Optimal Transfer Trajectories to the Haumea System

Diogo M. Sanchez[1], Antonio F. B. A. Prado[2] and Alexander A. Sukhanov[3]
National Institute for Space Research, São José dos Campos, SP, 12227-010, Brazil

and

Tadashi Yokoyama[4]
State University of São Paulo, Rio Claro, SP, 13500-970, Brazil

This paper provides a preliminary study for a mission to the dwarf planet Haumea, focusing on the travel between the Earth and this Kuiper Belt Object (KBO). We generate a set of transfer trajectories from the Earth to Haumea using the patched-conic motion model taking into account close approaches with the planets, as well as direct transfers Earth-Haumea. Among several possibilities of trajectories, we study in details the Earth-Jupiter-Haumea transfer and the Earth-Saturn-Haumea transfer. For each one of these transfers we calculate the launch and arrival windows, showing the minimum total ΔV (launch phase) of the transfer and the excess velocity in the arrival at Haumea, for several times of flight. For the trajectory Earth-Jupiter-Haumea, we also found asteroids of the main belt which could be reached during the transfer. The mid-term 2020-2035 was proposed for the launch of this mission. Since the exploration of the Haumea system involves close approaches to the two moons of Haumea, we made an additional study of the effect of these close approaches. The exploration of the Haumea system can provide important information about the formation of the KBOs and, consequently, about the Solar System itself. We expect that our results will be useful for a real mission in the future.

Nomenclature

EH = direct Earth to Haumea transfer
EJH = Earth – Jupiter – Haumea transfer
ESH = Earth – Saturn – Haumea transfer
ΔV = minimum total transfer velocity
V_∞ = velocity of approach of the spacecraft
ΔV_c = variation in the velocity of the probe required to approach an asteroid
T_F = flight time
C = angular momentum
E = energy
r = distance between two primaries
r_1 = distance from the probe to the first primary
r_2 = distance from the probe to the second primary
R_p = periapsis distance
x,y,z = coordinates of the probe
x_1,y_1 = coordinates of the first primary
x_2,y_2 = coordinates of the second primary
V_p = velocity of the probe at the periapsis
V_r,V_t = radial and transverse components of the velocity of the second primary

---

[1]Ph.D. Student, INPE-DMC, Av. dos Astronautas, 1758, S. J. dos Campos, SP, Brasil.
[2]Head of the Space Mechanics and Control Division, INPE-DMC, Av. dos Astronautas, 1758, S. J. dos Campos, SP, Brasil, Associate Fellow.
[3]Engineer, INPE-DMC, Av. dos Astronautas, 1758, S. J. dos Campos, SP, Brasil.
[4]Titular Professor, IGCE-DEMAC, Av. 24A, 1515, Rio Claro, SP, PO box 178, Brasil.

V_2 = velocity of smaller body of the close approach
μ = mass parameter
Ψ = angle of approach of the probe

I. Introduction

THE dwarf planet Haumea have intrigued the scientific community since its discovered in the year 2003, because of the peculiarities of its shape. A Kuiper Belt Object (KBO), Haumea is a tri-axial oblate ellipsoid with approximated dimensions of 500 x 750 x 1000 km with a high value for the density (2.6 g/cm³), which imply in a small ice fraction[1]. These peculiarities, and its high spin (period of 3.9154 h), indicate that Haumea is a parent of a collisional family which includes its two moons, Hi'iaka and Namaka. Several papers with simulations of this collisional family were published in the last years (like the work of Volk and Malhotra, published in 2012[2]). Therefore, the exploration of the Haumea system can provide important information about the formation of the KBOs and, consequently, about the Solar System itself. Due to these reasons, the Haumea system is a compelling target for missions in the next decades[3]. Following the example of the Dawn mission[4] – that orbited the asteroid Vesta during one year (to map the surface of the asteroid, to generate a gravity map and gathering several other important information about the system), and now it is flying in the direction of the dwarf planet Ceres (with arrival predicted for the year 2015) –, a mission to Haumea, with similar objectives of the Dawn mission, is limited by the incoming excess velocity (V_∞), because the increase of this velocity increases the costs of the capture, which is an essential part of this type of mission. Since one of the major goals of the mission is to observe seasonal variations at the surface of the Haumea, a capture of the probe is required such that the observations can be made for longer times. As we will show in the results, the incoming velocity near Haumea is inversely proportional to the flight time (as expected) and can reaches high values (up to 20 km/s). This magnitude of incoming velocity is a technical challenge for the capture. In order to overmatch this technical barrier, we propose that the probe be composed by a main vessel, which will carry several nano satellites that will be delivered when the probe is near the Haumea system. The nano satellites would be captured by the system, while the main vessel would become a communication center (like a space beacon), making a communication bridge between the nano satellites and the Earth.

The present work is divided in two parts. In the first part, we designed part of the mission to Haumea with focus in the travel between the Earth and Haumea, searching for a balance between optimal total velocity for the launch phase and incoming excess velocity at Haumea. We generated a set of transfer trajectories from the Earth to Haumea using the patched-conic model taking into account close approaches with the planets. We found launch dates to start the mission in the mid-term 2020–2035, and the transfer trajectories are optimal with respect to the fuel consumption (represented by the minimum total ΔV). The probe passes through a number of bodies as large as possible during these trajectories. The accretion of bodies in the trajectory between the Earth and Haumea allow the increasing of the velocity of the probe via gravity assisted maneuvers, turning the mission more interesting in terms of lowering the consumption of fuel, as well as in terms of scientific returns, since the fly-bys helps to obtain data from the bodies visited. The method of the patched conics is a well-known method and it was used for the planning of several interplanetary missions, like a mission to the Sun using gravity assists from the inner planets[5], a mission to Neptune[6], etc. The description of the method can be found in several publications, e. g., Escobal, 1968[7]. References 8 to 14 show a series of examples of missions which uses this technique.

In the second part of this work, we assume that the nano satellites launched from the main vessel into the Haumea System will orbit Haumea when they make close approaches with one of its moons (Hi'iaka and Namaka) to make observations. Those passages modifies the orbit of the probes, so the mapping of those effects is required in two ways: to use them for maneuvers inside the system and to avoid earlier escapes from the system. The close approaches with the two moons are performed for several initial conditions. The resulting trajectories are classified by the modifications in the orbit of the probes made by the close approaches, in particular studying the increases and decreases of the two-body energy of the probe-Haumea, to verify captures and escapes from the system, and the modification of the sense of the orbit (direct to retrograde or vice-versa). These trajectories can be defined by three variables: V_∞, the velocity of the probes with respect to the moon, when approaching this body; ψ, the angle between the periapsis of the approach trajectory and the line defined by the primaries, which is named "angle of approach"; and R_P, the periapsis distance. The planar restricted elliptical three-body problem is used as the mathematical model here.

The equations of motion are numerically integrated in both senses of time (positive and negative), until the probes reach a large distance from the moon, such that it is possible to disregard its gravity. At those points, the energy and the angular momentum are calculated before and after the close approach and the possible sixteen classes of trajectories are defined based on the calculations just made of the energy and angular momentum at both points of the integration. The results are shown in letter-plots that tells the behavior of the orbits in terms of energy and sense of the orbit. At this point it is possible to find the orbits that result in escape from the system, that are the ones that needs to be avoided. Orbits that can help the transfers of the probes between two orbits are also mapped.

