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Gravity Assisted Maneuvers for Asteroids using Solar Electric Propulsion

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Abstract: *Apophis considered by Greeks as chaos, the God of destruction, in current time's returns to the scenario as a destructive threat potential, an asteroid that orbits a region at risk of colliding with Earth. The asteroid located in 2004, with the name of 2004MN4 was named Apophis, and several researchers and institutes are monitoring and designing missions with the goal to search more information about the asteroid. This work will be approached to optimizing interplanetary missions maneuvers using solar electric propulsion for Apophis and others asteroids. Gravity Assisted Maneuver to reduce mission costs, exploiting the high specific impulse and the high of electric propulsion for performs the maneuver.*

Optimized trajectories will be analyzed in a spacecraft that leaves Earth on a Low Earth Orbit (LEO) and arrives on an asteroid, Apophis this case, using ion propulsion and can do: a direct trajectory; a gravitational maneuver on Earth, a gravitational maneuver on Earth (EGA) and on Mars (EMGA). An indirect optimization method will be used in the simulations.

1. Introduction

Approximately 1000 asteroids are known currently whose orbits approaching significantly from Earth heliocentric orbit, constituting a threat to the planet because can cross the Earth's orbit. These asteroids are usually designated by the initials NEA (Near Earth Asteroid). The data of the asteroids used in simulations were collected from the JPL database, which is part of NASA project "Near Earth Object Search Program" of the Jet Propulsion Laboratory (JPL) located in Pasadena, California.

Electric propellants are being substantially used to realize the propulsion of orbit correction maneuvers of the satellite and as primary propulsion in missions toward asteroids. Indirect optimization methods are suitable for the low thrust trajectories that are used in simulations. A finite force is applied during a finite time and is necessary to integrate the equation of state over time to know its effect. Several results exist in literature, starting with the works of Tsien (1953) and Lawden (1955). Other results and references can be found in Prado (1989), Prado and Rios-Neto (1993), Casalino and Colasurdo (2002), Santos (2009). The most used method in this model is the so called "primer-vector theory", developed by Lawden (1953 and 1954), according to Prado et. All (2006), Santos (2008 and 2009). In this paper, theory of optimal control is applied and a procedure based on the Newton Method to decide the boundary problems is developed. The Pontryagin's Maximum Principle (PMP) is used to maximize the Hamiltonian associated to the problem and evaluates the optimal structure of the "switching function".

The spacecraft leaves the Earth's sphere of influence with a hyperbolic velocity whose optimal magnitude and the direction will be supplied by the optimization procedure. The initial mass is directly related to the magnitude of the hyperbolic velocity, assuming that a chemical

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thruster is used to leave a low Earth orbit (LEO). Out of the Earth's sphere of influence, the electric propellants is activate and the available power is proportional to the square of the distance from the sun; the propulsion is provided by one or two "PPS 1350 ion thrusters and Phall 1 (UNB)".

For simulation purposes were chosen the asteroids 2004MN4 (Apophis) and 2002TC70, which are part of the Group of asteroids have orbits near the Earth, NEOs (Near Earth Object), and also due to the size of the semi-major axis (a) and low inclination (i) in relation to the axis of the ecliptic, which are parameters that favor the maneuvers that will be simulated.

2. Solar Electric Propulsion (SEP)

The solar electric propulsion could be the best option for the transports of the future due to its high specific impulse when compared to the chemical propulsion. Electric propellants are being extensively used to assist the propulsion of terrestrial satellites for the maneuvers of orbit correction and as primary propulsion in missions toward other bodies of the solar system.

Both NASA and ESA have launched spacecrafts which used SEP (Solar Electric Propulsion) as the primary propulsion system; NASA's DS1 and ESA's Smart-1 to the moon to comet Borrelly.

Indirect optimization methods are suitable for the low thrust trajectories that are used in simulations. A finite force is applied during a finite interval of time and it is necessary to integrate the state equation along the time to know its effect. Several results exist in literature, starting with the works of Tsien (1953) and Lawden (1955). Other results and references can be found in Prado (1989), Prado and Rios-Neto (1993), Casalino and Colasurdo (2002), Santos (2006). The most used method in this model is the to called "primer-vector theory", developed by Lawden (1953 and 1954). In this paper, theory of optimal control is applied and a procedure based on the Newton Method to decide the boundary problems is developed. The Pontryagin's Maximum Principle (PMP) is used to maximize the Hamiltonian associated to the problem and evaluates the optimal structure of the "switching function".

