56 Influence of Unmodeled Accelerations in the Orbit Determination

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Summary. In the process of high accuracy orbit determination of artificial satellites, frequently a sinusoidal residual behaviour is observed during its error analysis. Actually, it is the result of unmodeled residual accelerations, which present frequencies near or multiple to the satellite period and appear by different reasons. In case of geopotential it may be caused by truncation of the harmonics of the geopotential field; whereas in the solar radiation pressure, the possible causes are mismodelled attitude, self-shadowing, and differences between physical and simplified derived models. Assuming that we cope with the unmodeled accelerations, which have no direct physical reasons, or that the modelling effort is not worthwhile, the main target here is to analyse these anomalous accelerations empirically.

56.1 Introduction

The problem of orbit determination consists essentially of estimating parameters values that completely specify the body trajectory in the space, processing a set of information from this body. Such observations can be collected through a tracking network on Earth or through sensors, like the GPS receiver onboard Topex/Poseidon (T/P).

The Global Positioning System (GPS) is a powerful and low cost means to allow computation of orbits for artificial Earth satellites. The T/P satellite is an example of using this system for space positioning.

The orbit determination of artificial satellites is a nonlinear problem in which the disturbing forces are not easily modelled, like geopotential and direct solar radiation pressure. Through an onboard GPS receiver, it is possible to obtain measurements (pseudo-ranges) that can be used to estimate the state of the orbit.

Usually, the iterative improvement of the position parameters of a satellite is carried out using the least squares methods. In this work, the algorithm was implemented through orthogonalisation [1].

56.2 Disturbing Effects Considered

The disturbing effects are included according to the physical situation presented and to the accuracy that is intended for the orbit determination.

Earth gravitational field and the attraction force associated with it are studied in the case of an artificial satellite. The geopotential is a force of gravitational origin that disturbs the orbits of Earth artificial satellites. Earth gravitational field represents one of the main perturbations on the motion of artificial satellites [2].

Solar radiation pressure is engendered throughout a continuous flux of photons that stumble at satellite surface, which can absorb or reflect such flux. This incident photons rate origins the solar radiation pressure force. The components of solar radiation pressure force can be expressed in several systems. Throughout these systems, the orbital elements of the satellite can be connected with Sun's position. This procedure was used here, for the direct solar radiation pressure model adopted for the Topex/Poseidon satellite, developed by Marshall and Luthcke [3].

Sun and Moon perturbations on the Earth satellites' orbits are due to Sun–Moon gravitational attraction, and may be meaningful the further the satellite is distanced from the Earth. Since the orbital variations are from the same type, whether the attractive body is Sun or Moon, they should be studied without distinguishing the third body. The simplest version of the three bodies restricted problem was adopted: the three bodies restricted-plane-circular problem, for which motion equation is in Prado and Kuga [4].

56.3 Unmodeled Accelerations

Some spacecraft missions require precise orbit knowledge to support payload experiments. Sometimes after launch, ground-based orbit determination (OD) solutions do not provide the level of accuracy expected. After verifying all known dynamic models, there may be a residual signature in the orbit as a result of unmodeled accelerations. This leads to attempt to estimate anomalous accelerations during the orbit fit, if sufficient data exist. If successful, the acceleration estimates can improve the fit residuals, and also results in better orbital position estimates, as can be seen in Soyka and Davis [5].

Unmodeled accelerations may have many reasons: truncation of geopotential field; limitations of modelling solar pressure, Earth albedo, Earth infrared radiation, drag, and others. Some of these accelerations can be corrected through the use of higher fidelity dynamic and physical modelling, while others require post-launch calibration.

The use of periodic accelerations, with a period near once per revolution of the satellite orbit, has been used within precision orbit determination programs to improve the accuracy of the derived ephemeris.

56.3.1 Anomalous accelerations modelling

When defining an anomalistic or periodic acceleration, one must consider three aspects: the subarc interval, the type of function, and the coordinate frame.

56.3.1.1 Subarc interval

The subarc interval is the time of duration or number of revolutions for a given acceleration to be active. As its name implies, it is usually a subset of the total arc. A reason to break an arc into a subarc is to allow for better overall fits.

56.3.1.2 Type of function

The underlying mathematical function of an acceleration function is usually a constant, a sine, or a cosine function.