Close approach trajectories has many applications and the literature shows that point. The usual objective is to reduce the fuel consumption of the probe. Broucke[16] gives more details about this technique. Examples of missions are shown in Carvell[17], which made a three-dimensional close approach in Jupiter to change the inclination of the orbital plane of the probe; Casalino et al.[8], which combined low thrust with close approaches; D'Amario et al[9], which studied in details the Galileo mission. A famous and important mission was the Voyager trip, a grand tour of the Solar System made by two probes that made important discoveries in the giant planets. This mission is described in more details by Minovich[14] and Kohlhase and Penzo[11]. Other particular aspects of this problem is considered in the literature. One example is the inclusion of an impulsive maneuver during the passage[18]. Another example is the consideration of elliptic motion for the primaries[19]. The study of a cloud of particles[20,21] or the problem of finding trajectories to the Sun based in close approaches in the inner planets of the Solar system[6]. Graphical methods to study this problem can be found in Strange and Longuski[12] and in MacConaghyet al.[13]. A study similar to the one shown here was made by Prado[22], but using the restricted circular three-body problem in the triple asteroid 2001SN$_{263}$. We expect that our results will be useful for a future real mission to Haumea system.

II. The Mission to Haumea System Options and Results

The orbital characteristics of the Haumea system, which are shown in Table 1, cause restrictions in the choice of launch windows for a mission to this KBO. A first intuitive option for a mission to Haumea is a direct Earth to Haumea transfer trajectory (*EH*). In this case, the probe would fly from the Earth to Haumea without passing through other planets. For this option, the optimal launch date (in the time interval 2020–2035) is 2029, January 16th, with a minimum total velocity of 8.280 km/s, and V_∞ = 4.377 km/s. The minimum total velocity is the summation of the launch velocity and midcourse maneuvers (eventual corrections in the trajectory), so it reflects the launch velocity, since the midcourse maneuvers are almost negligible. Although the low value of the excess velocity, the flight time, in this case, is equal to 71.15 years. This value of flight time turns impracticable the use of this trajectory. Figure 1 shows the projection of this trajectory in the plane of the ecliptic and in a plane perpendicular to the ecliptic. We can see that the entire transfer occurs in the plane of the ecliptic.

Table 1. Parameters of the Haumea System

Object	Mass[1], kg	Semi-major Axis[1], km	Eccentricity[1]	Inclination[1], degrees	Radius, km	Orbital Period
Haumea*	4.006×10^{21}	6.46020×10^{9}	0.19368	28.22281	690	283.28 y
Hi’iaka	1.79×10^{19}	49880	0.0513	126.356	195	49.44 d
Namaka	1.79×10^{18}	25657	0.249	113.013	100	18.27 d

*With respect to the ecliptic plane.

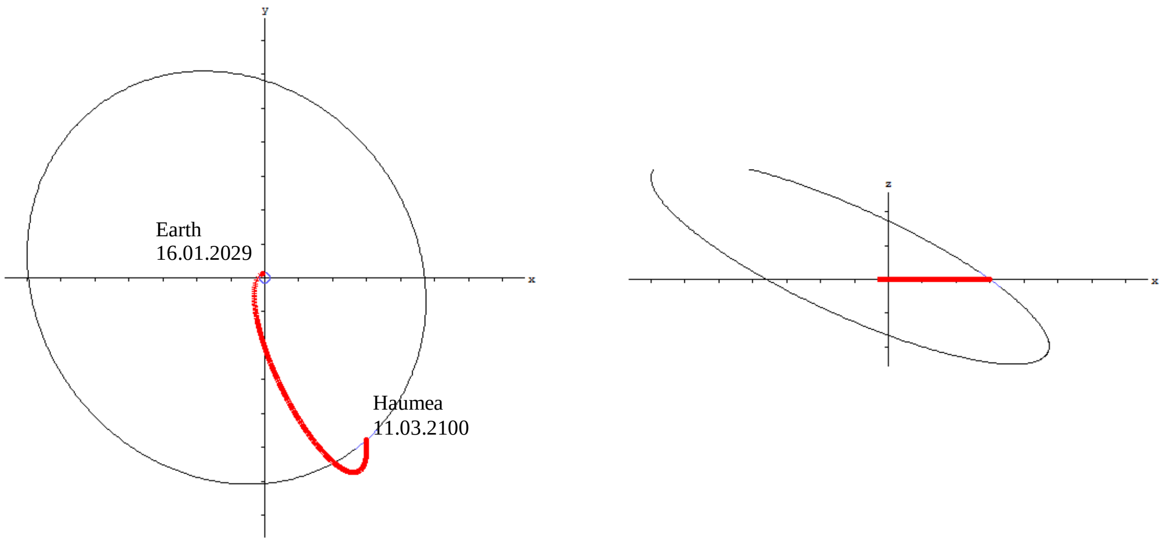

Figure 1. ***EH*** **trajectory in the plane of the ecliptic (left), and in the plane perpendicular to the ecliptic (right).**

Figure 2 shows the *EH* trajectory with a restriction for the flight time in 35 years. We can see that the trajectory is out of the ecliptic plane, due to the position of the Haumea at the arrival date of the probe. Then, for a feasible transfer to Haumea (in the sense of the maximum time of flight of 35 years), all the trajectories will be out of the ecliptic plane. For the transfer shown in Fig. 2, the launch date is 2034, January 29th, and the minimum total velocity is equal to 17.898 km/s and V_∞ = 4.787 km/s. Although the velocity near Haumea is low, the velocity of launch is too high. If we decrease the time of flight, an increasing of the velocity of launch will occurs, turning impossible the direct transfer to Haumea.

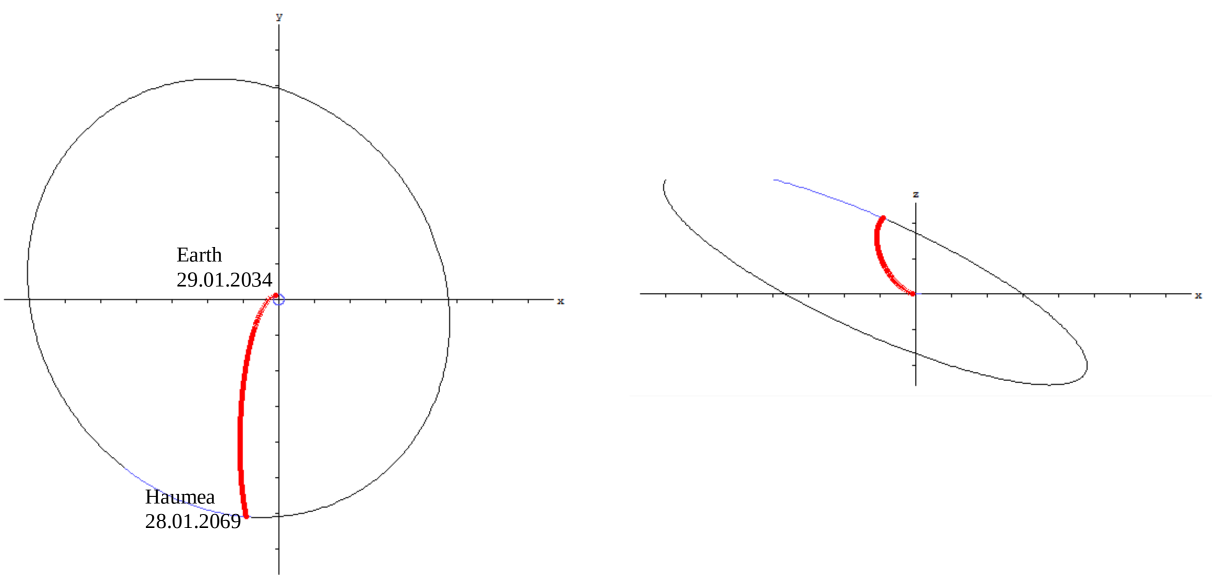

Figure 2. ***EH*** **trajectory in the plane of the ecliptic (left), and in the plane perpendicular to the ecliptic (right), for a flight time of 35 years.**

Other possibilities of transfers arise with the inclusion of passing by the planets in the trajectory. The addition of those celestial bodies brings the possibility of gravity assisted maneuvers between the planet and the probe, which changes the heliocentric energy of the probe. The result is a non-impulsive maneuver which may increase or decrease the heliocentric energy of the probe (depending on the geometric details of the encounter) and, consequently reduce the consumption of fuel to accomplish the mission. We tested transfers using close approaches in Jupiter and Saturn. For the time interval considered, Venus, Uranus and Neptune are not in good orbital positions. Another point is that a trajectory with a combined close approaches with Jupiter and Saturn is not possible due to the orbital positions of these planets in the time interval considered.