The spacecraft leaves the Earth's sphere of influence with a hyperbolic velocity whose optimal magnitude and the direction will be supplied by the optimization procedure. The initial mass is directly related to the magnitude of the hyperbolic velocity, assuming that a chemical thruster is used to leave a low Earth orbit (LEO). Out of the Earth's sphere of influence, the electric propellants is activate and the available power is proportional to the square of the distance from the sun; the propulsion is provided by one or two "PPS 1350 ion thrusters and Phall1 thrusters (UNB)".

3. Description of the Problem

The spacecraft will be considered a point with variable mass m and trajectory will be analyzed using the patched-conics approach. The time required by the spacecraft to leave the Earth's sphere of influence is neglected and, in this formulation, only equations of motion in the heliocentric reference system will be considered. The spacecraft is influenced by the Sun gravitational acceleration $\vec{g}(r)$ and the propulsion system of the vehicle implements a thrust T. With this formulation, a maneuver of Earth flyby can be used to gain energy and velocity, that provokes a discontinuity in the relative state variables in the velocity.

The variables are normalized using the radius of the Earth's orbit, the corresponding circular velocity, and the mass of the spacecraft in stationary orbit as values of reference.

The solar electric Propulsion will be considered, therefore, the available power and thrust varies with the square of the distance from the sun.

In the problem, the thrust is the only control during the heliocentric arcs, and it will be optimized to get the minimum consumption, that is measured by the final mass of the spacecraft. Since the thrust appears linearly in the equation of motion, a bang-bang control, that consists of alternating ballistic arcs with arcs of maximum thrust will be required. The trajectory

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is composed by a succession of ballistic arcs (zero-thrust) and arcs of maximum thrust, where the optimal direction will be supplied by the optimization procedure.

The boundary conditions are imposed in satisfactory way at the junctions between trajectory arcs.

The integration initiates when the spacecraft leaves the Earth's sphere of influence, at the position $\vec{r}_i = \vec{r}_{\oplus}(t_i)$ that coincides with the Earth's position, considering the velocity $\vec{v}_i$ free. The hyperbolic velocity is given by $\vec{v}_{\infty i} = \vec{v}_i - \vec{v}_{\oplus}(t_i)$, assuming that a rocket thruster is used to leave the Low Earth Orbit (LEO) with an impulsive maneuver; the vehicle mass on LEO is specified. The increment of velocity (ΔV) demanded to provide the hyperbolic velocity is $\Delta V = \sqrt{v_{\infty i}^2 + v_e^2} - v_c$, where v_e and v_c are the escape and circular velocity at the LEO radius (Santos et. All., 2009.)

The initial mass at the exit from the Earth's sphere of influence is,

$$m_i = (1+\varepsilon)e^{-\frac{\Delta V}{C'}} - \varepsilon \qquad (1)$$

where,

$\varepsilon(1-m_i)$ is the jettisoned mass of the exhausted motor, which is proportional to the propellant mass. The spacecraft intercepts the Earth and accomplishes Gravity Assisted Maneuvers (Santos et al., 2005). The position of the vehicle $\vec{r}_{\pm} = \vec{r}_{\oplus}(t_{\pm})$ is constrained and the magnitude of the hyperbolic excess velocity $\vec{v}_{\infty\pm} = \vec{v}_{\pm} - \vec{v}_{\oplus}(t_{\pm})$ is continuous $v_{\infty+}^2 = v_{\infty-}^2$ [2].

If the minimum height constraint on the flyby is requested, a condition on the velocity turn angle is added:

$$\vec{v}_{\infty+}^T \vec{v}_{\infty-} = -\cos(2\phi) v_{\infty-}^2 \qquad (2)$$

where,

$$\cos(\phi) = \frac{v_p^2}{\left(v_{\infty-}^2 + v_p^2\right)} \qquad (3)$$

v_p is the circular velocity at the low distances allowed for a Planet.

$$\vec{v}_{\infty\pm} = \vec{v}_{i\pm} - \vec{v}_{\otimes} \qquad (4)$$

Where $\vec{v}_{\otimes}$ is the velocity´s Planet.

At the final point (subscript f), the position and velocity vectors of the spacecraft and the asteroid coincide,

$$r_f = r_A(t_f) \qquad (5)$$

$$v_f = v_A(t_f) \qquad (6)$$

The theory of optimal control provides the control law and necessary boundary conditions for optimality.

