can be seen in References [15] to [17]. Figure 3 shows the states of the genetic algorithm and its evolution after 500 evaluations of the objective function. The genetic algorithm provided satisfactory solutions when compared with the solutions of the literature. The population comprises of 500 individuals and up to 250 generations of individuals in those simulations. Maneuver 2 shows an interesting feature of the algorithm. Although the specified number of impulses was four, the algorithm found a better solution that uses only three impulses, so it generated an impulse with zero cost.

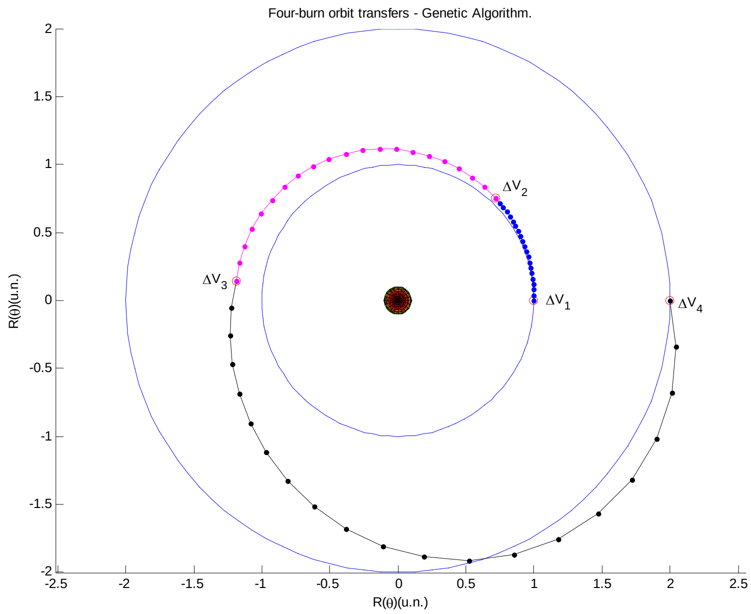

Figure 2. Four-burn orbital rendezvous, simulation 1: r_0 =1, r_f = 2 .

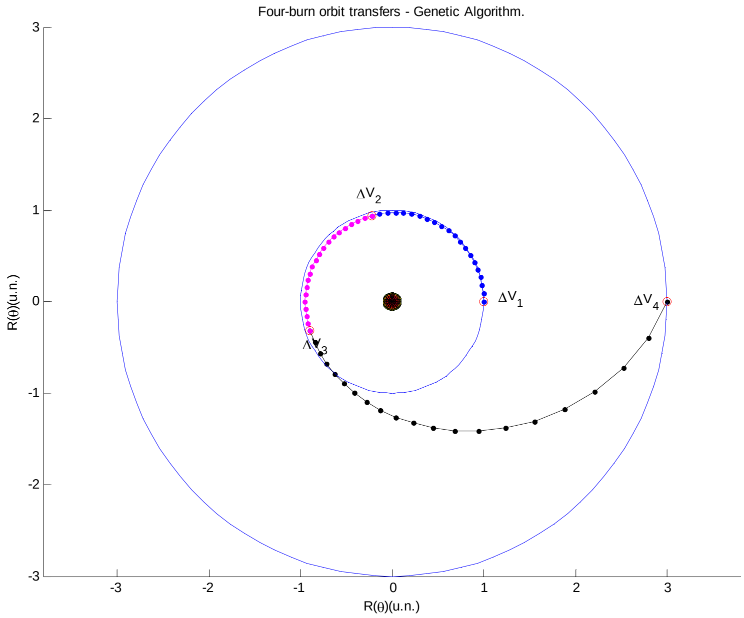

Figure 3. Four-burn orbital rendezvous, simulation 2: $r_0 = 1$, $r_f = 3$.