Figure 3 shows the trajectory *EJH*, a mission option in which the probe, after the departure from the Earth, makes gravity assisted maneuvers with Jupiter to reach Haumea, for a maximum flight time of 35 years. For this trajectory, we found $\Delta V = 7.519$ km/s, $V_\infty = 5.241$ km/s, flight time of 33.04 years, and optimal launch date 2026, October 24th.

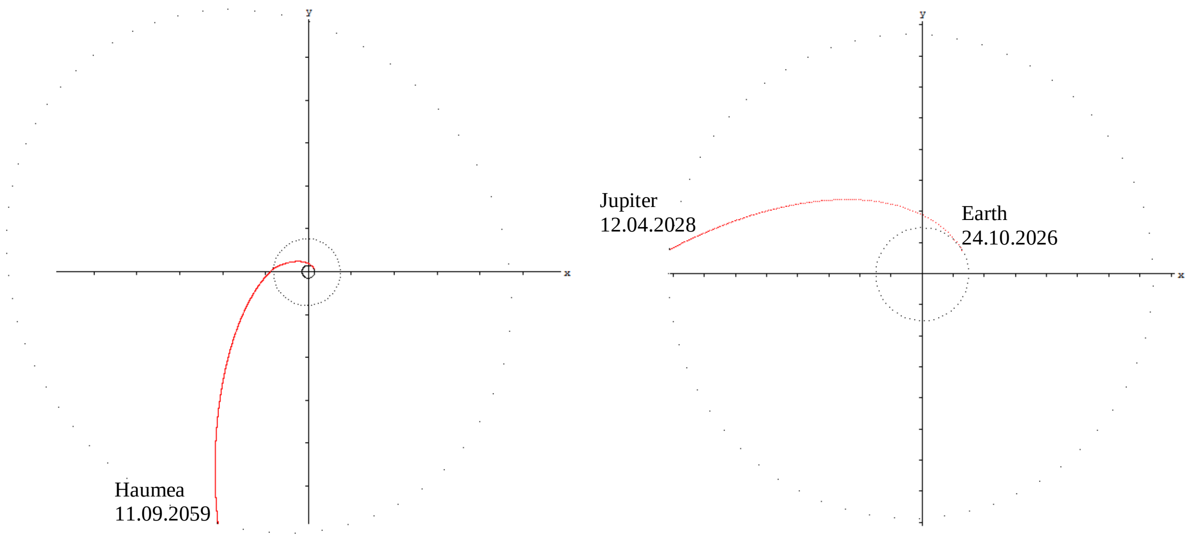

Figure 3. *EJH* trajectory in the plane of the ecliptic, with maximum time of flight limited to 35 years (left), and a magnification of the inner orbits of the trajectory (right).

Taking Saturn instead of Jupiter and also limiting the maximum flight time to 35 years, we have $\Delta V = 7.449$ km/s, $V_\infty = 5.880$ km/s, $T_F = 34.99$ years, and optimal launch date 2030, August 1st. Comparing the results of *EJH* and *ESH* for maximum flight time of 35 years, we can see that the values are almost the same, although the optimum launch date for *ESH* is four years after the optimum launch date for *EJH*. Figure 4 shows the trajectory *ESH* for maximum flight time of 35 years. As we mentioned before, a mission option with Jupiter and Saturn in the same trajectory is not possible in the proposed time interval.

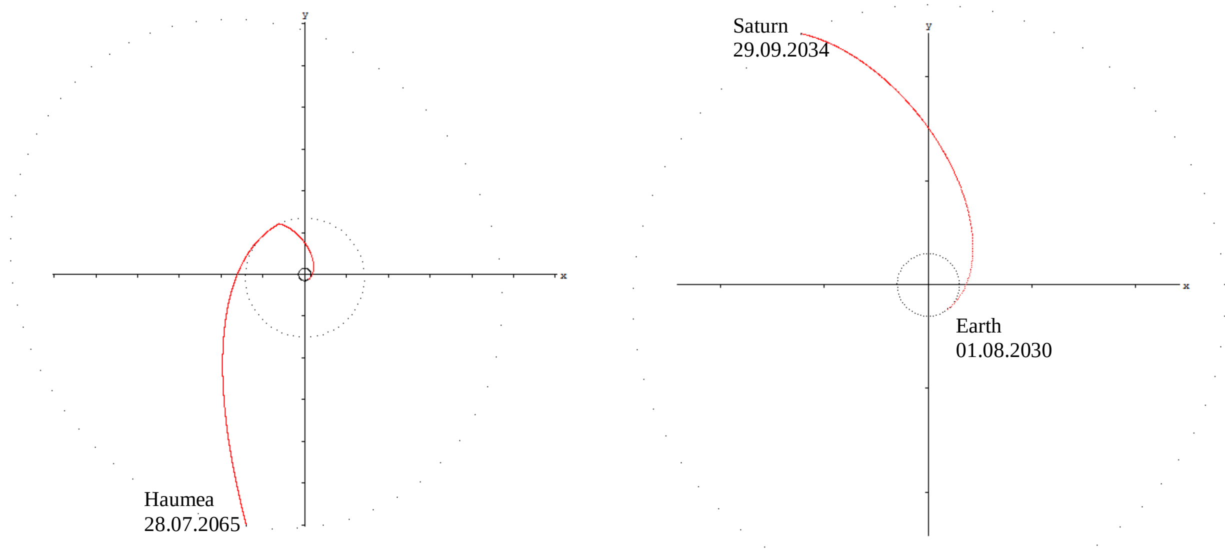

Figure 4. *ESH* trajectory in the plane of the ecliptic, with maximum time of flight limited to 35 years (left), and a magnification of the inner orbits of the trajectory (right).

A flight time of 35 years is not practical for this type of mission. Narrowing the flight time to 15 years we found ΔV = 8.345 km/s, V_∞ = 14.833 km/s, T_F = 14.99 years, and optimal launch date 2026, October 28th for *EJH*, and ΔV = 13.982 km/s, V_∞ = 20.959 km/s, T_F = 14.99 years, and optimal launch date 2031, August 22nd for *ESH*. For flight time of 15 years, the trajectory *EJH* is the best choice, even though the excess velocity is 14.833 km/s. Since our proposal is that a swarm of nano satellites is delivered by the probe (main vessel) and those nano satellites are the bodies that will be captured by the Haumea system, the large breaking velocities are not so problematic, since the mass of each nano satellite is small. Figures 5 and 6 show the *EJH* and *ESH* trajectories, respectively, for a flight time of 15 years. Figure 7 shows the minimum total transfer velocity and the velocity near Haumea as a function of the flight time for the trajectories *EJH* and *ESH*. We can note that the velocities are similar for flight times above 25 years; however, the trajectory *EJH* stands out for low flight times. Table 1 summarizes a comparison between the results of the trajectories *EH*, *EJH* and *ESH*.

Table 1. Optimal launch date for several transfers.