The constant function is the most basic: a constant force in a specific direction. And the periodic functions (sine or cosine) have amplitude, frequency, and phase associated with them. The periodic functions are written as:

$$\text{accel} = A\sin(\omega t + \phi_A) \quad \text{or} \quad \text{accel} = B\cos(\omega t + \phi_B) \tag{56.1}$$

where A and B are amplitudes, ω is the frequency, t is the time elapsed since the start of the periodic function reference point or subarc interval, ϕ_A and ϕ_B and are the phase offsets. Either of these accelerations can be rewritten as:

$$\text{accel} = A'\sin(\omega t) + B'\cos(\omega t) \tag{56.2}$$

where for a sine acceleration with phase, $A' = +A\cos\phi_A; B' = +A\sin\phi_A$, and, for a cosine acceleration with phase, $A' = -B\cos\phi_B; B' = +B\sin\phi_B$. When estimated, the amplitudes A' and B' will adjust themselves to produce an effective phase offset.

56.3.1.3 Coordinate frame

The selection of the start of the subarc can be important, especially for non-circular orbits. Conventionally, equator crossings, argument of perigee, mean anomaly or orbit angle have been used as reference point.

56.4 Force Model

In a first step, the effect of including the considered perturbations in orbit propagation was analysed, before orbit determination through least squares estimation. Figure 56.1 shows the behaviour of the error, in metres, in RNT (radial, normal, and transverse) system, along a 24 h period, which is a meaningful interval of time in case of orbit propagation.

In case of orbit determination, the force model includes perturbations due to high order geopotential (50 × 50), with harmonic coefficients from JGM-2 model, due to direct solar radiation pressure, and due to Sun–Moon gravitational attraction.

The obtained data were evaluated through one parameter: error in position. Such parameter was later translated to RNT components of orbit fixed system [6].

First, only geopotentials were considered for the mentioned periods of orbit determination. After that, the direct solar radiation pressure force acting on Topex and the way such force acts on satellite orbit determination were analysed.

Figure 56.2 shows the behaviour of the error in position, given in metres and along time, given in seconds, considering only geopotential, and geopotential and direct solar radiation pressure effects, shown in two different curves, and Figure 56.3 includes Sun–Moon perturbation in the force model. The three following graphics used data from 18/11/1993.

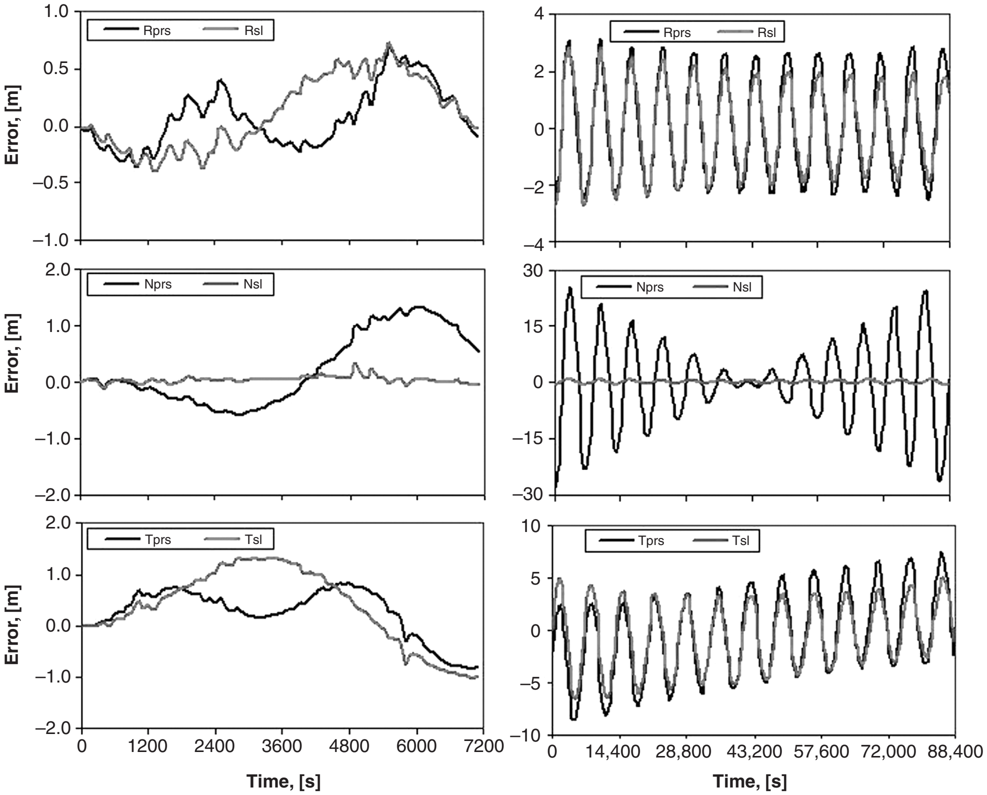

Figure 56.1 Orbit propagation per 24 h period for the days 11/18/93 (on the left side), and 11/19/93 (on the right side)

In the legends of Figures 56.1–56.3, ‘R’ means radial component; ‘N’ normal component; and ‘T’ transverse component of orbit fixed system. The subscript ‘geo’ means perturbations due to geopotential only; ‘prs’, perturbations due to geopotential and direct solar radiation pressure; and ‘sl’, all the perturbations considered.

