Low Thrust Maneuvers for Artificial Satellites

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Abstract: - An orbital maneuver is an important phase of a space mission. The idea is to change the orbit of a spacecraft, in order to be able to complete the mission goals required. It can be done to insert a spacecraft in its nominal orbit or during the mission to compensate undesired effects of perturbations. To perform the orbital maneuvers, a software that calculates an optimal maneuver is developed. This method will be used as a reference for comparison and analises of the suboptimal methods to be used on board. This method id based on an analytical development that generate equations that can be computed in a shorter time, allowing real time applications. In all the simulations, low thrust is used to make the maneuvers.

Key-Words: - Orbital maneuvers, low thrust, astrodynamics, artificial satellites, orbital dynamics.

1 Introduction

The problem of calculating orbital maneuvers is a very important topic in Orbital Mechanics. Thus, the problem of transferring a spacecraft from an orbit to another has grown in importance in recent years. Applications of this study can be found in various space activities, such as placing a satellite in geostationary orbit, the maneuvers of a space station, orbit maintenance of a satellite, among others. More information about this type of maneuvers can be found in references [1], [2] and [3].

In actual applications, there may be a need to make an additional maneuver, both for a transfer orbit or only for periodic corrections of lesser magnitude. This issue of transfer is to change the position, velocity and mass of the satellite from its current status to a new state pre-determined. The transfer may be completely constrained or partially free (free time, free final velocity, etc.). In the most general case, the choice of direction, sense and magnitude of the thrust to be applied should be made, respecting the limits of the available equipment. To carry out this transfer, it is intended to use optimal or sub-optimal continuous maneuvers [4], [5]. So, to fulfill that task, two methods for calculating maneuvers were developed.

The first of them will get an optimization without worrying about the processing time. The second method is sub-optimal and it will approach the directions of application of the thrust to allow a faster calculation of the control.

The optimum method will be used to compare the consumption obtained by the sub-optimal method, which involves simplifications for each special situation, in order to obtain a high processing speed, favoring the possibility of using it in real-time. In both methods, it will be assumed that the magnitude of the thrust to be applied is constant and small and the search will be to find its direction. This direction can be free (optimal method) [6] or with some kind of constraints (sub-optimal method).

2 Sub-optimal Method

The goal of this topic is to develop a sub-optimal method with high-speed computing for the calculation of orbital maneuvers based on continuous thrust and small magnitude. The idea is to have a method that generates quick result and, if possible, with a result in terms of cost of fuel not much different from the optimal method described above. This method should be used in cases of transfers with small magnitude, which usually are more frequent in the steps that follow the insertion of the spacecraft in its nominal orbit. To solve this problem, it was chosen in the literature a base method to make expansions and adjustments to the needs of this work. The method is described below.

A near optimal method for calculating orbital transfer and with minimum time (so, minimum consumption, as the magnitude of the thrust is constant, what implies that the time of application of thrust and consumption are directly proportional) around the Earth for spacecraft with electric solar propellant was developed by [7]. It used a technique of direct optimization to solve the problem of optimal control, with approaches toward application of thrust. The optimal trajectories calculated by the direct approach present very close results to the optimal trajectories obtained from variational calculation.

The equations of motion for the vehicle when the thrust is acting are shown below. The equations are written in terms of non-singular equinotial elements to cover both circular and planar orbits (i = 0 °, 180). The relation between the equinotial elements (a, h, k, p, q, F) and the classical orbital elements (a, e, i, W, W, E) is given by:

$$h = e\,\mathrm{sen}(\omega + \Omega) \quad (1)$$

$$k = e\cos(\omega + \Omega) \quad (2)$$

$$p = \tan\left(\frac{i}{2}\right)\mathrm{sen}\,\Omega \quad (3)$$

$$q = \tan\left(\frac{i}{2}\right)\cos\Omega \quad (4)$$

$$F = \Omega + \omega + E \quad (5)$$

where: a = semi-major axis, e = eccentricity, i = inclination, W = longitude of the ascending node, w = argument of perigee, E = eccentric anomaly and F = eccentric longitude.