Transfer Type	Launch Date	Excess Velocity V_∞, km/s	Minimum Total ΔV, km/s	Flight Time, years
EH	16.01.2029	4.377	8.280	71.15
EH	29.01.2034	4.787	17.898	35.00
EJH	24.10.2026	5.241	7.519	33.04
ESH	01.08.2030	5.880	7.449	34.99
EJH	28.10.2026	14.833	8.345	14.99
ESH	22.10.2031	20.959	13.982	14.99

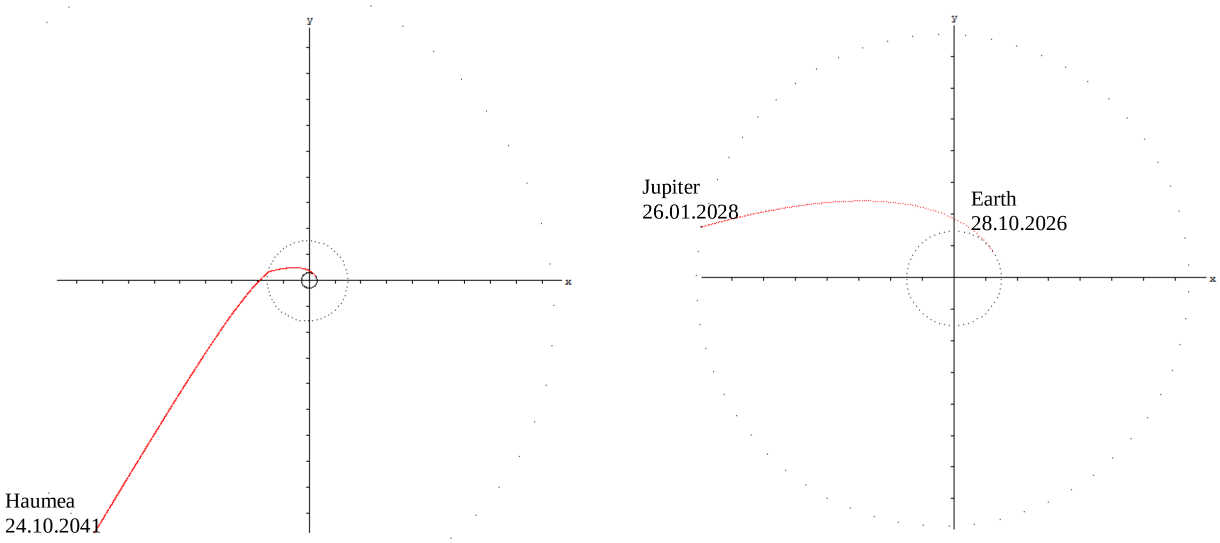

Figure 5. ***EJH*** **trajectory in the plane of the ecliptic, with maximum time of flight limited to 15 years (left), and a magnification of the inner orbits of the trajectory (right).**

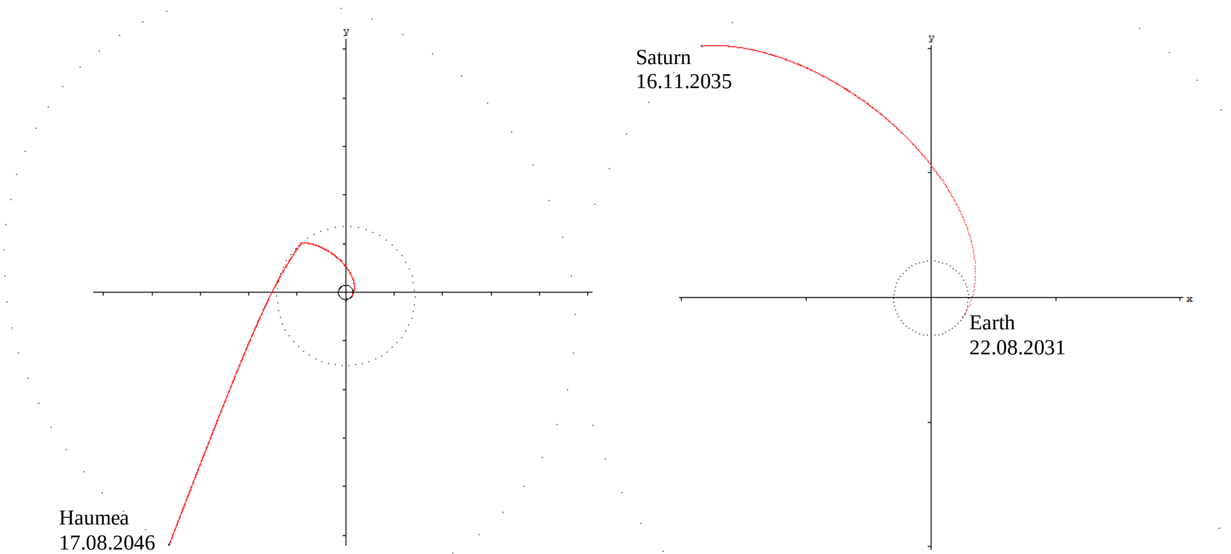

Figure 6. ***ESH*** **trajectory in the plane of the ecliptic, with maximum time of flight limited to 15 years (left), and a magnification of the inner orbits of the trajectory (right).**

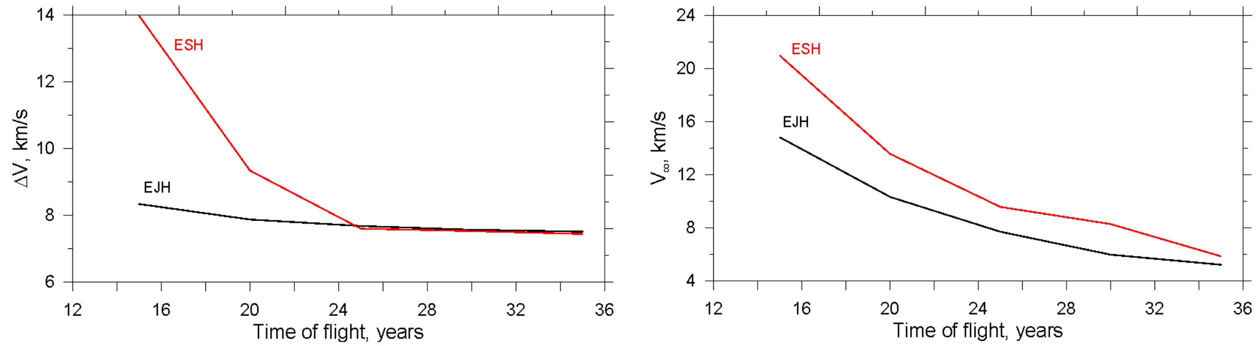

Figure 7. Left: Minimum total transfer velocity (launching and mid-courses phases) as a function of the time of flight for the trajectories *EJH* and *ESH*, for the launch windows 24-28 October, 2026 (*EJH*), and July, 2030 – August, 2031 (*ESH*). Right: Excess velocity near Haumea as a function of the time of flight for the trajectories *EJH* and *ESH*, for the launch windows 24-28 October, 2026 (*EJH*), and July, 2030 – August, 2031 (*ESH*).

Taking the best trajectory found up to now, the *EJH*, we reduce the time of flight to 12 years and we search for asteroids of the main belt (between Mars and Jupiter) that could be reached by the probe when it is travelling between Earth and Jupiter. We limit the velocity necessary to make the correction in the orbit in order to approach the asteroids to 0.5 km/s, and we search asteroids in a radius of 5 million kilometers around the trajectory of the probe. We found 19 asteroids which could be reached, and Table 2 shows the date of approach, the name, the diameter, the distances of the closest approaches, and the variation in the velocity of the probe necessary to approach the asteroid, for each of them. Among all the asteroids found, 1098 Hakone is the only one with known diameter, and Fig. 8 shows an *EJH* trajectory with a close approach by the Hakone Asteroid. In this trajectory, we found ΔV = 8.783 km/s, V_∞ = 19.373 km/s (near Haumea), ΔV_c = 0.919 km/s, T_F = 11.99 years, and optimal launch date 2026, November 1st. In fact, one could plan *EJH* trajectories with close approaches by several asteroids which are presented in Table 2.

