Consecutive Collision Orbits to Obtain EGA Maneuvers

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Abstract: Studying optimal space maneuvers that searches the minimum fuel consumption for interplanetary missions is an important field of research for development of space technologies. This paper analyzes optimal maneuvers of a spacecraft that leaves one celestial body and goes back to this same body, using this return passage to perform a gravity assisted maneuvers using the mother planet to change its velocity, energy and angular momentum. During this approach, the space vehicle place itself in another orbit of interest of the mission. The dynamics used to solve this problem is the traditional model of the Restricted Three Body Problem, so it is assumed that the three bodies involved are mass points and don't suffer external disturbances. Using the gravitational attraction and the geometric configuration of the bodies involved, the passage next to the body causes a considerable change in velocity (ΔV), energy (ΔE) and angular momentum of the spacecraft. Several orbits are simulated.

Key-Words: Astrodynamics, Swing-By, Collision Orbits and Celestial Mechanics.

1. Introduction

Considering the problem of orbital maneuvers, many alternatives exist in the literature. An important possibility is the low thrust maneuver, where a force with low magnitude is used during a finite time. There are many papers in the literature like references [1] to [8]. A second idea is based in impulsive thrust, where the thrust has an infinity magnitude. References [9] to [15] explain this idea. After that, the gravitational capture has been considered. The perturbation of a third-body [16] is used to decrease the fuel consumption of the maneuver. References [17] to [21] are good examples of this approach. When considering missions to the solar system, they produces a very high cost to the missions. To deal with this fact, the Gravity Assisted Maneuver, also known as "Swing-By maneuver" is used to help to design interplanetary missions. A mathematical treatment of this problem is shown here using two-body dynamics. This approach is usually known as "patched conics", refering to the fact that Keplerian orbits are conic sections. Examples of this approach are in references [22] to [29].

In the middle of the XIX century, astronomers and mathematicians knew this type of maneuver. Later on, analytic equations were found and numerical results that describe the Swing-By, especially in the problem of capture of comets by Jupiter were made.

The use of this Maneuver is very important in reducing the costs of space missions. It is a maneuver where the space vehicle uses a close approach with a celestial body to modify the velocity, energy and angular momentum of the spacecraft. These maneuvers can be used to decrease the fuel expenditure in missions that request an Earth escape, like in the case of interplanetary trips. In that case, the vehicle just leaves the Earth with energy enough to enter in an elliptic orbit that crosses with the orbit of the Moon in some point.

The classical methods of maneuvers use the propulsion model with infinite impulse, has also done in the present research. All the maneuvers considered here use two impulses to complete the transfer (Prado e Broucke [34]). Only the limiting case $\mu = 0$ is considered in this paper.

2. The Swing-by maneuvers

The dynamics of the two bodies is used in the present formulation, considering that the system is formed by three bodies. It is possible to say that:

1. The body m_1, with finite mass, is located in the center of mass of the cartesian system;

2. M_2, a smaller body, can be a planet or a satellite of m_1, in a keplerian orbit around m_1;
3. A body m_3, a space vehicle with infinitesimal mass, is traveling in a keplerian orbit around m_1, when it makes an encounter with m_2.

This encounter changes the orbit of M_3 and, by the hypothesis assumed for the problem, it is considered that the orbits of M_1 and M_2 do not change.

Using the "patched conics" approximation, the equations that quantify those changes are available in the literature.

The standard maneuver can be identified by the following three parameters (Fig. 1) (Prado [31]):

i) $\left|\vec{V}_\infty\right|$, the magnitude of the velocity of the spacecraft with respect to M_2 when approaching the celestial body;

ii) r_p, the distance between the spacecraft and the celestial body during the closest approach;

iii) ψ_A, the angle the approach.

