

Perspectives for Testing Quantum Aspects of Gravity using LISA

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Abstract. LISA should be able to detect the gravitational waves from the QNM ringdown of supermassive black holes in the $10^5 - 10^8$ solar mass range. On the other hand, it is reasonable to think that any quantum theory of gravitation should impose the quantization of the energy levels of these QNM. Here we discuss the possibility of distinguishing quantum aspects of gravity using LISA to observe QNM overtones of highly excited supermassive black holes.

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1. Introduction

Gravitational waves can not escape from within the event horizon of a black hole. However, outgoing waves which originate at the spacetime outside the horizon are possible and are the so called quasi-normal modes (QNMs). There is an extensive amount of literature on QNMs [1, 2, 3, 4, 5, 6], which include good reviews [7, 8].

QNMs of perturbed black holes are the characteristic damped oscillations which display the signatures of these objects. It is expected that they will provide the definitive observational evidence on the existence of black holes. In fact, since QNMs depends uniquely on black hole's mass and angular momentum, they will give us the possibility to determine these parameters by performing black hole gravitational wave spectroscopy. Besides, it has been speculated that QNMs might provide not only the classical signature of black holes but also give some information about their quantum aspects [9]. This opens an interesting possibility to testing quantum aspects of gravity by observing QNMs from existing astrophysical black holes. LISA and other laser interferometer antenna projects in space, such as BBO and DECIGO, seem to be the best choices for observing gravitational waves coming from these QNMs, because the expected signal-to-noise ratio can be enormous in many possible scenarios.

Based on Bohr's correspondence principle, it is possible to think of the high overtones of the ringing frequencies of a black hole as being equally spaced [10]. On the other hand the lowest QNM frequency might be related to the mean irreducible mass associated with the quantum ergosphere [11].

Therefore the quantum aspects of gravity might be imprinted either on the asymptotic behavior of the high overtones (on their relative frequency separation) or on the wavelength of the fundamental ones (their absolute frequencies), or on both.

QNMs excited by coalescence of supermassive black holes (SMBHs) from galaxy mergers or from newly formed SMBHs, among other astrophysical events, are very important sources for LISA [12].

LISA should be able to detect the gravitational waves from the QNM ringdown of supermassive black holes in the $10^5 - 10^8$ solar mass range throughout the observable Universe [13], as well as having the potential to perform no-hair tests [14].

Here we analyze the perspectives for testing quantum aspects of gravity using LISA.

2. Quasinormal Modes of Supermassive BHs and Possible Astrophysical Scenarios of Excitation

From theoretical calculations it appears that there is a high probability that most energy is emitted in the fundamental mode of the quadrupole ($l=2$) [15], leaving the other overtones of the quadrupole and the other multipoles with a much smaller portion of the total energy emitted. The physical explanation of this might be related to the time scale of the free falling ("plunge") of bodies from the innermost stable circular orbit ($\sim 6GM/c^3$) into a BH and the last stable orbit period ($\sim 2\pi \cdot 6GM/c^3$), which are both around the period of the fundamental mode of the quadrupole ($\sim 2.7 \cdot 6GM/c^3$). In other words, the time scales of most of the physical processes that excite the BH are closely tuned to the fundamental ($n=0, l=2$) QNM. This makes the amount of energy that goes to the other modes very sensitive to the details of the astrophysical event that produces excitation. For a "point test particle" of mass m falling radially into a Schwarzschild black hole of mass $M \gg m$, Davis et al. 1971 found the distribution of the total energy to the multipoles $l=2, l=3, l=4, l=5, l=6$ to be about 0.879, 0.105, 0.0134, 0.00191, 0.000268, respectively.

Expressions for the frequencies of QNMs with very large imaginary parts can be found in Anderson 93 [16], Nollert 93 [17], Liu 95 [18], Nollert 99 [7], Kokkotas 99 [8], and Abdalla 07 [19].

Two important ingredients that should be taken into account are the rotation of BHs [20, 21] and the accretion of matter (dust shells and thick accretion disks) onto the black hole [22, 23, 24, 25].