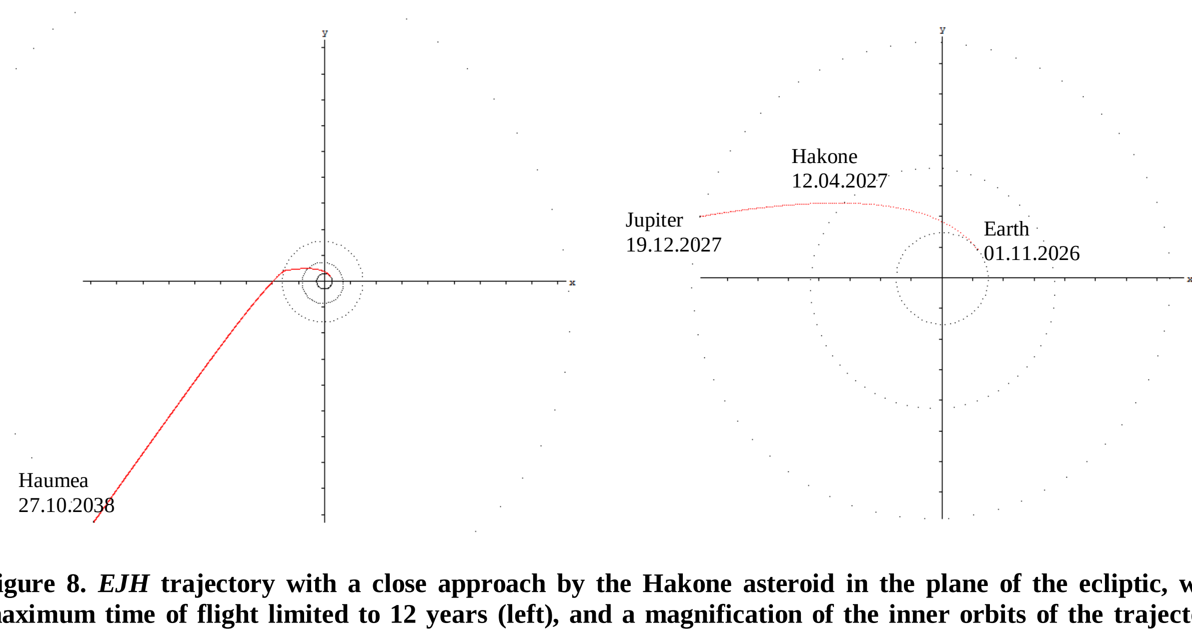

Figure 8. *EJH* trajectory with a close approach by the Hakone asteroid in the plane of the ecliptic, with maximum time of flight limited to 12 years (left), and a magnification of the inner orbits of the trajectory (right).

Table 2. Asteroids of the main belt that could be reached in the *EJH* transfer.

Date of Approach	Asteroid	Diameter, km	Distance, × 10^6 km	ΔV_c, km/s
27.02.2027	74754 1999 RO198	-	2.239	0.468
27.02.2027	80793 2000 CX86	-	1.415	0.296
4.03.2027	102834 1999 VF190	-	0.712	0.149
12.03.2027	43449 2000 YC83	-	1.801	0.410
5.04.2027	150795 2001 RR52	-	2.182	0.425
10.04.2027	1098 Hakone	24.7	2.010	0.394
8.04.2027	43614 2002 AT187	-	2.430	0.469
21.04.2027	19127 Olegefremov	-	1.791	0.333
27.04.2027	24198 Xiaomengzeng	-	2.246	0.417
21.04.2027	69762 1998 QS5	-	1.610	0.304
21.04.2027	126935 2002 EN146	-	2.584	0.481
28.04.2027	86928 2000 HJ62	-	0.920	0.166
30.04.2027	118225 1996 TP18	-	2.643	0.469
11.05.2027	8495 1990 QV1	-	1.687	0.273
22.05.2027	48777 1997 QE5	-	2.277	0.410
30.05.2027	26921 Jensallit	-	2.270	0.377
4.06.2027	71298 2000 AH62	-	2.540	0.447
31.05.2027	133576 2003 UE49	-	2.881	0.459
9.06.2027	112577 2002 PA53	-	2.466	0.397

III. Mapping Swing-by Trajectories within the Haumea System and Results

The dynamical system has four bodies, assumed to be points of mass: Haumea, Hi'iaka, Namaka, and the probe, which has negligible mass. The two moons are orbiting Haumea in elliptical non-coplanar orbits. We assume that one moon does not affect the motion of the probe when it is moving near the other moon, so the system formed by Haumea, the closest moon and the probe can be modeled by the planar restricted elliptic three-body problem.

The goal is to measure energy and angular momentum of the probes with respect to Haumea before and after the close approach by one of the moons. The modifications of the trajectories are studied, with emphasis in finding escapes and captures.

There are several options for reference systems to study the elliptic restricted three-body problem.[23,24] We choose to use the inertial system, which has the origin in the center of mass of the system (Haumea and the moon, Ni'iaka or Namaka, depending on the case studied). The horizontal axis is the line connecting the primaries and the vertical axis is the line perpendicular to the horizontal axis. In this system, the primaries are in elliptic orbits that can be described by:

$$x_1 = -\mu r \cos(\nu), \tag{1}$$

$$y_1 = -\mu r \sin(\nu), \tag{2}$$

$$x_2 = (1-\mu) r \cos(\nu), \tag{3}$$

$$y_2 = (1-\mu) r \sin(\nu), \tag{4}$$

where μ is the ratio between the mass of the moon and the mass of the moon plus the mass of Haumea, r is the distance between Haumea and the moon, which is given by:

$$r = \frac{(1-e^2)}{1+e\cos(\nu)}, \tag{5}$$

and ν is the true anomaly of the moon in its orbit around Haumea. The equations of motion of the probe are:

$$\ddot{x} = -\frac{(1-\mu)(x-x_1)}{\mathrm{r}_1^3} - \frac{\mu(x-x_2)}{\mathrm{r}_2^3}, \tag{6}$$

$$\ddot{y} = -\frac{(1-\mu)(y-y_1)}{\mathrm{r}_1^3} - \frac{\mu(y-y_2)}{\mathrm{r}_2^3}, \tag{7}$$

where r_1 and r_2 are the distances between the probe and Haumea and the probe and the moon, respectively, given by:

$$r_1^2 = (x - x_1^2) + (y - y_1^2), \tag{8}$$

$$r_2^2 = (x - x_2^2) + (y - y_2^2). \tag{9}$$

The energy of the probe can be calculated by:

$$E = \frac{1}{2}(x + \dot{y})^2 + (\dot{x} - y)^2 - \frac{1-\mu}{r_1} + \frac{\mu}{r_2}. \tag{10}$$