For a spacecraft moving in the gravitational field and subject to the propulsive force, the equations of motion are as follows:

$$x' = a_T M\hat{\alpha} \quad (6)$$

In equation 6 the state vector is x = [a, h, k, p, q] T and the sign (') indicates the derivative with respect to time. The vector (3 x 1) is a unit vector along the direction of thrust application. The value aT is the magnitude of the thrust acceleration, given by:

$$a_T = \frac{2\eta P_0}{mgI_{sp}} \quad (7)$$

where h is the efficiency of the propulsion system, P0 is the initial power given propulsion system, m is the mass of the vehicle, g is the gravity acceleration at sea level and Isp is the specific impulse. The equation of state for F is not included because it has been used the average of orbital elements and thus only elements which vary slowly are considered.

The processing time is significantly reduced when using orbital averages. As all orbital elements used are variables that vary slowly, due to the fact that the force of thrust has little magnitude, it can be used major steps of integration, in the order of days. The equation of motion of the spacecraft can be approximated by calculating the increment of each orbital element in a period and dividing by such a time. Therefore, the variation in time of the equinotial elements by complete orbit with the propeller acting can be obtained from the equation:

$$\bar{\mathbf{x}}' = \frac{1}{T}\int_{-\pi}^{\pi} \bar{\mathbf{a}}_T \overline{\mathbf{M}}\hat{\alpha}\frac{dt}{dF}dF \quad (8)$$

where is the approximation of the state and T is the orbital period. The bar on top of variables means that they were evaluated using the average state vector. The integration represents the change in orbital elements in a revolution with the orbital elements kept constant, unless the eccentric longitude F, which is varied between -p e +p.

Setting the direction for the thrust application by the components (id, jd, kd), the product shown inside the integral symbol can be obtained. As the acceleration is held constant, it means that this value can be placed outside of the integral symbol. Thus, the analytical equations used for the terms corresponding to each of the elements are shown below, where id, jd and kd represent the three components of the direction vector of the thrust application.

For the semi-major axis (a):

$$\dot{a} = \frac{2\,jdA_1}{\sqrt{\frac{\mu}{a^3}}\left(1 - k\cos F - h\,\mathrm{sen}\,F\right)} + \frac{2idA_2}{\sqrt{\frac{\mu}{a^3}}\left(1 - k\cos F - h\,\mathrm{sen}\,F\right)} \quad (9)$$

For the element h:

$$\dot{h}=\frac{id\sqrt{1-h^2-k^2}\left(a*B_1-B_2\right)}{a^2\sqrt{\frac{\mu}{a^3}}}+\frac{jd\sqrt{1-h^2-k^2}\left(-B_3+a\left(B_4+B_5\right)\operatorname{sen}F\right)}{a^2\sqrt{\frac{\mu}{a^3}}}+$$
$$+\frac{kkd\left(-a*p*B_6+a*q*B_7\right)}{a^2\sqrt{1-h^2-k^2}\sqrt{\frac{\mu}{a^3}}} \tag{10}$$

For the element k:

$$\dot{k}=-\frac{jd\sqrt{1-h^2-k^2}\left(a*C_1+C_2\right)}{a^2\sqrt{\frac{\mu}{a^3}}}-\frac{id\sqrt{1-h^2-k^2}\left(a\left(C_3\cos F+C_4\right)+C_5\right)}{a^2\sqrt{\frac{\mu}{a^3}}}+$$
$$-\frac{hkd\left(-a*p*C_6+a*q*C_7\right)}{a^2\sqrt{1-h^2-k^2}\sqrt{\frac{\mu}{a^3}}} \tag{11}$$