Having those variables, it is possible to obtain δ, the total deflection angle, by using the equation (Broucke [1]): $\delta = \arcsin\left(1/\left(1+\frac{r_p V_\infty^2}{\mu_2}\right)\right)$;

A complete description of this maneuver and the derivation of the equations can be found in Broucke [1]. The final equations are reproduced below.

$$\Delta E = -2V_2 V_\infty \sin(\delta)\sin(\Psi_A) \quad (1)$$

$$\Delta C = -2V_2 V_\infty \sin(\delta)\sin(\Psi_A) \quad (2)$$

$$\Delta V = \left|\Delta\vec{V}\right| = 2\left|\vec{V}_\infty\right|\sin(\delta) = 2V_\infty \sin(\delta) \quad (3)$$

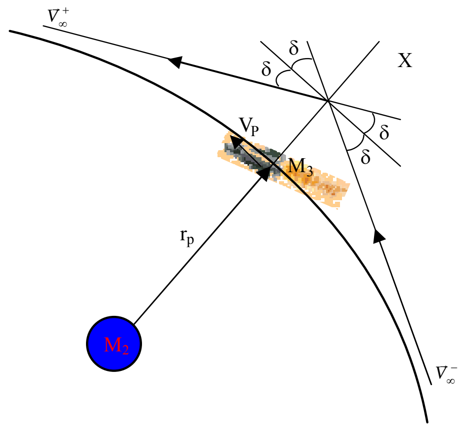

Figure 1 – Swing-By Maneuver.

The problem will be modeled using the dynamics of the two bodies, which means that μ = 0, implying in the reduction of the problem with three bodies in a problem with two bodies. In this way, the equations derived by Kepler can be used to find the solutions. Two impulses will be used in the transfer maneuver. It is assumed that the three bodies involved are mass points and do not suffer external disturbances (Fig. 2).

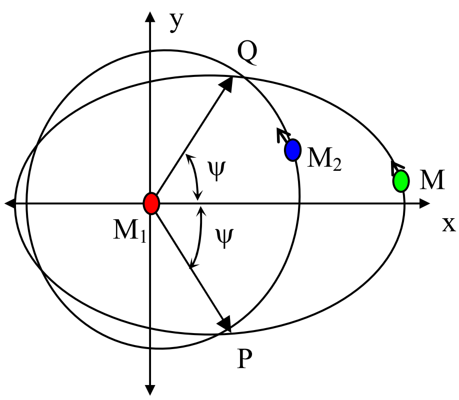

Figure 2 - Consecutive Collision Orbits.

3. The Consecutive Collision Orbits Problem

M_1 and M_2, are the two primaries with masses (1 - μ) and μ, respectively. M_2 is in a circular orbit around M_1. The space vehicle M_3 leaves M_2 from a point P (t = ψ_0). It follows a trajectory around M_1 that meets again with M_2 in a point Q (t = ψ_f), where $\psi_0, \psi_f \in [0,2\pi]$ and $\psi_f > \psi_0$. The values of ψ_0 and ψ_f are not necessarily symmetrical (Santos [32]).

Hénon [33] studied this problem and published graphs with solutions for the case of circular orbits. Howel [34] published solutions for the elliptic case, where the transfers were symmetrical with respect to the periapsis. Prado e Broucke [30] also published solutions for this problem, in the same situations, using the Lambert method. Hitzl and Hénon [36] also published results in this topic. The results were analyzed and disposed in form of tables and graphs.

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4. Mathematical Formulation of the Consecutive Collision Orbits Problem

The Hénon [33] problem, formulated as a Lambert problem, can be described in the way shown below.

i) The position of M_3 is known at $t = \psi_0$ (point P, initial point of the transfer orbit). The position vector R_1 can be specified as a function of the angle ψ_0, where:

$$R_1 = \frac{a(1-e^2)}{1+e\cos(\Psi_0)} \tag{4}$$

It is the same value for M_2 and M_3, because at the initial moment ($t = \psi_0$) M_2 and M_3 occupy the same position.

ii) The position of M_3 at $t = \psi_f$ (point Q, final point of the orbit). The position vector R_2, similar to the item above, it is described by the equation:

$$R_2 = \frac{a(1-e^2)}{1+e\cos(\Psi_f)} \tag{5}$$

iii) The total transfer time is given by $\Delta t = \omega(\Psi_f - \Psi_0)$. Remember that the angular velocity of the system (ω) is unit, so ψ can be considered to be the time as well as the angle.

iv) The total angle $\Delta\psi$, that the spacecraft must travel to go from P to Q, for the case where the orbits are elliptic, has several possible values.