This equation shows if the orbit of the probe is open or closed, by examining the sign of the energy (positive means open orbit and negative means closed orbit). For the angular momentum, required to verify the sense of the orbit, we use:

$$C = x^2 + y^2 + x\dot{y} - y\dot{x} \tag{11}$$

The final algorithm for the calculations has the following steps:

i) Values are given for R_p, V_∞, ψ;
ii) The initial position and velocity are given by:

$$x_i = (1-\mu)r\cos(\nu) + r_p\cos(\psi + \nu), \tag{12}$$

$$y_i = (1-\mu)r\sin(\nu) + r_p\sin(\psi + \nu), \tag{13}$$

$$V_{xi} = V_r\cos(\nu) - V_t\sin(\nu) - V_p\sin(\psi + \nu), \tag{14}$$

$$V_{yi} = V_r\sin(\nu) + V_t\cos(\nu) + V_p\cos(\psi + \nu), \tag{15}$$

where V_p is the velocity of the probe when passing by the periapsis, obtained from V_∞ and the conservation of energy of the system probe-moon, and V_r and V_t are the radial and transverse components of the velocity of the moon in an inertial frame. They are obtained from:

$$V_r = \frac{(1-\mu)e\sin(\nu)}{\sqrt{1-e^2}}, \tag{16}$$

$$V_t = \frac{(1-\mu)(1+e)\cos(\nu)}{\sqrt{1-e^2}}; \tag{17}$$

iii) The equations of motion are numerically integrated for positive times until the probe reaches a large distance from the moon. Then energy (E_+) and angular momentum (C_+) after the maneuver are obtained from Eqs. (10-11);
iv) The process is repeated and the equations of motion are numerically integrated in negative times, using the initial conditions given by Eqs. (12-15), and energy (E_-) and angular momentum (C_-) before the maneuver are obtained, when the probe is one more time far from the moon, again using Eqs. (10) and (11);
v) These data makes it possible to verify when a capture ($E_- > 0$ and $E_+ < 0$) or an escape ($E_- < 0$ and $E_+ > 0$) happens. Other information, like the change of the sense of the orbit are also obtained at this point.

The results can be seen in plots that show the effects of the close approaches in the orbits of the probe, with emphasis in captures and escapes. Table 3 shows the classes of orbits based in the alteration of the orbit of the probe, in terms of open/closed or direct/retrograde. The plots have the velocity of approach (km/s) in the vertical axis and the angle of approach ψ (degrees) in the horizontal axis. The interval used for ψ is 180-360 degrees. There is one plot for each periapsis distance. There is symmetry in this problem and so it is not necessary to shown the results for ψ from 0 to 180 degrees. This is the interval of gains in energy, so only escape trajectories are shown. The trajectories ending in captures are in the mirror image of the plots, for values of ψ in the interval 0-180 degrees. See reference 22 for more details. Regarding the periapsis distance, 1.1, 5.0, 10.0, and 15.0 radius of the moon were used. Figures 9-12 show the results. The parameters used here are:

a) Distance from the moon to the points considered far enough from it such that it is possible to neglect its gravity field: 0.5 canonical units[23];
b) Distance to consider the probe far from the Haumea system: 2.0 canonical units;
c) The physical data of the bodies were already shown in Table 1.

Table 3.Rules for the assignment of letters to orbits.

After: Before:	Direct Ellipse	Retrograde Ellipse	Direct Hyperbola	Retrograde Hyperbola
Direct Ellipse	A	E	I	M
Retrograde Ellipse	B	F	J	N
Direct Hyperbola	C	G	K	O
Retrograde Hyperbola	D	H	L	P

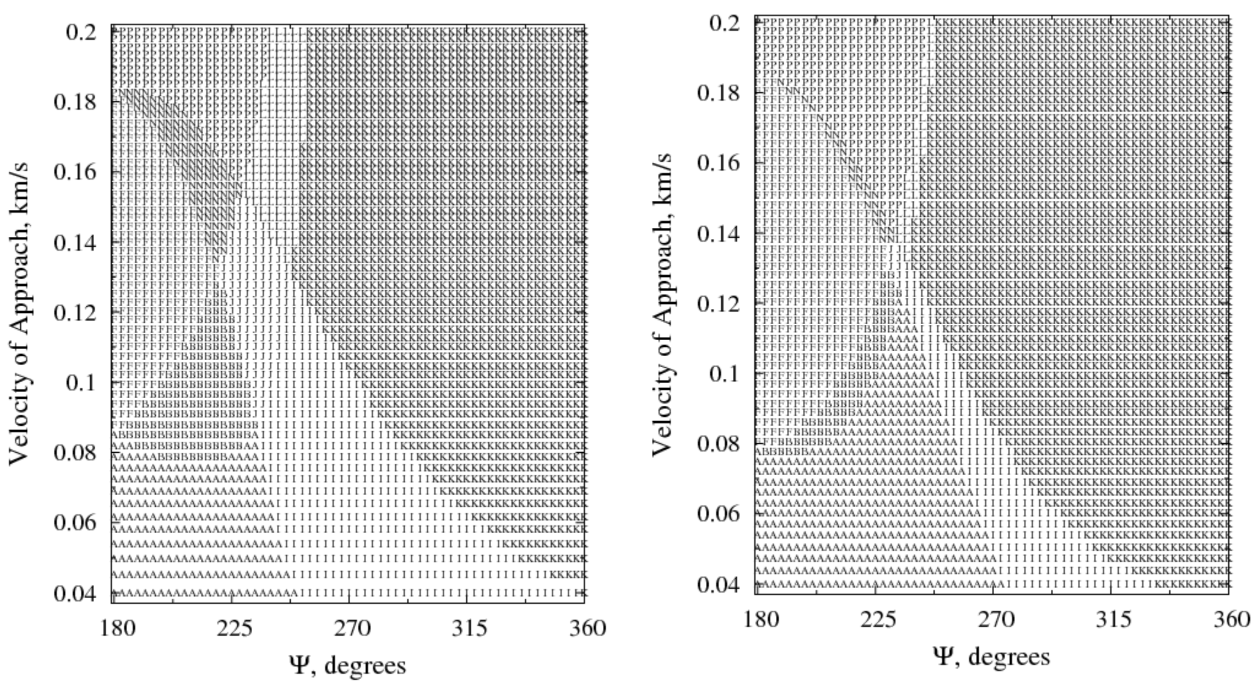

Figure 9. Orbits after the probe after the passage by Hi'iaka for R_P = 1.1 (left) and 5.0 (right) radius of Hi'iaka.

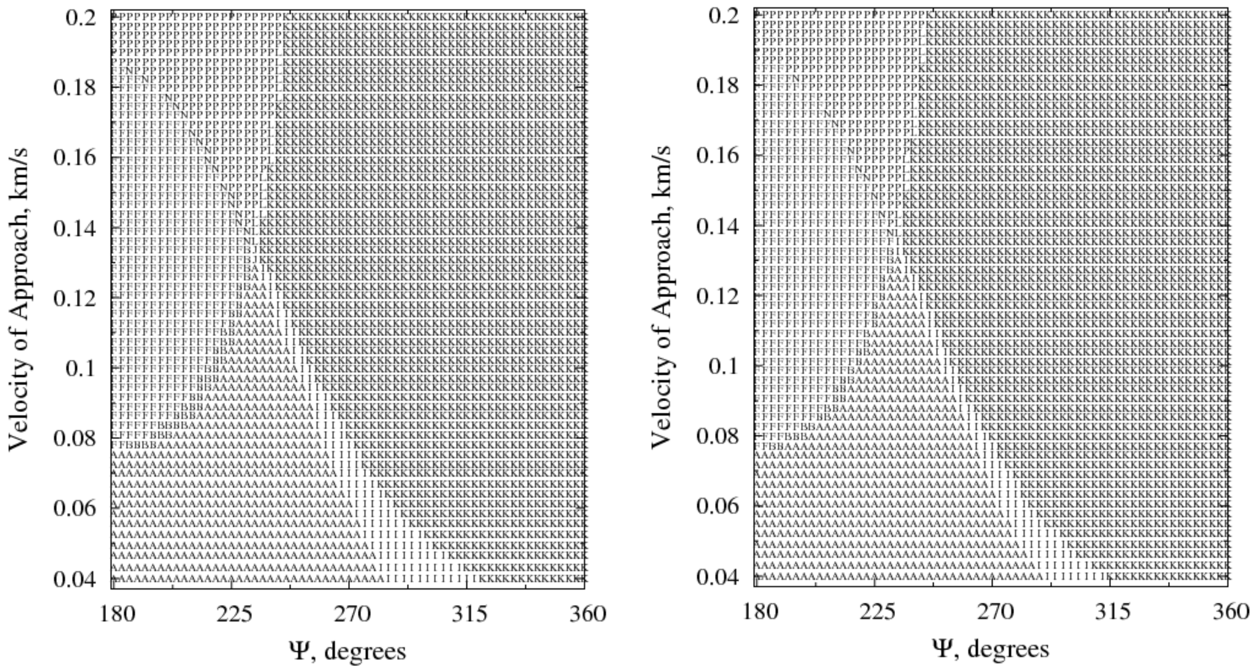

Figure 10. Orbits after the passage by Hi'iaka for R_P = 10.0 (left) and 15.0 (right) radius of Hi'iaka.