For the element p:

$$\dot{p}=\frac{kd(1+p^2+q^2)\left(-h+\frac{hk\cos F}{1+\sqrt{1-h^2-k^2}}+\left(1-\frac{k^2}{1+\sqrt{1-h^2-k^2}}\right)\operatorname{sen}F\right)}{2a\sqrt{1-h^2-k^2}\sqrt{\frac{\mu}{a^3}}} \tag{12}$$

For the element q:

$$\dot{q}=\frac{kd(1+p^2+q^2)\left(-k+\left(1-\frac{h^2}{1+\sqrt{1-h^2-k^2}}\right)\cos F+\frac{hk\operatorname{sen}F}{1+\sqrt{1-h^2-k^2}}\right)}{2a\sqrt{1-h^2-k^2}\sqrt{\frac{\mu}{a^3}}} \tag{13}$$

These equations are written in terms of equinotial orbital elements. It is also possible to write them according to the traditional keplerian elements. In this case they are:

$$\dot{a}=\frac{-\left(C_8\right)\hat{j}\cos F+e^2\hat{j}\cos\left[F-2\left(\Omega+\omega\right)\right]+\hat{i}\left(\left(C_8\right)\operatorname{sen}F+e^2\operatorname{sen}\left[F-2\left(\Omega+\omega\right)\right]\right)}{\left(1+\sqrt{1-e^2}\right)\sqrt{\frac{\mu}{a^3}\left(-1+e\cos\left[F-\Omega-\omega\right]\right)}} \tag{14}$$

$$\dot{h} = \frac{1}{8a\sqrt{1-e^2}\sqrt{\frac{\mu}{a^3}}} *$$

$$\Bigg(8(1-e^2)\hat{i}\Bigg(-1+\frac{e\,\mathrm{sen}\,F\,\mathrm{sen}(\Omega+\omega)}{1+\sqrt{1-e^2}}+\frac{e^3\cos(\Omega+\omega)^2\,\mathrm{sen}\,F\,\mathrm{sen}(\Omega+\omega)}{\sqrt{1-e^2}\left(1+\sqrt{1-e^2}\right)^2}+$$

$$+\frac{e^3\cos F\cos(\Omega+\omega)\,\mathrm{sen}(\Omega+\omega)^2}{\sqrt{1-e^2}\left(1+\sqrt{1-e^2}\right)^2}+$$

$$\frac{e\,\mathrm{sen}(\Omega+\omega)\left(\left(e^2-2\left(1+\sqrt{1-e^2}\right)\right)\mathrm{sen}\,F-e^2\,\mathrm{sen}[F-2(\Omega+\omega)]\right)}{2\left(1+\sqrt{1-e^2}\right)^2(-1+e\cos[F-\Omega-\omega])}\Bigg)-$$

$$\Big(e\sqrt{1-e^2}\,\hat{j}\Big(-e\left(e^2-2\left(1+\sqrt{1-e^2}\right)\right)\mathrm{sen}(2F)+\left(-8+4e^2-8\sqrt{1-e^2}\right)\mathrm{sen}(F-\Omega-\omega)+$$

$$+6e\,\mathrm{sen}[2(F-\Omega-\omega)]-4e^3\,\mathrm{sen}[2(F-\Omega-\omega)]+$$

$$+6e\sqrt{1-e^2}\,\mathrm{sen}[2(F-\Omega-\omega)]+2e\,\mathrm{sen}[2(\Omega+\omega)]-$$

$$+2e^3\,\mathrm{sen}[2(\Omega+\omega)]+2e\sqrt{1-e^2}\,\mathrm{sen}[2(\Omega+\omega)]-8\,\mathrm{sen}(F+\Omega+\omega)+$$

$$+6e^2\,\mathrm{sen}(F+\Omega+\omega)-$$

$$+8\sqrt{1-e^2}\,\mathrm{sen}(F+\Omega+\omega)+2e^2\sqrt{1-e^2}\,\mathrm{sen}(F+\Omega+\omega)-2e^2\,\mathrm{sen}[F-3(\Omega+\omega)]-$$