Next, Table 56.1 shows the maximum and minimum values of the obtained errors for each of the perturbations considered.

As Table 56.1 shows, for short period, solar radiation pressure decreases up to 43% the radial component value and up to 16% the transverse. And for the long one, it reduces up to 42% the radial component value and up to 30% the transverse. The solar radiation pressure does not act meanwhile on normal component.

Observing the results on radial and transverse components, the inclusion of Sun–Moon attraction it is not justified. Although, its inclusion becomes necessary as it is responsible for the conspicuous decrease in the amplitudes of the normal component. For short period, Sun–Moon attraction decreases up to 70% the normal component value, against 3% of solar radiation pressure, and for long period, it reduces up to 78% the normal component value, against 4% when the Sun–Moon attractive effects are not considered.

Table 56.1 Maximum and minimum values of errors

Error		2 h			24 h		
Model	value	*R*	*N*	*T*	*R*	*N*	*T*
GCO	Maximum	1.476	4.442	2.188	5.703	26.258	9.266
	Minimum	−0.754	−2.283	−2.089	−4.667	−28.628	−12.431
GEO; PRS	Maximum	0.835	4.490	1.891	4.896	25.595	7.699
	Minimum	−0.709	−2.211	−0.163	−2.690	−27.918	−8.699
GEO; PRS; Sun–Moon	Maximum	0.857	0.385	1.338	2.772	0.803	62.896
	Minimum	−0.613	−0.199	−1.045	−3.928	−0.757	−0.381

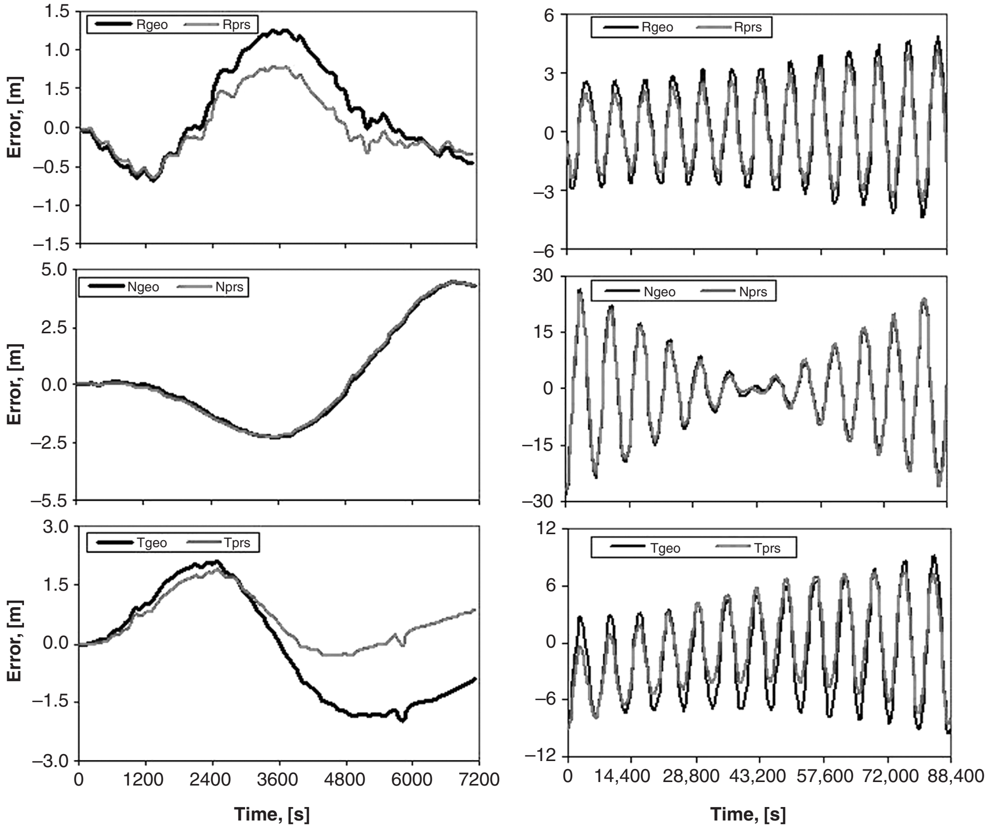

Figure 56.2 Errors in position, given in RNT coordinates, for 2 h (on the left) and 24 h (on the right), considering perturbations due to geopotential and direct solar radiation pressure