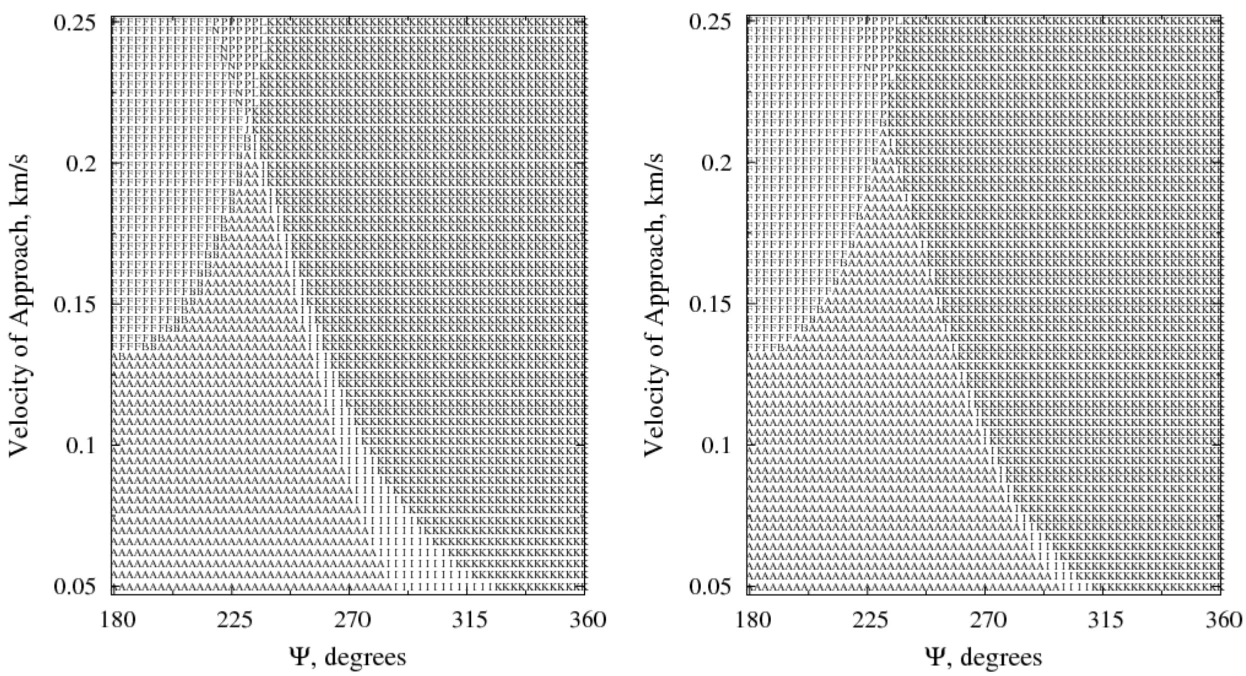

Figure 11. Orbits after the passage by Namaka for R_P = 1.1 (left) and 5.0 (right) radius of Namaka.

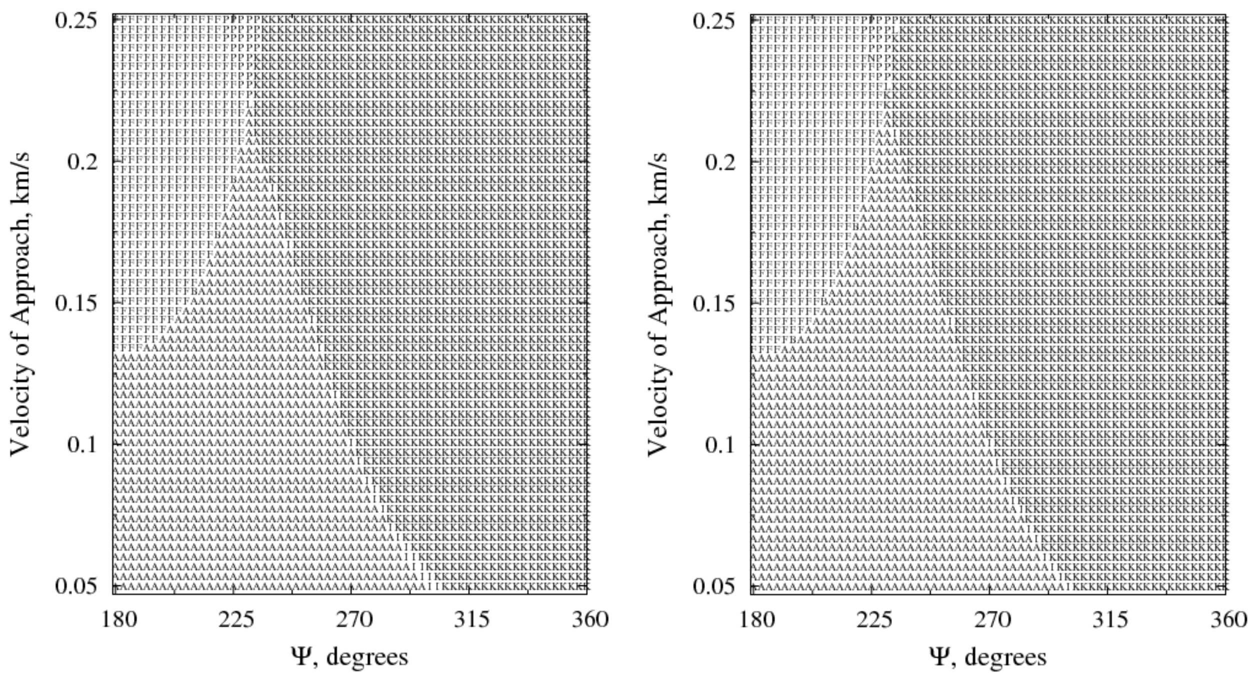

Figure 12. Orbits after the passage by Namaka for R_P = 10.0 (left) and 15.0 (right) radius of Namaka.

The first aspect noted is that the two moons generate different results, because their masses and eccentricities are different, with Namaka being more eccentric. In general, Hi'iaka gives stronger effects due its larger mass. The figures are dominated by orbits represented by the letter K, that are concentrated in the right parts of the figures. They are direct hyperbolic orbits before and after the close approach. This is expected, because the masses of the moons are too small to make larger modification of the orbit. At the left-top part of the figures, when considering Hi'iaka (Figs. 9 and 10), letter P dominates, which means retrograde hyperbolic orbits before and after the passage. At the same region of the Figs. 11 and 12 (for Namaka), that means a retrograde ellipse before and after the passage. Namaka also presents regions of letter P at the top of the same figures. Family L also represents open orbits, composed by retrograde hyperbolic orbits before the passage and hyperbolic direct orbits after that. The closed orbits are concentrated in the family F (retrograde ellipses before and after the passage) and also in the more interesting family B, composed by elliptical orbits that change the sense from retrograde to direct. Orbits represented by the letter A are also closed all the time, but changes the sense from direct to retrograde. Those orbits are important if a modification of the sense of the orbit is useful for the mission. The more important orbits are the ones that generates escapes, which is equivalent to captures in the symmetric region. They are present in the families I (direct elliptical orbit before the passage and direct hyperbolic orbit after that), J (retrograde elliptical orbit before and direct hyperbolic orbit after the maneuver), N (retrograde elliptical orbit before and retrograde hyperbolic orbit after that). They are more frequent for Hi'iaka than for Namaka and also more numerous when the probe passes closer to the moon, as expected. It is clear that, for Hi'iaka, after a periapsis distance of 5.0 radius of Hi'iaka those types of orbits are rare and, after a periapsis distance of 15.0 radius of Hi'iaka, there is almost no more escapes. Regarding the Namaka, those limits are even smaller, with almost no escapes after a periapsis distance of 5.0 radius of Namaka is reached.