$$+2e^2\sqrt{1-e^2}\,\mathrm{sen}[F-3(\Omega+\omega)]+e^3\,\mathrm{sen}[2(F-2(\Omega+\omega))]\Big)\Big)/$$

$$\left(\left(1+\sqrt{1-e^2}\right)^2(-1+e\cos[F-\Omega-\omega])\right)-\frac{4e\hat{k}\cos(\Omega+\omega)}{1+\sqrt{1-e^2}}*$$

$$\frac{\left(\left(e^2-2\left(1+\sqrt{1-e^2}\right)\right)\mathrm{sen}[F-\Omega]+e\left(e\,\mathrm{sen}[F-\Omega-2\omega]+2\left(1+\sqrt{1-e^2}\right)\mathrm{sen}\,\omega\right)\right)\tan\frac{i}{2}}{1+\sqrt{1-e^2}}\Bigg) \qquad (15)$$

$$\dot{k} = \frac{1}{8a\sqrt{1-e^2}\sqrt{\frac{\mu}{a^3}}} *$$

$$\Bigg(-8(1-e^2)\hat{j}\Bigg(-1+\frac{e\cos F\cos(\Omega+\omega)}{1+\sqrt{1-e^2}}+$$

$$+\frac{e\cos(\Omega+\omega)\left(\left(e^2-2\left(1+\sqrt{1-e^2}\right)\right)\cos F+e^2\cos[F-2(\Omega+\omega]]\right)}{2\left(1+\sqrt{1-e^2}\right)^2(-1=e\cos[F-\Omega-\omega])}+$$

$$-\frac{e^3\cos(\Omega+\omega)^2\,\mathrm{sen}\,F\,\mathrm{sen}(\Omega+\omega)}{\sqrt{1-e^2}\left(1+\sqrt{1-e^2}\right)^2}+\frac{e^3\cos F\cos(\Omega+\omega)\,\mathrm{sen}(\Omega+\omega)^2}{\sqrt{1-e^2}\left(1+\sqrt{1-e^2}\right)^2}\Bigg)+$$

$$+\Big(e\sqrt{1-e^2}\,\hat{i}\Big(-e(e^2-2)\left(1+\sqrt{1-e^2}\right)\Big)\mathrm{sen}(2F)+\left(8-4e^2+8\sqrt{1-e^2}\right)\mathrm{sen}(F-\Omega-\omega)+$$

$$-6e\,\mathrm{sen}[2(F-\Omega-\omega)]+$$

$$+4e^3\,\mathrm{sen}[2(F-\Omega-\omega)]-6e\sqrt{1-e^2}\,\mathrm{sen}[2(F-\Omega-\omega)]+2e\,\mathrm{sen}[2(\Omega+\omega)]+$$

$$-2e^3\,\mathrm{sen}[2(\Omega+\omega)]+2e\sqrt{1-e^2}\,\mathrm{sen}[2(\Omega+\omega)]-8\,\mathrm{sen}(F+\Omega+\omega)+$$

$$
\begin{aligned}
&+6e^2\,\mathrm{sen}(F+\Omega+\omega)+\\
&-8\sqrt{1-e^2}\,\mathrm{sen}(F+\Omega+\omega)+2e^2\sqrt{1-e^2}\,\mathrm{sen}(F+\Omega+\omega)-2e^2\,\mathrm{sen}\left[F-3(\Omega+\omega)\right]+\\
&-2e^2\sqrt{1-e^2}\,\mathrm{sen}\left[F-3(\Omega+\omega)\right]+e^3\,\mathrm{sen}\left[2\left[F-2(\Omega+\omega)\right]\right]\Big)\Big)/\\
&\left(\left(1+\sqrt{1-e^2}\right)^2\left(-1+e\cos\left[F-\Omega-\omega\right]\right)\right)+\\
&+\frac{4e\hat{k}\left(\left(e^2-2\left(1+\sqrt{1-e^2}\right)\right)\mathrm{sen}(F-\Omega)\right)}{1+\sqrt{1-e^2}}+\\
&\left.+\frac{e\left(e\,\mathrm{sen}\left[F-\Omega-2\omega\right]+2\left(1+\sqrt{1-e^2}\right)\mathrm{sen}\,\omega\right)\mathrm{sen}(\Omega+\omega)\tan\left(\frac{i}{2}\right)}{1+\sqrt{1-e^2}}\right)
\end{aligned}
\tag{16}
$$