First of all, it is necessary to consider two possible choices for the transfer: the one that uses a direction of the shortest possible angle between P and Q ("short way") and the one that uses the direction of the longest possible angle between these two point ("long way").

After considering these two choices, it is also necessary to consider the possibilities of multirevolution transfers. In this case, the spacecraft leaves P, makes one or more complete revolutions around M_1, and then it goes to Q. Thus, combining those two factors, the possible values for $\Delta\psi$ are $[\psi_f - \psi_0 + 2m\pi]$ and $[2\pi-(\psi_f - \psi_0) + 2m\pi]$. There is no upper limit for m, and this problem has an infinite number of solutions, except in the case where the orbit of M_3 is parabolic or hyperbolic, where $\Delta\psi$ has a unique value.

The solution of the Lambert problem is the Keplerian orbit that contains the point P and Q and that requires the given transfer time $\Delta t = \Delta\psi = \psi_f - \psi_0$ for the spacecraft to travel between these two points. In this paper, we used the Gooding's Lambert routine to solve the Lambert problem (Gooding [36]).

Possible applications for this technique are interplanetary research in the Solar System, a basis for a transportation system between the Earth (M_1) and the Moon (M_2) where no orbit correction is required, etc.

5. Hypotheses for a Earth's Swing-By

1. The system is formed by two bodies in elliptic orbits and a third massless body moving under the action of the gravitational forces;
2. The origin of the system is placed in the center of mass. The horizontal axis is the line M_1 and M_2 and the vertical axis is perpendicular to that line;
3. The spacecraft leaves P, crosses the horizontal line (Sun - Earth), goes to the apsis and then reaches the point Q, where the close approach occurs (Fig. 2);
4. After the close approach, the spacecraft modify the velocity, energy and angular momentum;
5. We used the system of canonical units. This formulation implies that the unit of distance is the distance between M_1 (Sun) and M_2 (Earth); the angular velocity (ω) of the motion of M_1 and M_2 is assumed to be unitary; the mass of the smaller primary (M_2) is given by $\mu = \frac{m_2}{m_1 + m_2}$, (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is $(1 - \mu)$, to make the total mass of the system unitary; the gravitational constant is one;

   In this system of units, the gravitational parameter of the Earth is $\mu_t = 2.9970165E-6$.

5.1. Earth's Swing-By (EGA)

After the spacecraft performs the Consecutive Collision Orbits, it takes advantage of the passage by the body M_3 close to the primary M_2 to make a Swing-By to change its energy.

5.1.1. Simulation e = 0.4

Some simulations for the case Earth-Satellite, considering the closest approach of the space vehicle with the Earth equal to the value of 1.2 radius of the Earth were performed.

Table 1 and Figures 3 – 5, show solutions with maximum ΔV, found from the solutions of the problem proposed. For certain values of ψ we find values of maximum gains (measured by ΔV and ΔE). Note that the solutions are not unique.

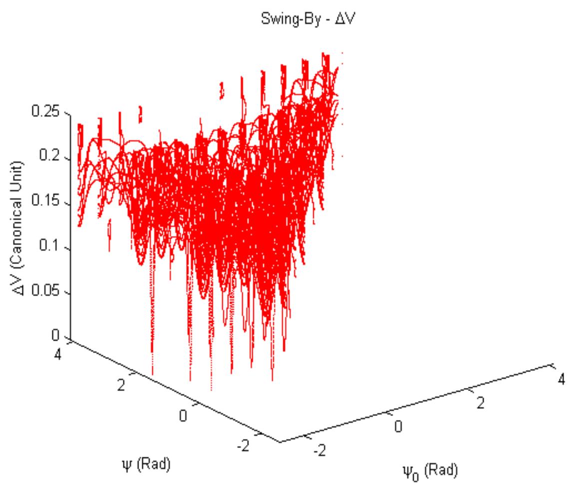

Figure 3 - Variation of velocity (ΔV) after the *Swing-By* with e = 0.4, ψ_o = - 2.5 rad, ψ_f = 4 rad.

Varying the values of the angles (ψ_0), we found values of maximum gains for the variation of the energy (ΔE) and velocity (ΔV).