We should analyze how they will affect the precision of the QNM frequency measurement and, therefore, on our capability of testing quantum aspects of gravity with these measurements.

QNMs of SMBHs can be strongly excited and emit gravitational waves under a finite number of possibilities. We may consider the following:

- the formation of a SMBH from a protostar;
- the coalescence of two smaller SMBHs;
- the capture of stars by a SMBH;
- the falling of large amounts of baryonic matter (rocks, dust, and gas) into the SMBH.

It is possible that there are other exotic possibilities (maybe involving cosmic strings or dark matter), but we will restrict our analysis to the above set.

After a quick inspection of the above list, we can conclude that almost all major possible sources of excitation of the QNMs of SMBHs have time scales related to the free falling time (“plunge”) from the innermost stable circular orbit (from $\sim 3 R_{Sch} = R_{ISCO}$) and the last stable orbit periods. The exception might be the formation of the SMBH from a supermassive very low metallicity protostar and the cases close to equal-mass SMBHs.

3. Analysis and Discussion Considering Future Observations from LISA

Using numerical relativity simulations of non-spinning binary black holes mergers Berti et al. 2006 and 2007 [13, 14] analyzed the problem of detecting ringdown waveforms and of estimating the source parameters, showing that LISA has the potential to perform no-hair tests of general relativity. They computed the expected signal-to-noise ratio for ringdown events, the relative parameter estimation accuracy, and the resolvability of different modes. They also discussed the extent to which uncertainties on physical parameters, such as the black hole spin and the energy emitted in each mode, will affect the ability of performing black hole spectroscopy. Ioka and Nakano 2007 [26] also studied the problem of higher perturbative order of QNMs in binary BH mergers. They found that the second-order QNMs ($l=4$) have frequencies twice those of the first-order ones ($l=2$) and the GW amplitude is up to $\sim 10\%$ of that of the first order one, in agreement with the previous findings of Davis et al. 1971 [15]. They also compared these characteristic GW amplitude curves (first-, second-, and with third-order) with the sensitivity curves of LISA and Ultimate DECIGO.

How feasible is it testing quantum aspects of gravity using LISA and others laser interferometer projects in space?

One really promising thing when we talk about these detectors is: they will measure gravitational waves from astrophysical events with huge signal to noise ratios, especially when they come from SMBHs. Some of these events will be seen at the border of the observable universe.

In order to perform our analysis we are going to choose one event with high signal to noise ratio and, likewise, with smaller errors for the determination of the astrophysical parameters such as rotation (spin). Our SMBH, therefore, should have its QNMs in the highest sensitivity band of LISA, namely from ~ 2 to 20 mHz, which is in agreement with the results found by Berti et al. 2006 [13]. Our SMBH will have 3.7×10^6 solar masses, which is the mass of the putative BH at the center of our Milky Way. For scenarios at high redshifts the total mass should go down from the 3.7×10^6 solar mass value by a factor of $(z + 1)$.

In Figure 1 we plotted the characteristic GW amplitude (empirical) curves for the fundamental ($n=0$) and the seven first excited overtones plus the 70th excited overtone for the first-order ($l=2$, quadrupole) QNM of a Schwarzschild BH for three possible astrophysical scenarios. The excited overtones 8th to 69th were omitted in order to avoid overloading the graph with curves. The LISA sensitivity curve plotted is for bursts and $S/N \sim 5$. This means that the fundamental and the first 70 overtones for $l=2$ are detectable with this chosen sensitivity threshold. A Kerr BH would have its QNMs shifted in frequency to the right, to higher frequencies.