$$
\dot{p}=-\frac{\hat{k}\sec\left(\frac{i}{2}\right)^2\left(\left(e^2-2\left(C_9\right)\right)\mathrm{sen}\,F+e\left(2\left(C_9\right)\mathrm{sen}(\Omega+\omega)+e\,\mathrm{sen}\left[F-2(\Omega+\omega)\right]\right)\right)}{4a\sqrt{1-e^2}\left(C_9\right)\sqrt{\frac{\mu}{a^3}}}
\tag{17}
$$

$$
\dot{q}=\frac{\hat{k}\left(C_{10}\cos F+e\left(-2\left(C_9\right)\cos(\Omega+\omega)+e\cos\left[F-2(\Omega+\omega)\right]\right)\right)\sec\left(\frac{i}{2}\right)^2}{4a\sqrt{1-e^2}\left(C_9\right)\sqrt{\frac{\mu}{a^3}}}
\tag{18}
$$

The calculus of integration shown in Equation 8 generates equations that are too long for real applications, especially taking into account the need to implement them in real time for a maneuver. Thus, although these equations, in their complete form shown here, generate a new method for calculating orbital maneuvers, their use will be focused on individual cases. This implies to define a reference orbit, which may be the final desired orbit, the initial orbit of the spacecraft or even an average of those two orbits. As it will be considered only maneuvers with small amplitudes, this restriction will not bring great losses in terms of accuracy.

Then, with these approaches made, numerical values can be used, so that, these functions are only functions of F and numerical constants. From there, the integral used in equation 8 can be calculated and it is obtained a simple analytical equations for the variation of each orbital element considered as a function of direction and magnitude of the thrust applied. Thus, the problem of obtaining the lowest fuel consumption in a maneuver can be defined as to find the optimal direction for thrust application that minimizes:

$$J = t_f \tag{19}$$

Subject to the mean equations of motion and the initial condition:

$$\overline{x}(0) = x_0 \tag{20}$$

and also subject to the ties in the final state:

$$\psi\left[\overline{x}(t_f), t_f\right] = \overline{x}(t_f) - x_f = 0 \tag{21}$$

3 Results

Several maneuvers were simulated to test the methods developed [8]. The first two maneuvers involve the same initial orbit, but the directions of thrust application and time of the operation are different, with the goal of reaching a final orbit farther. The third maneuver involves a greater range of variation in a semi-major axis and it is good to demonstrate the applicability of the method in situations like this.

3.1 Maneuver 1

In this specific case, the semi-major axis was changed and eccentricity and the argument of perigee of the orbit were kept constants. The changes were small in magnitude (about 47 meters in a semi-major axis, the main objective of the maneuver) to be compatible with the method developed. The argument of perigee was kept constant as 90 degrees, but it could be any value.

Table 1 shows the elements of the initial orbit and the components of the orbit to be achieved after maneuver 1.

Table 2 shows the input data required for the first maneuver simulation with the optimal method: orbital elements of the initial orbit, the vehicle characteristics (initial mass, the magnitude of thrust, initial position of the vehicle, true anomaly), the condition imposed to the final orbit and estimation of the solution to start the process of iteration (beginning and end of propulsion, angles of pitch and yaw and their rates of initial change and an estimation of fuel consumption).

Table 1: Elements of initial and final orbit for maneuver 1.