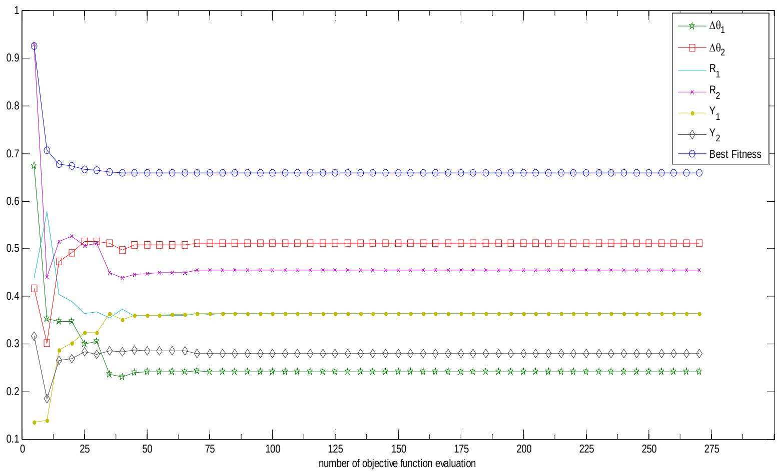

Figure 4 - The variables of the problem and the best fitness, simulation 1, with 4 burns.

After that, a different situation was assumed to test the Genetic Algorithm. Intermediate constraints were added to force the spacecraft to pass by several intermediate orbits before reaching the final desired one. To solve the problem, the engine was assumed to be also impulsive, but the number of impulses was increased to allow the satisfaction of the intermediate constraints (passing by specific orbits). Two different solutions were found and they are shown in Tables 2 and 3 and in Figures 5 and 6.

The consumption is large ($\Delta V_T = 2.234601$ for the first solution (Table 2) and $\Delta V_T = 3.249184$ for the second one (Table 3)), but it is a consequence of adding the intermediate constraints. These solutions differ from each other by intermediate constraints that we used, so the solutions have different transfer times and the trajectories of the spacecraft passes by different regions of the space, to be able to accomplish different goals for the mission.

Table 2 - Seven-burn orbit transfers, solution 1

ΔV_1	ΔV_2	ΔV_3	ΔV_4	ΔV_5	ΔV_6	ΔV_7	ΔV_8	ΔV_T
0.276038	0.116922	0.637252	0.215662	0.638076	0.000000	0.107102	0.243548	2.234601

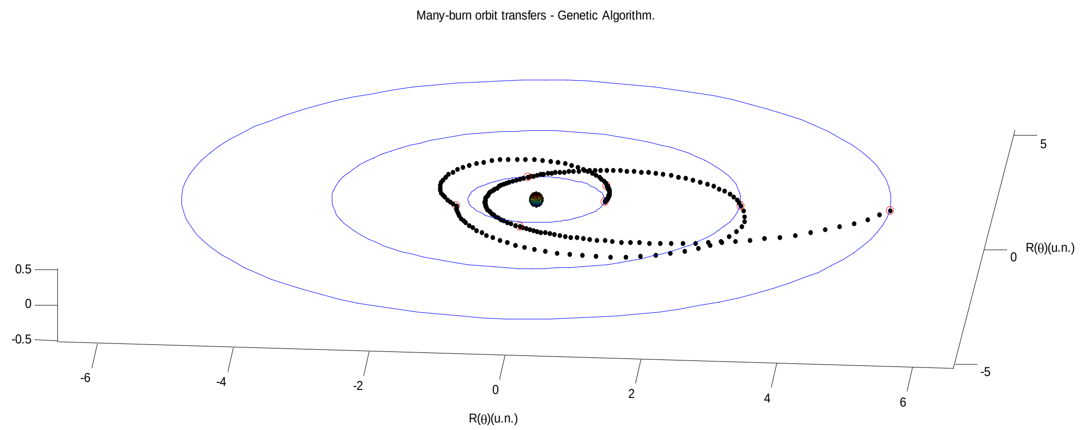

Figure 5. Seven-burn orbit transfers, $r_0 = 1$ (Earth), $r_f = 5.202803$ (Jupiter). Solution 1, with 7 burns and $\Delta V_T = 2.234601$.