We assumed for this calculation the parameters listed in Table 1. The total amplitude emitted in the form of GWs follows the known expression [27, 28, 29]:

$$h_{eff} = 2 \times 10^{-21} \left(\frac{\epsilon}{0.01} \right) \left(\frac{d}{10 Mpc} \right)^{-1} \left(\frac{\mu}{M_{solar}} \right), \text{ where } \epsilon \text{ was taken as } 0.01.$$

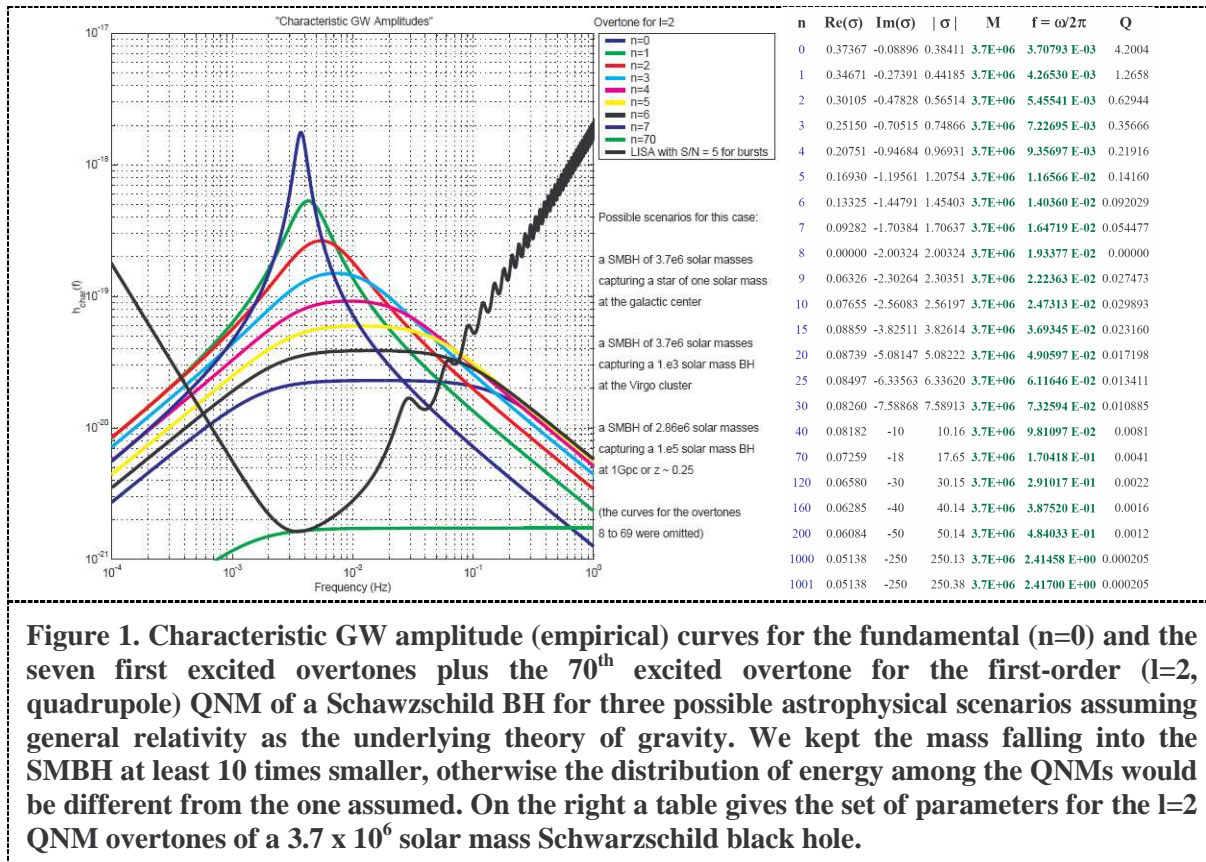


Figure 1. Characteristic GW amplitude (empirical) curves for the fundamental (n=0) and the seven first excited overtones plus the 70th excited overtone for the first-order (l=2, quadrupole) QNM of a Schwarzschild BH for three possible astrophysical scenarios assuming general relativity as the underlying theory of gravity. We kept the mass falling into the SMBH at least 10 times smaller, otherwise the distribution of energy among the QNMs would be different from the one assumed. On the right a table gives the set of parameters for the l=2 QNM overtones of a 3.7 x 10⁶ solar mass Schwarzschild black hole.