Initial orbit	**Condition in the final orbit**
Semi-major axis 7259,650 km	Semi-major axis 7259,697 km
Eccentricity 0,0629	Eccentricity 0,0629
Inclination 66,52º	Arg. of perigee 90º
Long. of ascending node 110º	
Arg. of perigee 90º	

Optimal case

Table 2: Data for the maneuver 1 using the optimal method.

Spacecraft initial data	Total mass (vehicle + fuel) 2500 kg
	Available thrust = 1 N
	Initial position = 0
	True anomaly = 0º
Initial estimate of solution	Start of the engime = 0º
	Stop of the engine = 5º
	Initial pitch angle = 0º
	Initial yaw angle = 0º
	Initial rate of variation in pitch = 0
	Initial rate of variation in yaw = 0
	Fuel needed to maneuver = 2 kg

Sub-optimal case

Turning the initial keplerian elements to non singular elements, according equations 1 to 5:

a = 7,25965x106 m

h = -0,0220906

k = -0,0588826

p = 0,614085

q = -0,230383

Therefore, now the numerical values of the orbit can be used, which will be used as the reference orbit, carrying out the integration shown in equation 8 and in this way, it is possible to obtain a set of equations that provide the variation of each of the elements used to describe the orbit by orbital revolution with the propellers acting all the time. Therefore, they will become the equations of motion of the spacecraft with the assumptions adopted. These equations, already taking into account the fact that it was a planar maneuver, so, kd = 0, as a function of the components of the vector that defines the direction of thrust applied, are:

daa= acel*(1987180*id+5296820*jd) (22)

dha= acel*(-0,832233*id+0,00831686*jd) (23)

dka= acel*(-0,00831686*id+0,851282*jd) (24)

dpa= =0 (25)

dqa= =0 (26)

where accel is the acceleration imposed by the satellite propellant.

To show in detail the usefulness of these equations, figures 1 to 3 show the variation of the elements by orbit as a function of the direction of the thrust applied. It is possible to get many informations about the effect of the direction of the thrust applied in orbital elements. Figure 1, made for the situation where the direction of the thrust applied is a constant, shows that there is a value of the component x for which the semi-major axis shows a maximum variation. This value is around 0.35. That figure may be used for a prior assessment of the direction of thrust applied depending on the objectives of the mission.

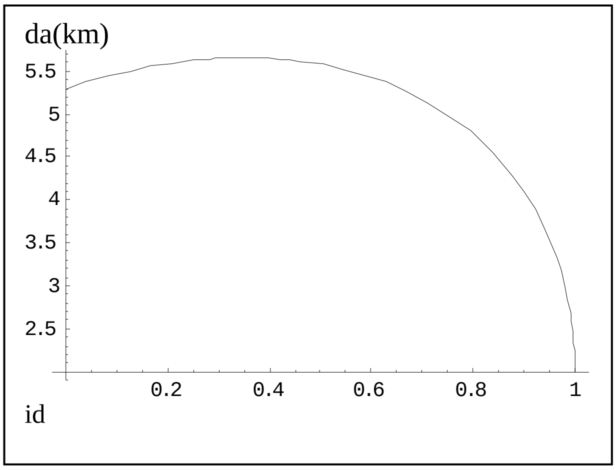

Fig. 1: Variation of the semi-major axis as a function of the direction of the thrust applied.

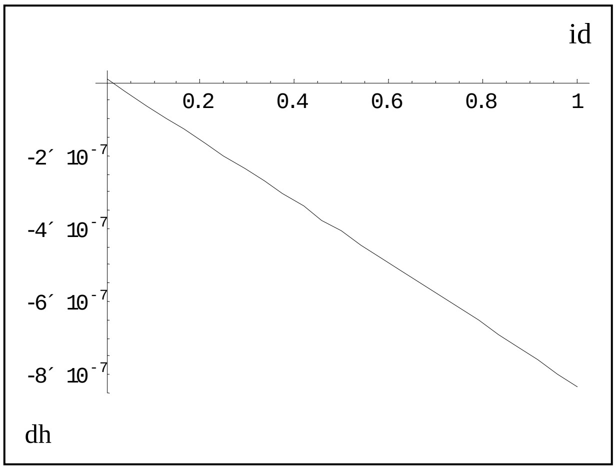

Fig. 2: Variation of the orbital element h as a function of the direction of the thrust applied.