Table 3 - Seven-burn orbit transfers, solution 2.

ΔV_1	ΔV_2	ΔV_3	ΔV_4	ΔV_5	ΔV_6	ΔV_7	ΔV_8	ΔV_T
0.014055	0.000000	0.235369	0.226667	0.632009	0.982334	0.949656	0.223149	3.249184

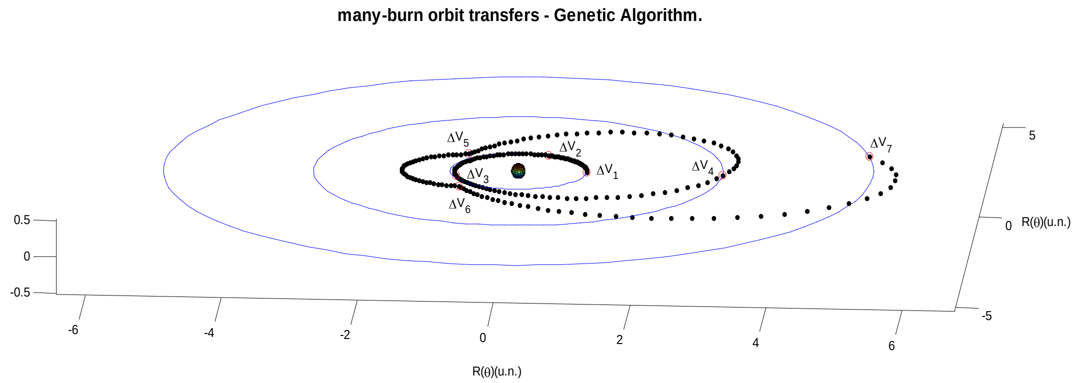

Figure 6. Seven-burn orbit transfers, $r_0 = 1$ (Earth), $r_f = 5.202803$ (Jupiter). Solution 2: with 7 burns and $\Delta V_T = 3.249184$.

Then, we implemented a low thrust algorithm. This type of maneuver assumes that the engine can deliver a continuous but low thrust to the spacecraft. Similar problems are studied in several papers in the literature, like shown in References [18] to [22]. The main advantage is that it consumes a lot less fuel, but at an expense of larger times for the maneuver, as well as more complex implementation of the hardware. The choice of which technique to use depends on the mission requirements and on the engines available for the spacecraft.

In the present formulation, the main ideas shown in References [21] to [22] are used and, to avoid singularities in the equations of motion, the following variables are used:

$$X_1 = [a(1-e^2)/\mu]^{1/2} \tag{17}$$

$$X_2 = e\cos(\omega-\phi) \tag{18}$$

$$X_3 = e\sin(\omega-\phi) \tag{19}$$

$$X_4 = (\text{Fuel Consumed})/m_0 \tag{20}$$

$$X_5 = t \tag{21}$$

$$X_6 = \cos(i/2)\cos((\Omega+\phi)/2) \tag{22}$$

$$X_7 = \sin(i/2)\cos((\Omega-\phi)/2) \tag{23}$$

$$X_8 = \sin(i/2)\sin((\Omega-\phi)/2) \tag{24}$$

$$X_9 = \cos(i/2)\sin((\Omega+\phi)/2) \tag{25}$$

$$\phi = \underline{f} + \omega - s; \tag{26}$$

where:

a = semi-major axis;

e = eeccentricity;

i = inclination;

Ω = argument of the ascending node;

ω = argument of perigee;

$\underline{f}$ = true anomaly;

s = range angle;

μ =gravitational constant;

m_0 = initial mass of the spacecraft.