For both moons, there are regions of escapes in almost all the interval of ψ. The ranges in terms of velocity of approach goes from around 40 to 200 m/s for Hi'iaka and from 50 to 250 m/s for Namaka. Of course, when using larger values for the periapsis distance, the frequency of escape trajectories decreases. The general conclusion is that the moons of Haumea are not efficient to participated in the capture of the probe when it is coming from the Earth, but they can help the maneuvers in the Haumea system. It is also important to verify if the orbits of the probes will not end in escapes.

IV. Conclusion

In this paper we analyzed transfer orbits to the Haumea system, by taking into account the minimum total ΔV, the excess velocity near Haumea and the flight time. We found that the *EJH* transfer is the best choice for the launch of a probe if a small flight time is required. For a flight time of 12 years we found 19 asteroids in the main belt which could be reached during the flight, so increasing the scientific gains of the mission. We suggest that nano satellites would be delivered by a main probe in the Haumea system, in order to avoid the decrease of the higher incoming velocity of the probe for the total mass for smaller flight times. This technique makes possible the exploration of the system for longer periods, since the nano satellites can be captured by the system with lower costs, due to their smaller masses. Thinking in this strategy, we map orbits with close approaches in the Haumea system, given emphasis for the search of captures and escapes trajectories. The use of different initial conditions identify several important regions, including the cases where the orbits modify the sense of the motion or ending in captures or escapes. It is shown that, even with the small masses of the moons of Haumea, trajectories ending in escapes, that represent a danger for the mission, due to the early escape from the system. The general characteristics of the trajectories passing by both moons are similar, but Hi'iaka has a larger mass, so it is able to generate larger modifications in the orbits.

Acknowledgments

The authors wish to express their appreciation for the support provided by grants \# 473387/2012-3 and 304700/2009-6, from the National Council for Scientific and Technological Development (CNPq); grants \# 2011/09310-7, 2012/21023-6 and 2011/08171-3, from São Paulo Research Foundation (FAPESP) and the financial support from the National Council for the Improvement of Higher Education (CAPES).

References

[1]Ragozzine, D., and Brown, M. E., "Orbits and masses of the satellites of the dwarf planet Haumea (2003 EL61)," *The Astronomical Journal*, Vol. 137, 2009, pp. 4766-4776.

[2]Volk, K., and Malhotra, R., "The effect of orbital evolution on the Haumea (2003 EL_{61}) collisional family," *Icarus* Vol. 221, 2012, pp. 106-115.

[3]Grundy, W. M. et al., "Exploration strategy for the ice dwarf planets 2013-2022," SBAG Community White Paper.

[4]Russel, C. T. et al., "Dawn completes its mission at 4 Vesta and prepares for 1 Ceres,"*Asteroids, Comets, Meteors 2012*, Proceedings of the conference held May 16-20, 2012 in Niigata, Japan. LPI Contribution No. 1667, id.6085.

[5] Solórzano, C. R. H., Sukhanov, A. A., and Prado, A. F. B. A., "Optimization of Transfers to Neptune," *Nonlinear Dynamics and Systems Theory*, Vol. 8, No. 2, 2008, pp. 205-2011.

[6]Sukhanov, A. A., "Close approach to Sun using gravity assists of the inner planets," *Acta Astronautica*, Vol. 45, 1999, pp. 175-185.

[7]Escobal, P. R., *Methods of Astrodynamics*, 1st ed., John Wiley & Sons Inc., New York, 1968, Chap. 4.

[8]Casalino, L., Colasurdo, G.,and Pastronne, D., "Optimal low-thrust scape trajectories using gravity assist." *Journal of Guidance, Control and Dynamic*s, Vol. 22, No. 5, 1999, pp. 637-642.

[9]D'Amario, L. A., Byrnes, D. V., andStanford, R. H., "Interplanetary trajectory optimization with application to Galileo."*Journal of Guidance, Control, and Dynamics*, Vol. 5, No. 5, 1982, pp. 465-471.

[10]Dunham, D., and Davis, S., "Optimization of a multiple lunar- swing-by trajectory sequence." *Journal of Astronautical Sciences*, Vol. 33, No. 3, 1985, pp. 275-288.

[11] Kohlhase, C. E., and Penzo, P. A., "Voyager mission description."*Space Science Reviews*, Vol. 21, No. 2, 1977, pp. 77-101.

[12]Strange, N. J.andLonguski, J. M.,, "Graphical method for gravity-assist trajectory design." *Journal of Spacecraft and Rockets*, Vol. 39, No. 1, 2002, pp. 9-16.

[13]McConaghy, T. T., Debban, T. J., Petropoulos, A. E., and Longuski, J. M., "Design and optimization of low-thrust gravity trajectories with gravity assist." *Journal of Spacecraft and Rockets*, Vol. 40, No. 3, 2003, pp. 380-387.

[14]Minovich, M. A., "A method for determining interplanetary free-fall reconnaissance trajectories." Pasadena: JPL , Aug. 23, 1961. 47 p. (JPL Tec. Memo 312-130).

[15] Striepe, S. A., and Braun, R. D., "Effects of a Venus Swing-By periapsis burn during an Earth-Mars trajectory."*Journal of the Astronautical Sciences*, Vol. 39, No. 3, 1991, pp. 299-312.

[16]Broucke, R. A.,"The Celestial Mechanics of Gravity Assist."AIAA Paper 88-4220, 1988.

[17]Carvell, R. "Ulysses -the Sun from above and below." *Space*, vol 1, 1985, p. 18-55.

[18]Prado, A. F. B. A.,"Powered swing-by."*Journal of Guidance Control and Dynamics*, vol. 19, No 5, 1996, pp. 1142-1147.

[19]Prado, A. F. B. A.,"Close-approach trajectories in the elliptic restricted problem."*Journal of Guidance, Control and Dynamics*, vol. 20, No 4, 1997, pp. 797-802.
[20]Gomes, V. M.; Prado, A. F. B. A.,"Swing-by maneuvers for a cloud of particles with planets of the Solar system."*WSEAS Transactions on Applied and Theoretical Mechanics*, vol. 3, No 11, 2008, pp. 859-868.
[21]Gomes, V. M.; Prado, A. F. B. A., Justyna, G., "Dynamics of Space Particles and Spacecrafts Passing by the Atmosphere of the Earth."*The Scientific World Journal*, vol. 2013, pp. 1-6, 2013.
[22]Prado, A.F.B.A., "Mapping Swing-By Trajectories in the Triple Asteroid 2001SN_{263}," *SpaceOps2014*, Pasadena, USA, 5-9 May 2014.
[23]Szebehely, V.G., Theory of Orbits, Academic Press, New York,1967
[24]Broucke, R. A., "Stability of Periodic Orbits in the Elliptic, Restricted Three-Body Problem," *AIAA Journal*.Vol. 7, No. 6, pp. 1003-1009, 1969.