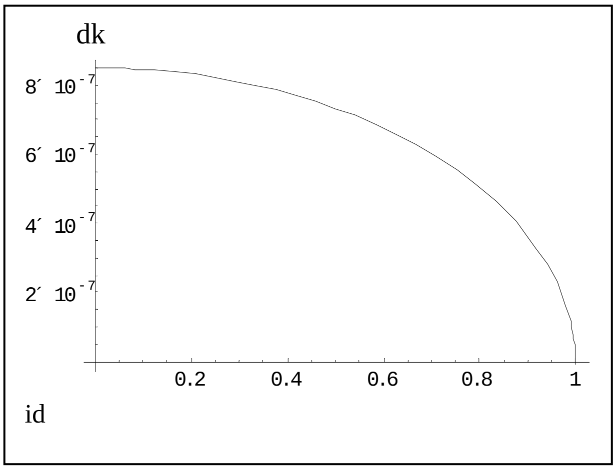

Fig. 3: Variation of the orbital element k as a function of the direction of the thrust applied.

As p and q are constants, graphic is not shown.

With these equations, the software Mathematica is used to solve the optimization problem and to obtain the optimal solution. Several assumptions can be made about the direction of the thrust applied. The simplest of them is assuming a constant direction. So, the problem becomes to find the value of id that generates the minimum fuel consumption, because kd = 0 (planar maneuver) and jd is obtained by the condition that the vector that defines the direction of the thrust applied is unit. The solution found is id = 0.38. Considering linear or parabolic relations can reduce the consumption so much and the time of maneuver obtained, but it is not studied in this part of the work.

Table 3 shows the final orbit achieved by the spacecraft after the maneuver, for the optimal and sub-optimal method, as well as the fuel consumed and time of make the maneuver.

Table 3: Final keplerian elements obtained for maneuver 1.

Final elements	**Optimal method**	**Sub-optimal method**
Semi-major axis (km)	7259,697	7259,697
Eccentricity	0,062887	0,062887
Inclination (°)	66,52	66,52
Long. of ascending node (°)	110	110
Arg. of perigee (°)	90	90
Consumption (kg)	0,1652	0,2808
Duration of the maneuver (min)	302	512

3.2 Maneuver 2

For maneuver 2, the semi-major axis was also changed and the eccentricity and the argument of perigee of the orbit were kept constant. The changes were of a magnitude slightly higher than in the previous case, about 120 meters in semi-major axis, the main objective of the maneuver, also aiming to be compatible with the method developed. The argument of perigee was kept constant in value 90 degrees, but could be any value.

Table 4 shows the elements of the initial orbit and the conditions imposed for maneuver 2.

Table 4: Elements of initial and final orbit for maneuver 2.

Initial orbit	Condition in the final orbit
Semi-major axis 7259,650 km	Semi-major axis 7259,770 km
Eccentricity 0,062900	Eccentricity 0,062900
Inclination 66,52º	Arg. of perigee 90º
Long. of ascending node 110º	
Arg. of perigee 90º	

Optimal case

Table 2 also shows the parameters used for maneuver 2, with the optimum method.

Sub-optimal case

As the initial orbit (which is also used as the reference orbit) is the same as the previous example, both the initial orbital elements and the approximate equations of motion are the same.

So, the software Mathematica is again used to solve the problem of optimization and to get the optimal solution. Once more it will be assumed a constant direction. In this way, the problem becomes to find the value of id that generate the minimum fuel consumption, because kd = 0 (planar maneuver) and jd is obtained by the condition that the vector that defines the direction of the thrust applied is unit. The solution found is id = 0.41.

Table 6 shows the final orbit achieved by the spacecraft after the maneuver, for the optimal and sub-optimal methods, as well as the fuel consumed and the time of the maneuver.