In the graphs and Tables (Figs. 3 and 4, Table 1) are visualized the variation of the velocity (ΔV). It is visible that there are several maximum ($\Delta V \approx 0.242028$) and minimum gains ($\Delta V \approx 0.08$). There are also values for the angle of approach (ψ_A) for which a mission that is looking for fuel savings is not possible.

For the energy variation (Figs. 5 and 6, Table 1) we also found maximum ($\Delta E \approx 0.242026$) and minimum ($\Delta E \approx -0.15$) points. The values of the angle of approach (ψ) that gives you a larger gain of energy is around 47.4º, with angle of approach (ψ_A) around 269.8º.

Figures 7 and 8 shows the balance of the velocity variation, i.é. the value of ΔV total, that is the result of the ΔV value obtained from the Swing-by less the ΔV spent with the operation maneuver to get the multiple encounters.

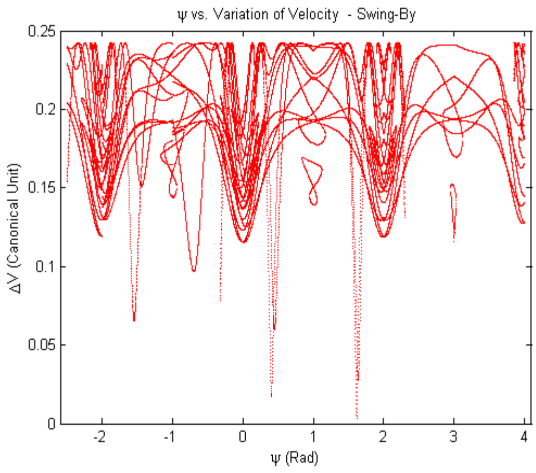

Figure 4 - ψ vs. Variation of velocity (ΔV), for e = 0.4, ψo = -2.5 rad e ψf = 4 rad.

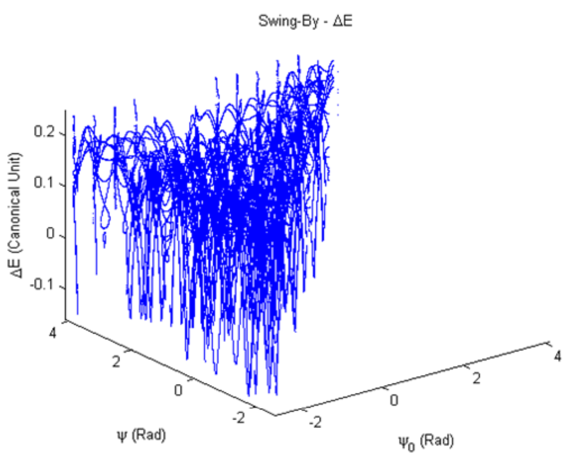

Figure 5 - Variation of Energy (ΔE) after the *Swing-By* with e = 0.4, ψ_o = -2.5 rad e ψ_f = 4 rad.

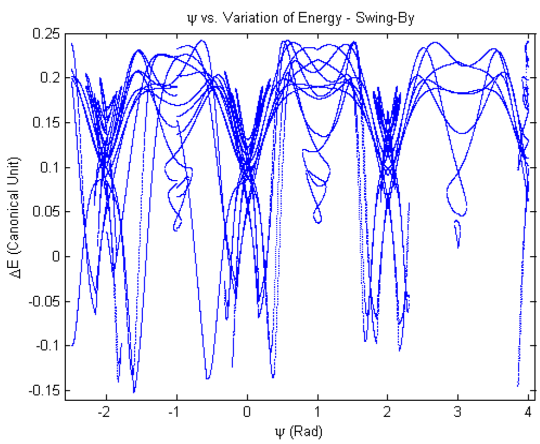

Figure 6 - ψ vs. Variation of Energy (ΔE), for e = 0.4, ψo = -2.5 rad e ψf = 4 rad.

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In Figure 7 verify the balance of ΔV, depending on angle ψ. Note that most of the solutions shown in this figure are negative. These solutions represent the values of angle ψ, where a mission which seeks the minimum fuel consumption will not be productive, because the ΔV obtained from the Swing-By with the Earth is less than the ΔV consumed to perform the maneuver of multiple encounters.