4. Optimization Procedures

The objective is to use the theory of optimal control to maximize the spacecraft final mass. Dynamical equations are,

$$\begin{aligned} \dot{\vec{r}} &= \vec{v} \\ \dot{\vec{v}} &= \vec{g}(\vec{r}) + \frac{\vec{T}}{m} \\ \dot{m} &= -\frac{\vec{T}}{c} \end{aligned} \qquad (7)$$

Applying the theory of optimal control, the Hamiltonian function is defined as (Lawden, 1954) :

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$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t (\vec{g} + \frac{\vec{T}}{m}) - \lambda_m \frac{\vec{T}}{c} \qquad (8)$$

An indirect optimization procedure is used to maximize the payload. According to Pontryagin's Maximum Principle the optimal controls maximize H.

The nominal thrust T_o at 1 AU, and the electrical power are (Santos, 2009),

$$P_0 = \frac{T_0 c}{2\eta}$$
$$T_{Max} = \frac{T_0}{r^2} \qquad (9)$$

Optimal control theory provides differential equation for the adjoint equations of the problem (Euler-Lagrange). Adjoint equations and the necessary optimal conditions are find in (Santos et all. 2008, [Eq. 15 – 18]).

At the initial point:

1. $\vec{r}_0 = \vec{r}_\oplus$;
2. $m_o = 1 - bV_\infty - cV_\infty^2$
3. $(\vec{v}_0 - \vec{v}_\oplus)^2 = \vec{v}_{\infty 0}^2$;
4. Equations 16 and 18 provide optimal control with λ_{ro} and T_{ro} free;
5. the necessary condition optimal of the state is $\vec{\lambda}_{v0}$ (primer vector) be parallel to the hyperbolic velocity;

At flyby (Santos et all. 2008, [Eq. 15 – 18]):

1. the equations (15 and 16) are used to obtain the transversality conditions, that implicates in determining the arc time used;
2. at the equations (17 and 18) the $\vec{\lambda}_{vi}$ is parallel to the hyperbolic velocity, before and after of free flyby maneuver; the magnitude is continuous;
3. the states of Hamiltonian remain continuous through the *flyby* maneuvers;
4. when the minimum height constraint of the *flyby* is requested, a condition on the velocity turn angle is added (Eq. 2 and 3).

At the final point:

1. $\vec{\lambda}_{vf}$ is parallel to the hyperbolic velocity, $\vec{\lambda}_{rf}$ is parallel to the radius and $\vec{\lambda}_{rf}^t \vec{v}_f + \vec{\lambda}_{vf}^t \vec{g} = 0$;
2. the final values of $\vec{\lambda}_{mf}$ and H_f depends on the control model that was considered in the maneuver;
3. the adjoint variable $\vec{\lambda}_v$ is zero during the whole trajectory.

5. Numerical Analysis With Phall 1 (UNB)

The researchers of the Plasma Laboratory of the Physics Institute of the Brasilia University (UNB), since 2002, pledge in the study and development of a propellant that uses a plasma propulsion system produced by current Hall, based on Stationary Plasma Thrusters (SPT). They use permanent magnets with generating the magnetic field, reducing the electricity consumption.

The characteristics of the spacecraft propulsion system are:

1. the mass of the spacecraft with an altitude of 200 km in circular LEO is 2133.3 Kg; specific impulse I_s = 1607s; specific energy ε = 0.06; T = 2 ·126 mN (thruster Phall 1 - UNB); nominal thruster T_o = 1 UA; The time: *time* = 0 corresponds to the date 01/01/2000.

