## USING CLENSHAW SUMMATION FOR RECURSIVE COMPUTATION OF HIGH ORDER AND DEGREE GEOPOTENTIAL FOR SPACE APPLICATIONS

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An artificial satellite subject to the attraction of the Earth is disturbed due to non-spherical distribution and nonsymmetrical Earth mass. This uneven distribution of mass is expressed by the so-called spherical harmonic coefficients of the Earth potential. For a faster computation, the acceleration derived from the potential is obtained by a series expansion in terms of these harmonics, the fully normalized Legendre polynomials and their derivatives, and several recursions associated with the longitude, geocentric latitude and altitude of the center of mass of the satellite. This paper analyzes the detailed aspects of disturbances in artificial satellites, related with the modeling of the Earth's gravitational potential as well as numerical implementation of a recursive algorithm to calculate the acceleration of the geopotential based on the Clenshaw summation. In general, one uses recursive equations of high degree and order to calculate the Legendre polynomials in order to obtain faster processing and numerical accuracy. However, the recursions can yield numerical errors at each step of the recursion so that for higher orders and degrees of harmonics, the accumulated error may be quite pronounced. The computational implementation of the algorithm is carried out in a PC computer. With the implementation of this algorithm it is possible to calculate the geopotential acceleration for different orbits and different situations. Such approach aims at mitigating the numerical problems arising from usage of extended series expansion when computing recursively the Legendre polynomials. Once the favorable numerical properties are proven, the algorithm can be used in the solution of practical problems of orbital space mechanics and for the Brazilian Space Mission.

## REFERENCES

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