The partition of energies among the multimodes were assumed to follow the one proposed by Davis et al. 1971 [15], and the partition among the overtones were assumed to be proportional to the ratios $Q_n^2/\Sigma Q_n^2$, where $Q_n = \pi f_n t_n$ is the quality factor of the QNM overtone n, and ΣQ_n^2 is the sum of all (infinite) Q_n^2 . This partition is assumed valid only in the case the astrophysical events that excite the QNMs have their Fourier peak below the fundamental mode (n=0) frequency. This is not the case of equal-mass coalescences, for example, but it is when $m_{\text{captured}} \ll M_{\text{SMBH}}$. The characteristic GW amplitude curves plotted in Figure 1 are empirical, but are in agreement with the shape of the curves found by Ioka and Nakano 2007 [26].

The results might be off from a rigorous calculation, but they are satisfactory for the point we wish to make. It is clear from this graph that the fundamental overtone, when emitted, masquerades the shape of the other overtones, making it difficult for one to determine their nominal frequencies and Qs. Even using special techniques as the ones mentioned by Berti et al. 2006 and 2007 [13, 14], it will almost be impossible to determine frequencies of very high overtones with the required precision for measuring the frequency spacing. The high overtones form a kind of single “flat” background signal. Perhaps only the fundamental and the five first excited overtones will be determined with any satisfactory precision.

The situation might be a little bit better in the cases of close to equal-mass coalescences. Medium overtones (n ~ 10) might be more excited compared to the ones with the partition of energy assumed above, but still this does not help the very high overtones (n > 100) to be measured with precision.

If one wants to find quantum aspects of gravity from SMBH QNMs measured using LISA, one has probably to find them in the absolute frequencies and spacing among the fundamental and first overtones.

It is puzzling to note that the angular frequency (ω) of the fundamental QNM for $l=2$ is only 20% off from the interesting expression:

$$\omega (l=2;n=0) \sim \frac{\hbar}{\pi L_P^2 M}, \text{ where } \pi L_P^2 \text{ is the area of a sphere with a diameter equal to the Planck}$$

length, and M is the mass of the star;

which can be rewritten as:

$$\omega (l=2;n=0) \sim \frac{M_P}{\pi T_P M}, \text{ where } M_P \text{ is the Planck mass and } T_P \text{ is the Planck time.}$$

4. Hawking Radiation

There is one curious coincidence connecting the Hawking radiation, SMBHs, and the laser interferometer space antennas such as LISA, BBO, and DECIGO. Even though it is strange to talk about black body radiation for “thermal” temperatures of 10^{-11} K to 10^{-15} K, these are the temperatures where SMBHs in the $10^4 - 10^8$ solar mass range will peak in the sensitive band ($10^{-4} - 1$ Hz) of LISA, BBO and DECIGO for the Hawking gravitational radiation. The flux emitted in this band, however, is negligible and, so is the correspondent h . This is a pity, because the Hawking radiation on gravitational waves would be an interesting tool for probing quantum aspects of gravity. Only for very low mass black holes might the flux (of high frequency gravitational waves) be measurable some day.

5. Conclusion

Three astrophysical events involving Schwarzschild BHs were chosen as examples that would produce the same set of characteristic GW amplitude curves for LISA, with high signal to noise ratio. The curves were calculated empirically, assuming a partition of energies among the multimodes and among the overtones of the quadrupolar mode and that general relativity is the underlying theory of gravity (could other theories give significantly different results?). The signal to noise ratio was strong enough to keep up to the 70th excited overtone within the reach of the burst sensitivity curve for LISA with $S/N \sim 5$.

From these curves it was apparent that the fundamental overtone, when emitted, masquerades the shape of the other overtones, making it difficult for one to determine their nominal frequencies and Q_s . Even using special techniques it is unlikely that the frequencies of very high overtones can be determined with the required precision for measuring the frequency spacing.

From the measurements of SMBH QNM using LISA, it is likely that only the fundamental and first overtones will be available for one to look for quantum aspects of gravity. Perhaps the frequencies of the fundamental QNMs themselves are directly related to quantum principles.

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