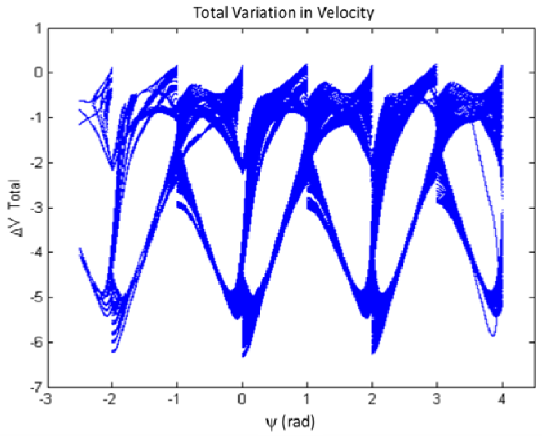

Figure 7 – The total ΔV obtained from Swing-By in maneuver of multiple encounters, for e = 0,4.

It is shown in Figure 7 the positive balance of ΔV according to the final angle of each maneuver (ψ). This figure shows that the solutions with total ΔV is positive, for the simulation with the eccentricity e = 0.4.

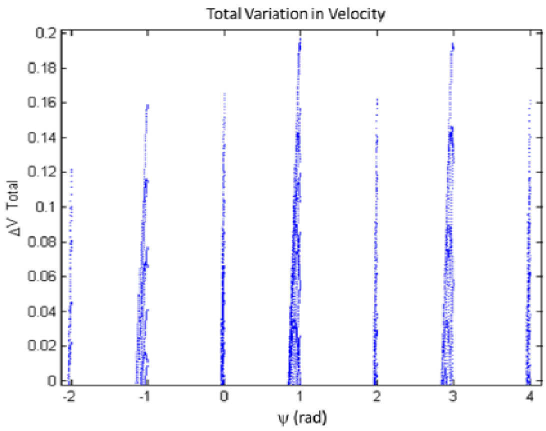

Figure 8 – Positive balance of ΔV obtained in Swing-By, for e = 0,4.

6. Conclusion

The gravity assist maneuver can provide a considerable variation of velocity and energy for a spacecraft, reducing the costs of a mission. In the graphs and table shown in this work, it is verified that this maneuver is a powerful tool that can also be used in interplanetary missions that requires that the spacecraft leaves a body and return later to this same body.

Based in the simulations made here, it is verified that the gain in the velocity (ΔV) and energy (ΔE) is directly proportional to the eccentricity of the orbit (e) of the space vehicle M_3.

The numerical values of the gain in velocity (ΔV) and energy (ΔE) were presented as a solution to the problem of Swing-by.

7. Acknowledgements

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Table 1 - *Swing-By* with gain ΔV > 0.242. ΔV and ΔE in canonical units

e = 0,4 ψ_o = -2,5 rad ψ_f = 4 rad		Angle of Approach		Variation of Velocity	Variation of Energy
ψ		ψ_A		ΔV	ΔE
rad	degree	rad	degree		
-0,579	-33,1743	4,702196	269,416	0,242018	0,242006
-0,577	-33,0597	4,709691	269,8454	0,242008	0,242007
0,82	46,9825	4,719034	270,3807	0,242019	0,242013
0,821	47,0398	4,718033	270,3234	0,24202	0,242016
0,823	47,1544	4,71604	270,2092	0,242022	0,24202
0,824	47,2117	4,715047	270,1523	0,242023	0,242022
0,825	47,2690	4,714058	270,0956	0,242024	0,242023
0,826	47,3263	4,713071	270,0391	0,242024	0,242024
0,827	47,3836	4,712088	269,9828	0,242025	0,242025
0,828	47,4409	4,711107	269,9265	0,242026	0,242026
0,829	47,4982	4,71013	269,8706	0,242026	0,242026
0,832	47,6701	4,707216	269,7036	0,242028	0,242024
0,833	47,7274	4,706251	269,6483	0,242028	0,242023
0,834	47,7847	4,705288	269,5931	0,242028	0,242022
0,835	47,8420	4,704329	269,5382	0,242028	0,24202
0,84	48,1285	4,69958	269,2661	0,242028	0,242008
0,841	48,1858	4,69864	269,2122	0,242028	0,242005
1,973	113,0446	4,699659	269,2706	0,242024	0,242004

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