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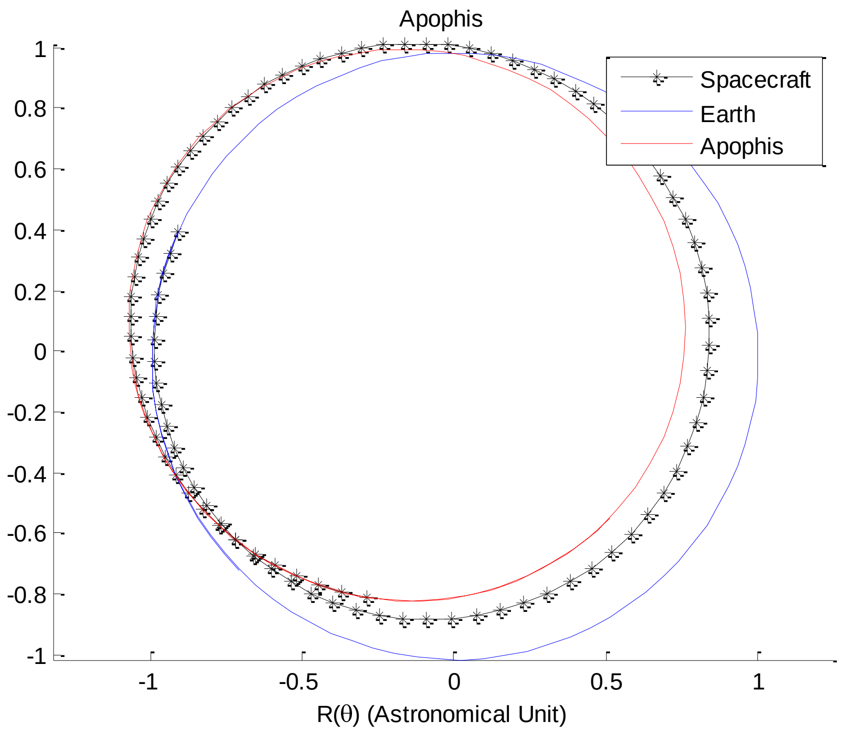

Figure 1 - Trajectory direct (without flyby) for the asteroid Apophis, Departure: 25/02/2014 and Arrival: 06/05/2015, time of maneuvers 435 days. .

The structure of the switching function that shows the thrust arc (red) and coast arc (blue), i.é., shows the alternation between the propulsion arcs and the arcs without propulsion (Figure 2); During the transfer maneuver happen variations in the eccentricity (e), semi-major axis (a) and hamiltonian and energy orbits.

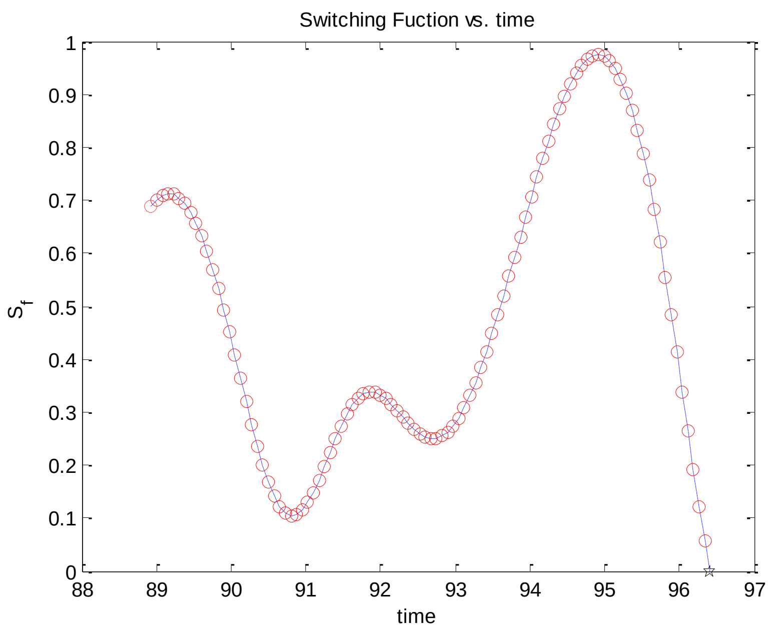

Figure 2 - The Switching Function in Trajectory direct for the asteroid Apophis. The switching function (S_f) shows the alternation between the propulsion arcs and the arcs without propulsion, in this case only the propulsion arc is shown (S_f > 0).

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6. Conclusion

The search for the best initial parameters for a mission is facilitated if the transfer orbit with free time is optimized first. Indirect optimization methods based on optimal control theory supply accurate solutions.

Orbits with Phall 1 had been analyzed using gravity assisted maneuvers and verified resulted optimistical for the implantation of probes using this technology, also being able to use this formularization in the future missions that use launch vehicle that is in development/improvement (VLS-2, Brazil), which can inject in LEO (low earth orbit) a satellite medium sized, thereafter, use the solar electric propulsion (SEP) or nuclear (NEP) to dislocate the vehicle for desired orbits, maximizing them with the maneuver that use assisted gravity.

The present analysis favor a guess at the tentative solution as the Earth's positions as departure and flyby are a priori known. The ideal asteroid has perihelion radius which is close to 1AU, a low-energy orbit and low inclination with relation to the ecliptical axis.

The performance parameters of Phall are competitive with known electromagnet Hall thrusters found on the literature.

The fuel consumption for a mission with multiples flyby's follows the criterion of the asteroid orbit.

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