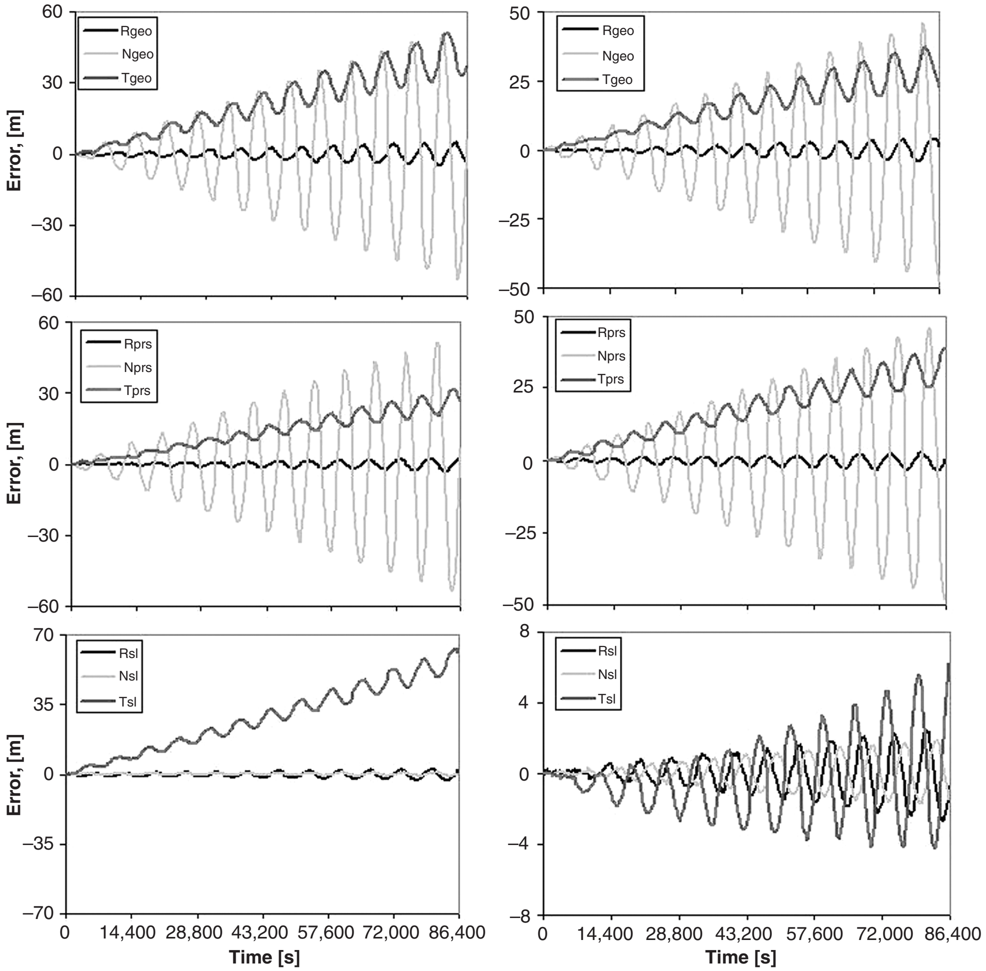

Figure 56.3 Errors in position, given in RNT coordinates, for 2 h (on the left) and 24 h (on the right), considering all the perturbations

56.5 Conclusions

Using signals of the GPS constellation and least squares algorithms using sequential Givens rotations as the method of estimation, the analysis period covered a short period (near once Topex period) and a long period orbit determination. Pseudo-range measurements were corrected from ionospheric effects, although the accuracy on orbit determination is not expressive. Real time requirements were not present; meantime, they were appropriate to keep low computational cost.

Throughout the results, it was found that least squares method through sequential Givens rotations and positioning using GPS showed trustfulness and accuracy enough for artificial satellite's orbit determination.

The results were compared with real data from Topex POE/JPL (Precision Orbit Ephemeris/Jet Propulsion Laboratory). For short period orbit determination, the magnitude of error in position varied from 4.6 to 4.2 m, and for long period, the magnitude varied from 13 to 30 m, according to the model's complexity increase.

Remaining errors have periodic nature, with a frequency near the orbital period, due to unmodeled residual accelerations, which appear by different reasons. In case of geopotential it may be caused by truncation of the harmonics of the geopotential field; whereas in the solar radiation pressure, the possible causes are mismodelled attitude, self-shadowing, and differences between physical and simplified derived models. Assuming that we cope with the unmodeled accelerations, which have no direct physical reasons, or that the modelling effort is not worthwhile.

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