Tabela 6 - Final keplerian elements obtained for maneuver 2.

Final elements	Optimal method	Sub-optimal method
Semi-major axis (km)	7259,77	7259,77
Eccentricity	0,0628876	0,0628876
Inclination (°)	66,52	66,52
Long. of ascending node (°)	110	110
Arg. of perigee (°)	90	90
Consumption (kg)	0,2520 kg	0,3900 kg
Duration of the maneuver (min)	601,8	1020,2

3.3 Maneuver 3

The following test will be a maneuver with greater magnitude, with a change of about 31 km in semi-major axis and 0.0026 in eccentricity and also around 38 degrees in the argument of perigee. Table 7 shows the input data of maneuver 3. These results show that this method can be applied to transfers involving changes of the order of tens of kilometers in a semi-major axis.

Table 7: Elements of initial and final orbit for maneuver 3.

Initial orbit	Condition in the final orbit
Semi-major axis 7738,87 km	Semi-major axis 7707,438 km
Eccentricity 0,00371677	Eccentricity 0,0011589
Inclination 66,0353º	Arg. of perigee 90º
Long. of ascending node 7,57006º	
Arg. of perigee 128,059º	

Optimal case

Table 8 shows the input data for the optimal method in the case of maneuver 3.

Table 8: Data for the maneuver 3 using the optimal method.

Spacecraft initial data	Total mass (vehicle + fuel) 2500 kg
	Available thrust = 1 N
	Initial position = 0
	True anomaly = 0º
Initial estimate of solution	Start of the engine = 0º
	Stop of the engine = 5º
	Initial pitch angle = 0º
	Initial yaw angle = 0º
	Initial rate of variation in pitch = 0
	Initial rate of variation in yaw = 0
	Fuel needed to maneuver = 12 kg

Sub-optimal case

Changing the initial keplerian elements to non singular elements, according to Equations 1 to 5:

a = 7,71544x10^6 m

h = 0,0011488

k = -0,000152672

p = 0,0856096

q = 0,644182

Therefore, now it is possible to use the numerical values of this orbit, which will be used as the reference orbit, carrying out the integration shown in equation 8 and, in this way, it is possible to obtain a set of equations that provide the variation of each of the elements used to describe the orbit by orbital revolution with the propellers acting all the time. Therefore, these will be the equations of motion of the spacecraft with the assumptions adopted. These equations, as a function of the components of the vector that define the direction of the thrust applied, are:

daa=acel*(7146070*id+949690*jd) (27)

dha=acel*(-0,936935*id-0,000068*jd) (28)

dka=acel*(0,00007081*id+0,936411*jd) (29)

dpa=0 (30)

dqa=0 (31)

With these equations, the software Mathematica is used to solve the optimization problem and to obtain the optimal solution. Assuming constant direction, the value found was id = 0.94.

Table 9 shows the final orbit achieved by the spacecraft after the maneuver, for the optimal and sub-optimal cases, the fuel consumed and the time to perform the maneuver.

Tabela 9 - Final keplerian elements obtained for maneuver 3.

Final elements	**Optimal method**	**Sub-optimal method**
Semi-major axis (km)	7707,4380	7707,4380
Eccentricity	0,0011589	0,0011589
Inclination (°)	66,0353	66,0353
Long. of ascending node (°)	7,57006	7,57006
Arg. of perigee (°)	90	90
Consumption (kg)	14,01	21,13
Duration of the maneuver (min)	322	504

4 Conclusion

It was studied and developed a method for the case of sub-optimal continuous maneuvers. This method is based on an analytical development, which generates equations that can be used for fast processing time, allowing its use in real time. The goal is to find the direction of the thrust applied to perform the orbital maneuvers, with the application of the linear direction of the applied thrust. The time and consumption are about 20% higher when compared to the ones obtained from the optimal method, so the suboptimal method can be used as a first estimate.

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