In those variables, the equations of motion are:

$$dX_1/ds = f_1 = SiX_1F_1 \tag{27}$$

$$dX_2/ds = f_2 = Si\{[(Ga+1)\cos(s)+X_2]F_1+GaF_2\sin(s)\} \tag{28}$$

$$dX_3/ds = f_3 = Si\{[(Ga+1)\sin(s)+X_3]F_1 -GaF_2\cos(s)\} \tag{29}$$

$$dX_4/ds = f_4 = Si\underline{Ga}F(1-X_4)/(X_1W) \tag{30}$$

$$dX_5/ds = f_5 = SiGa(1-X_4)m_0/X_1 \tag{31}$$

$$dX_6/ds = f_6 = - SiF_3[X_7\cos(s)+X_8\sin(s)]/2 \tag{32}$$

$$dX_7/ds = f_7 = SiF_3[X_6\cos(s)-X_9\sin(s)]/2 \tag{33}$$

$$dX_8/ds = f_8 = SiF_3[X_9\cos(s)+X_6\sin(s)]/2 \tag{34}$$

$$dX_9/ds = f_9 = SiF_3[X_7\sin(s)-X_8\cos(s)]/2 \tag{35}$$

where:

$$Ga = 1 + X_2\cos(s) + X_3\mathrm{sen}(s) \tag{36}$$

$$Si = (\mu X_1^4)/[Ga^3m_0(1-X_4)] \tag{37}$$

F, F_1 , F_2 , F_3 are the forces generated by the thrust, given by:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \tag{38}$$

$$\left|\vec{F}\right| = F \tag{39}$$

$$F_1 = F\cos(\alpha)\cos(\beta) \tag{40}$$

$$F_2 = Fsen(\alpha)\cos(\beta) \tag{41}$$

$$F_3 = Fsen(\beta) \tag{42}$$

where α is the angle between the perpendicular to the radius vector and the direction of the thrust and β is the out-of-plane angle of the thrust. The equations for the Lagrange multipliers are:

$$\frac{dp_1}{ds} = -\frac{4\sum_{j=1}^{9} p_j f_j + p_1 f_1 - p_4 f_4 - p_5 f_5}{X_1} \tag{43}$$

$$\frac{dp_2}{ds} = \frac{cos\,(s)}{Ga}\left[3\sum_{j=1}^{9} p_j f_j - p_4 f_4 - p_5 f_5\right] - Sip_2 F_1 - Sicos^2(s)(p_2 F_1 - p_3 F_2) - \\ -Sicos(s)sen(s)(p_2 F_2 + p_3 F_1) \tag{44}$$

$$\frac{dp_3}{ds} = \frac{sen\,(s)}{Ga}\left[3\sum_{j=1}^{9} p_j f_j - p_4 f_4 - p_5 f_5\right] - Sip_3 F_1 - Sicos(s)sen(s)(p_2 F_1 - p_3 F_2) \\ - Sisen^2(s)(p_2 F_2 + p_3 F_3) \tag{45}$$

$$\frac{dp_4}{ds} = -\left[\frac{\sum_{j=1}^{9} p_j f_j - p_4 f_4 - p_5 f_5}{m_0(1 - X_4)}\right] \tag{46}$$

$$\frac{dp_5}{ds} = 0 \tag{47}$$

$$\frac{dp_6}{ds} = \frac{-SiF_3[p_7 cos(s) + p_8 sen(s)]}{2} \tag{48}$$

$$\frac{dp_7}{ds} = \frac{SiF_3[p_6 cos(s) - p_9 sen(s)]}{2} \tag{49}$$

$$\frac{dp_8}{ds} = \frac{SiF_3[p_6 sen(s) + p_9 cos(s)]}{2} \tag{50}$$

$$\frac{dp_9}{ds} = \frac{-SiF_3[p_8 cos(s) - p_7 sen(s)]}{2} \tag{51}$$

The control to be applied to the spacecraft can also benefit from a substitution of variables, to avoid numerical problems. The following set of variables is used:

$$u_1 = s_0 \tag{52}$$

$$u_2 = (s_f - s_0)\cos(\beta_0)\cos(\alpha_0) \tag{53}$$

$$u_3 = (s_f - s_0)\cos(\beta_0)\sin(\alpha_0) \tag{54}$$

$$u_4 = (s_f - s_0)\sin(\beta_0) \tag{55}$$

$$u_5 = \alpha' \tag{56}$$

$$u_6 = \beta' \tag{57}$$

First order necessary conditions for the optimal problem can be written. For every instant of time we have:

$$\sin(\alpha) = q_2 / S' \tag{58}$$

$$\sin(B) = q_3 / S'' \tag{59}$$

$$\cos(\alpha) = q_1 / S' \tag{60}$$

$$\cos(B) = S' / S'' \tag{61}$$

where:

$$S' = \pm [\, q_1^2 + q_2^2 \,]^{1/2} \tag{62}$$

$$S'' = \pm [\, q_1^2 + q_2^2 + q_3^2 \,]^{1/2} \tag{63}$$

$$q_1 = p_1X_1 + p_2[X_2 + (Ga + 1)\cos(s)] + p_3[X_3 + (Ga + 1)\sin(s)] \tag{64}$$

$$q_2 = p_2Ga\sin(s) - p_3Ga\cos(s) \tag{65}$$

$$q_3 = - p_6[X_7\cos(s) + X_8\sin(s)]/2 + p_7[X_6\cos(s) - X_9\sin(s)] + p_8[X_6\sin(s) + X_9\cos(s)] + p_9[X_7\sin(s) - X_8 \cos(s)] \tag{66}$$

It is also possible to include constraints. Some of the most used ones can be represented by:

$$\mathbf{S}\,(\,.\,) \geq 0 \tag{67}$$

$$\frac{\left(a - a^*\right)}{\left|a_0 - a^*\right|} = 0 \tag{68}$$

$$\frac{\left[a(1+e) - a^*\left(1+e^*\right)\right]}{\left|a_0\left(1+e_0\right) - a^*\left(1+e^*\right)\right|} = 0 \tag{69}$$

$$\frac{(i-i^*)}{|i_0-i^*|}=0 \tag{70}$$

$$\frac{(\Omega-\Omega^*)}{|\Omega_0-\Omega^*|}=0 \tag{71}$$

$$\frac{(\omega-\omega^*)}{|\omega_0-\omega^*|}=0 \tag{72}$$

The first one represents generic inequality constraints, while the other five is used to specify an orbit.

After the implementation of this technique, the simulations showed in Table 1 was considered again, this time using this low thrust approach. The results are shown in Table 4. The consumptions are much lower, but it is necessary to have in mind that this situation is normal and constitute the most important characteristics of low thrust engines.

Table 4- Maneuvers between coplanar circular orbits using low thrust.

nº	**Simulation** (r_o=1)	**Consumption**
		Low Thrust Maneuvers
1	$r_f = 2$	0.0886
2	$r_f = 3$	0.1271
3	$r_f = 1.2$	0.0831
4	$r_f = 1.5$	0.0865

CONCLUSIONS

From the analysis of the results obtained, the genetic algorithm implemented here showed that this technique can obtain results for the proposed four impulsive rendezvous maneuvers. It means that it can be used in real cases. In particular, the algorithm is able to find solutions with a smaller number of impulses by making one or more of them with zero magnitude, if a maneuver with a lower number of impulses can be used.

It also can generate results in situations where intermediate constraints of passing by specific orbits are included. In this case, several burns are required and the consumption is larger, as expected. In the examples used here the number of impulses reached the number of seven and two solutions were found, by considering two different sets of intermediate constraints.

Then, a low thrust was used for the rendezvous missions. It shows the importance of this approach, which can find solutions with much lower fuel consumption, although it has some disadvantages like more time required for the maneuvers and more complex implementation of the hardware.

ACKNOWLEDGMENTS

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